# Trading for Bailouts*

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**Abstract**

Government interventions such as bailouts are often implemented in times of high uncertainty. Policymakers may therefore rely on information from financial markets to guide their decisions. We study a model in which a policymaker learns from market activity and traders have high private stakes in the intervention. We discuss how the presence of such traders affects intervention outcomes, and show that it reduces market informativeness and the efficiency of bailouts. Regarding normative implications, we show that a higher social cost of interventions and a gradual implementation of assistance can improve market informativeness and raise overall welfare.

**Keywords:** feedback effect, bailouts, market informativeness, trading, real efficiency.

**JEL classifications:** D83, G12, G14, G18, G28.

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# 1 Introduction

A fundamental question in financial economics concerns the informativeness of market prices. When financial market participants trade on private information, trading activity conveys information about underlying economic conditions. This fact motivates the usage of market prices to guide different types of real decisions. As a result, prices not only reflect the fundamental value of firms, but also affect it—a feedback effect (Bond, Edmans and Goldstein, 2012).

A particular context in which market activity is likely to provide useful information to decision makers is during financial crises, when regulators often need to decide quickly whether to provide assistance to firms in distress. This is illustrated by Warren Buffett's "Letter to Uncle Sam" in the New York Times after the Great Recession: "*When the crisis struck, I felt you would understand the role you had to play. But you've never been known for speed, and in a meltdown minutes matter. (...) You would have to improvise solutions on the run, stretch legal boundaries and avoid slowdowns, like Congressional hearings and studies*" (NYT, 2010). Given the scarcity of alternative information channels, financial markets can be a valuable and timely source of information for policymakers in times of distress. Especially, trades executed by large stockholders, who are likely to be well informed, could help assess the situation of firms before any intervention decisions.

In the days preceding bailout announcements there is usually substantial trading activity by firms' insiders, as documented by Jagolinzer et al. (2020) in the context of the Troubled Asset Relief Program (TARP). On an anecdotal level, large stockholders have often increased their participation before bailout episodes during the Great Recession, and those stock purchases have been accompanied by public statements reinforcing their positive assessment of firms' financial health. Saudi prince Alwaleed bin Talal increased his position in Citigroup from about 4% to 5% in November 2008 and publicly stated that he strongly believed that the firm's stock was dramatically undervalued (WSJ, 2008). Four days later, Citigroup received a massive government bailout. In the fall of 2008, Buffett's Berkshire Hathaway increased its stake in several financial institutions that were later the target of massive assistance under the TARP, such as Goldman Sachs, General Electric, Wells Fargo, Bank of America, American Express, among others. TARP-assisted companies constituted about 30% of Buffett's publicly disclosed stock portfolio at the time

(CNN, 2012). On that occasion, Buffett publicly announced his trust in the financial strength of some of those companies, and also elicited his belief that the government would "do the right thing" and provide assistance to firms in need in the near future (Reuters, 2009).[1]

Given that increases in stock ownership are costly signals sent by insiders—as they are betting on firms' success—policymakers could in principle interpret insider purchases as positive signals about firms' health. However, one concern is that large stockholders (and insiders in general) also have high stakes in government interventions. Their trades could reflect not only the information they possess about the firms' fundamentals, but also their expectations of a government bailout and even strategic motives to influence policymakers' decisions. It is therefore not obvious whether large trades by insiders in financial markets are informative signals to help guide policy.

Accordingly, this paper aims to shed light on the following questions: To which extent and in which circumstances can policymakers obtain useful information from the trading activity of large stockholders and insiders? How does the presence of those traders affect government interventions and their efficiency? How does the design of bailouts affect market informativeness and welfare?

To examine these issues, in Section 2 we present a parsimonious model in which an informed trader has high stakes in a government intervention. By intervening, a policymaker improves the cash flow of a firm. This intervention is socially desirable only when an economic fundamental (the state) is good, capturing that the policymaker would like to provide assistance to firms that are viable in the long run if provided with temporary support.[2] For instance, if a firm faces short-term liquidity shortage, the policymaker would like to assist it if it is illiquid but solvent. The policymaker observes the activity in a market in which the stocks of the firm are traded. As in Kyle (1985), there is a noise trader and a competitive market maker who meets the orders at the fair price. The key player in our setting is the informed trader, who derives a private benefit from the intervention. This benefit arises naturally when the trader is a creditor or blockholder of the firm, for instance. In the latter case, the private benefit scales with the initial block size.

[1] For instance, when investing $5bn in Goldman Sachs in October 2008, Buffett said in an interview that *"Goldman Sachs is an exceptional institution (...) It has an unrivaled global franchise, a proven and deep management team and the intellectual and financial capital to continue its track record of outperformance."* (CNN, 2008).

[2] Regarding the first banks to take part in TARP, Treasury Secretary Henry Paulson stated: *"These are healthy institutions, and they have taken this step for the good of the U.S. economy. As these healthy institutions increase their capital base, they will be able to increase their funding to U.S. consumers and businesses."* (Treasury, 2008).

We start with the positive implications of the model. In Section 3, we first characterize how trading behavior is affected by the private benefit of intervention. Without a private benefit, the large trader trades on her private information to maximize expected trading profits, which reveals the state to the policymaker as much as possible given the presence of the noise trader. With a large enough private benefit, however, the large trader has incentives to trade to avoid revealing the bad state in an attempt to increase the probability of an intervention. Thus, upon receiving bad news, the informed trader does not trade or even buys additional stocks of the firm if the benefit of intervention is high enough. We call this behavior *trading for bailouts*.

We show that a small amount of trading for bailout incentives can already have important consequences. If agents are ex ante pessimistic about the firm's situation, even a small private benefit or block size triggers the trading for bailouts behavior. The reason is that potential trading profits upon receiving bad news are limited in those cases, which reduces the opportunity cost of distorting trading decisions to influence policy.

Informed traders, however, do not always succeed in affecting the policy outcome. When the common prior about the firm's fundamental is good and hence the policymaker is ex ante prone to intervening, bailouts are more likely when the trader has a private benefit of intervention. When the policymaker is ex ante reluctant to intervene (since the good state is unlikely), bailouts may actually be even less likely when the trader has a private benefit. This benefit can end up shutting down an effective channel for the policymaker to learn from the market. Whether the policymaker's reduced reliance on market activity increases the probability of intervention depends on how the policymaker would act without any additional information.

We propose a simple measure of market informativeness and show that, in equilibrium, higher informativeness increases the efficiency of real decisions (i.e., whether to bail out the firm). A general insight is that the private benefit reduces market informativeness around a government intervention and hinders the efficiency of bailouts. The loss in efficiency arising from lower informativeness can be decomposed into losses from (i) intervening less often in the good state (higher type-I error); and (ii) intervening more often in the bad state (higher type-II error). We characterize which types of mistakes the policymaker makes under different market conditions.

Our main model yields novel testable implications regarding the effects of blockholding on market informativeness and bailout outcomes that may inform future empirical work. We consider changes in the trader's block size, or in any variable that drives the private benefit of the intervention. One key implication is that a larger block size reduces market informativeness *around government interventions* and reduces real efficiency because of stronger incentives to trade for bailouts.[3] Moreover, when the policymaker is ex ante prone to intervene, the ex-ante probability of intervention increases in the block size. When the policymaker is ex ante reluctant to intervene, by contrast, the effect of block size on the ex-ante probability of intervention is non-monotonic. On the one hand, the larger private benefit incentivizes more strategic trading to manipulate the belief of the policymaker and tends to increase the chance of an intervention. On the other hand, the policymaker becomes more skeptical about market information. Reducing its reliance on market activity, the policymaker places more emphasis on the prior that suggests no intervention.

We turn to normative implications in Section 4. We study how institutional features affecting the cost of policy implementation and how bailout design can improve market informativeness around bailouts, and consequently, increase welfare. We first show that an increase in intervention costs can improve market informativeness and welfare. For a large intervention cost, traders anticipate that the policymaker will be reluctant to provide assistance, and this may end up facilitating learning from the market. To effectively convince the policymaker to intervene when observing high enough aggregate orders, the trader must sell the stock with high enough probability when observing bad news, which improves learning. The gain in informativeness can more than compensate the higher implementation costs. This is likely to happen when the bad state is more likely and when the private benefit of informed traders is large.

Next, we study the effects of implementing assistance programs gradually. To do so, we modify the model by allowing the policymaker to offer a partial assistance package before observing market activity. After observing trading in financial markets, it can then decide whether to provide additional assistance. Such policy could be implemented by extending an unconditional credit line to financial institutions (e.g., the Federal Reserve discount window or liquidity assistance programs

[3] Our focus is not on informativeness in general but on informativeness around government interventions. There are reasons why blockholders might increase price informativeness in normal times. Please also see the discussion in Section 3.2.

offered by other central banks) or by simply implementing bailouts in consecutive rounds.

We show that offering an initial partial support can improve informativeness and welfare. The intuition is that partial assistance given ex ante reduces the residual benefit of additional support ex post, discouraging strategic trading and boosting informativeness. Despite part of the assistance being implemented with little information, this early decision allows the policymaker to learn more from the market and to implement any additional support more efficiently. Interestingly, providing some assistance early on can actually reduce the total expected size of the bailout, since it promotes learning in a later stage and thus avoids future wasteful interventions.

Section 5 discusses alternative settings. First, we describe a simple model in which a policymaker decides how much liquidity support to provide to a distressed financial institution, illustrating how our main setting maps into a specific application. We consider a situation in which a bank faces a liquidity shortage and a policymaker may provide liquidity if it considers that the social gains of avoiding inefficient liquidation of assets more than compensate for the costs of intervening. Second, we present an extension where orders are subject to mandatory disclosure and the policymaker can perfectly distinguish between orders placed due to liquidity reasons (noise) and those placed for strategic reasons. Our main results extend to this alternative setting.

**Literature.** Market prices may contain useful information for real decision makers—an idea that goes back to Hayek (1945). Evidence that decision makers look at market activity as a source of information has been documented in different contexts (e.g., Luo, 2005; Chen, Goldstein and Jiang, 2006; Bakke and Whited, 2010; Edmans, Goldstein and Jiang, 2012). A growing body of literature has incorporated the idea that agents may look at market prices to guide a decision that ultimately affects the value of securities (for instance, Dow and Gorton, 1997; Bond, Goldstein and Prescott, 2010; Lin, Liu and Sun, 2019; Banerjee, Davis and Gondhi, 2022).

The papers most related to ours are those with feedback effects and large strategic traders, such as Attari, Banerjee and Noe (2006), Goldstein and Guembel (2008), Khanna and Mathews (2012), Edmans, Goldstein and Jiang (2015), and Boleslavsky, Kelly and Taylor (2017). In Goldstein and Guembel (2008), an uninformed trader has incentives to short sell a firm's security to affect a managerial decision. The manager's misguided decision leads to a decrease in the real value of the firm and ends up generating trading profits for the uninformed short seller. In contrast, our paper concerns the strategic behavior of an informed trader with high stakes in an intervention and different forces are at play (apart from the focus on policy interventions instead of managerial decisions). To manipulate the decision maker's beliefs, the trader has incentives to sell the stock when she has no information in Goldstein and Guembel (2008), while the trader has incentives not to sell even upon observing bad news in our model.

In Edmans, Goldstein and Jiang (2015), a firm manager uses market activity to guide an investment decision. An informed trader trades the firm's security, and an asymmetric effect emerges: by trading on her information, the trader induces the manager to take the correct action, which always increases firm value; this increases incentives for her to buy on good news, but *decreases* incentives to sell on bad news. The main result is that there is an endogenous limit to arbitrage, and bad news is less incorporated into prices, leading to overinvestment. In the same spirit, Boleslavsky, Kelly and Taylor (2017) propose a model where an authority (e.g., a firm manager or policymaker) observes trading activity prior to deciding on an action that changes the state, thus affecting the security value. By assumption, the intervention removes the link between the initial state and firm value, and informed traders are harmed by the intervention since they lose their informational advantage. As in Edmans, Goldstein and Jiang (2015), price informativeness is also reduced, since informed investors have less incentive to sell the asset following bad news. Differently from those papers, in our setting the large trader derives a private benefit from the intervention (for instance, due to blockholding). Moreover, the intervention does not eliminate the informational advantage of the trader, as is the case in Boleslavsky, Kelly and Taylor (2017). Still, for different reasons, our model also generates asymmetric incentives to trade across states.

As discussed, one interpretation of the private of benefit is that the large trader holds some of the firm's stock. Attari, Banerjee and Noe (2006) and Khanna and Mathews (2012) study the role of blockholders in affecting learning and trading in financial markets, and hence our work is also related in that dimension. However, none of those papers study the interplay between financial markets and policy interventions. Our focus on trading around policy decisions explains why our model delivers different predictions regarding the effects of blockholding, as we detail next.

Khanna and Mathews (2012) argue that an informed blockholder can prevent value destruction from short-selling attacks by uninformed traders, as the ones in Goldstein and Guembel (2008). In their setting, the incentives of the decision maker (a manager) are aligned with the blockholder's, conditional on the state. In our model, by contrast, the incentives of the decision maker (a policymaker) and the blockholder are fully misaligned in bad states, in which the intervention is socially undesirable but profitable for the blockholder. Khanna and Mathews (2012) show that, by counteracting the trading manipulation by uninformed parties, blockholders increase real efficiency (and this effect takes place only when their block size is large enough). Hence, while their setting highlights a potentially beneficial role of blockholders from an efficiency perspective in some circumstances, our model points to potentially harmful effects of blockholding on efficiency when firms are the potential targets of government interventions.[4]

Attari, Banerjee and Noe (2006) study the interplay between *voice* and *exit* by blockholders, which are usually thought of as substitute instruments to affect firm management. The authors show that those can actually be complementary: Sell-offs by institutional investors can be informative to relationship investors and trigger activism. Differently from our paper, in their setting the informed institutional investor always trades according to her information: only the uninformed trader has incentives to trade manipulatively. Their setting is then more similar in spirit to Goldstein and Guembel (2008), in that all the action comes from the trader exploring her private information about being uninformed. Finally, the mechanisms in Attari, Banerjee and Noe (2006) rationalize pooling by institutional investors on the sell side of the market, while our results generate the prediction of excessive buying pressure by large shareholders around policy decisions.

Bond and Goldstein (2015) study policy interventions in a model of feedback. As opposed to our setting, there is a continuum of small traders that cannot move prices and, hence, cannot individually affect the policy outcome. In Bond, Goldstein and Prescott (2010), a decision maker also learns from a market price, but traders' decisions to trade are not modeled. Our paper also adds to the literature on the role of blockholders in general (e.g., Admati and Pfleiderer, 2009; Edmans, 2009; Edmans and Manso, 2010), emphasizing how their presence affects price

[4]In a contribution related to Khanna and Mathews (2012), Khanna and Sonti (2004) also find that strategic trading by blockholders can increase firm value when high stock prices facilitate investment.

informativeness around government interventions.

Finally, our paper speaks to the discussion in policy circles about how policymakers should use information from financial markets. Bernanke (2004) discusses how the Fed uses market information in different contexts. Gropp, Vesala and Vulpes (2006) provide evidence that financial markets can help predict bank distress, even controlling for accounting information, and give support to the use of market information as a screening device for regulators. Gunther, Levonian and Moore (2001), Curry, Elmer and Fissel (2003) and Kerry (2019) also empirically support the idea that stock markets improve regulators' information set. We contribute to this discussion by emphasizing that the strategic behavior of large shareholders can reduce the potential for the use of market information to guide policy, and propose measures that can mitigate this problem.

# 2 Model

There are two dates $t = 0, 1$, no discounting, and universal risk neutrality. The cash flow $v$ per unit of outstanding stock of a firm at $t = 1$ depends on a fundamental $\theta \in \{L, H\}$, which we refer to as the bad and good state, respectively, and an intervention by a policymaker $G \in \left[0, \overline{G}\right]$, where $\overline{G}$ represents the maximum scale of the intervention (in dollars). Each dollar of assistance increments the firm's cash flow by $\alpha_\theta$:

$$v(\theta, G) = R_\theta + \alpha_\theta G, \tag{1}$$

where $R_\theta$ is the part of the cash flow independent of the intervention. Without loss of generality, we normalize $\overline{G} \equiv 1$. Letting $\Delta_R \equiv R_H - R_L > 0$ and $\Delta_\alpha \equiv \alpha_H - \alpha_L$, we assume that the cash flow in the good state is above the cash flow in the bad state even if the policymaker intervenes as much as possible, $\Delta_R + \Delta_\alpha > 0$.

The fundamental $\theta$ is drawn at $t = 0$ but unobserved by the policymaker. The good state occurs with probability $\gamma \in (0, 1)$. We assume that the social cost of intervention is $c > 0$ per dollar of assistance, and the social benefit is $b_\theta > 0$ per dollar, with

$$b_H > c > b_L.$$

This implies that intervening is socially desirable only in the good state.[5] One interpretation is that only firms with good fundamentals are worth saving: bearing the intervention costs is only desirable when firms are viable in the long run if provided with temporary support. For instance, if a firm faces short-term liquidity shortage, the policymaker would like to assist it if it is likely to be solvent, although illiquid. Thus, the policymaker solves:

$$\max_{G\in[0,1]} G\left[\Pr\left(\theta = H|\mathcal{I}\right) b_H + \Pr\left(\theta = L|\mathcal{I}\right) b_L - c\right],$$

where $\mathcal{I}$ is the policymaker's information set. Whenever $\Pr\left(\theta = H|\mathcal{I}\right) > \overline{\gamma} \equiv \frac{c-b_L}{b_H-b_L} \in (0,1)$, the policymaker sets $G = 1$, and it sets $G = 0$ if that inequality is reversed. Hence, $\overline{\gamma}$ is the lowest probability assigned to the good state for which the policymaker is willing to intervene. For expositional simplicity, we normalize $b_L \equiv 0$ and $b_H \equiv b$ (so $\overline{\gamma} = \frac{c}{b}$) in the main model. (We endogenize those payoffs in an application in Section 5.1.)

Unless stated otherwise, we refer to an intervention as a binary variable $G \in \{0,1\}$. Having defined a continuous assistance variable will be important for some of our normative results in Section 4, but is immaterial to the equilibrium characterization and positive results because we allow for mixed strategies. A strategy in which the policymaker intervenes with probability $g \in (0,1)$ is equivalent to a strategy in which it gives out an assistance of size $g$ with probability one.[6]

Before deciding whether to intervene at $t = 1$, the policymaker learns from activity in a

[5] Consistently with the assumption of costly interventions, Flanagan and Purnanandam (2022) shows that the TARP bailouts were costly: recipients paid considerably lower returns to the taxpayers on a risk-adjusted basis. In a previous version of this paper, we also considered the case in which paying such costs was desirable only in the *bad* state and the broader message was unchanged, although equilibrium trades prior to a bailout go in an opposite direction. However, such assumption is inconsistent with our discussion in the introduction, that is, it does not square with the excessive buying behavior of large traders and with policymakers' reported concerns about the long-term viability of the firms being bailed out.

[6] Note that when $\Pr\left(\theta = H|\mathcal{I}\right) = \overline{\gamma}$ the policymaker is indifferent between any assistance level. However, since we assume universal risk neutrality and the firm's cash flow is linear in $G$, all players in the game only care about the expected size of the intervention. Hence, we can restrict attention to $G \in \{0,1\}$ when computing the equilibrium without loss of generality because mixed strategies are allowed for.

financial market (see Table 1 for a timeline). Shares of the firm are traded by a noise trader and an informed trader at $t = 0$. As in Edmans, Goldstein and Jiang (2015), traders can place three types of orders, where $-1$ represents a sell order, 0 represents no trade, and 1 represents a buy order. Although traders cannot buy or sell interior amounts, we allow for equilibria in mixed strategies, so traders may buy or sell with interior probability. This can be thought of as reflecting an intensive margin of trading. The noise trader is active for exogenous reasons (e.g., liquidity shocks) and places each order $z \in \{-1, 0, 1\}$ with equal probability regardless of the state. The informed trader observes $\theta$ and places an order $s \in \{-1, 0, 1\}$ to maximize her expected payoff. The key assumption here is that the informed trader has some relevant information unknown to the policymaker. As in Kyle (1985), there is a competitive market maker who observes the total order flow, $X = s + z$, sets the price $p$ to the expected value of the firm at $t = 1$, and executes the order at this price. The market maker uses the information contained in the order flow and rationally anticipates the policymaker's decision when setting the price.

Government interventions—such as bailouts of financial institutions—usually have large spillovers to some agents, including large stockholders and firm creditors, who can also participate in financial markets. To capture this, we assume that the informed trader derives a (potentially state-contingent) private benefit of the intervention, $\beta_\theta$.[7] That is, her payoff at $t = 1$ is

$$\pi = s\,(v - p) + \beta_\theta G. \tag{2}$$

An example where such private benefits arise naturally is in the context of outside blockholders, which are pervasive among U.S. firms (Holderness, 2009). When the trader has $\mu$ stocks of the firm at $t = 0$, the payoff from trading quantity $s$ is $(s + \mu)\,v - sp = s\,(v - p) + \alpha_\theta \mu G + \mu R_\theta$. Since $\mu R_\theta$ is exogenous, in this specific example of blockholding the trader's payoff can be represented as in Equation (2) by setting $\beta_\theta = \alpha_\theta \mu$.

[7]The assumption that the large trader with a private benefit of the intervention has useful information about the firm reflects the notion that such agents also have high incentives to acquire information about the firm.

| $t = 0$: Information and Trade | $t = 1$: Learning and Intervention |
|---|---|
| • State $\theta$ is realized and observed by the informed trader<br>• Traders place orders $(s, z)$<br>• Market maker sets price $p$ at which trade occurs | • Policymaker learns from financial market<br>• Policymaker decides on intervention $G$<br>• Payoffs are realized |

Table 1: Timeline of events.

# 3 Equilibrium and positive implications

We start by introducing some useful notation. A trading strategy for the informed trader is a probability distribution over orders $s \in \mathcal{S} = \{-1, 0, 1\}$ for each fundamental $\theta \in \Theta = \{L, H\}$ and is denoted by $l(s)$ and $h(s)$. An intervention strategy for the policymaker is a probability of intervening $g(X)$ for each total order flow $X \in \mathcal{X} = \{-2, -1, 0, 1, 2\}$. A price-setting strategy for the market maker is a function $p : \mathcal{X} \to \mathbb{R}$. Moreover, $q(X)$ is the probability the policymaker and the market maker assign to the good state $H$ upon observing the order flow $X$.

We study perfect Bayesian equilibrium. In our setting, such an equilibrium consists of (i) a trading strategy for the informed trader that maximizes her payoff given all other strategies and her information about the realized $\theta$; (ii) an intervention strategy that maximizes the policymaker's payoff given all other strategies and the order flow; (iii) a price-setting strategy that allows the market maker to break even in expectation given all other strategies and the order flow; and (iv) beliefs $q(X)$ consistent with Bayesian updating on the equilibrium path. Moreover, we impose that beliefs off the equilibrium path satisfy the Intuitive Criterion (Cho and Kreps, 1987).

**Lemma 1.** *In the good state, the informed trader always buys,* $h(1) = 1$.

In the good state, the informed trader only has incentives to buy: she expects to make positive trading profits and to influence the policymaker to intervene by conveying positive information about the state. Since the informed trader always buys in the good state, we classify possible equilibria based on the informed trader's action in the bad state:

(i) Sell equilibrium $(S)$: the informed trader always sells in the bad state, $l(-1) = 1$.

(ii) Inaction equilibrium ($I$): the informed trader does not trade in the bad state, $l(0) = 1$.

(iii) Buy equilibrium ($B$): the informed trader always buys in the bad state, $l(1) = 1$.

(iv) Equilibria in mixed strategies are denoted by combinations of $S$, $I$, and $B$. For example, $SB$ denotes an equilibrium in which $l(-1) > 0$, $l(1) > 0$, and $l(0) = 0$.

As a benchmark, we characterize the equilibrium set without a private benefit of intervention.

**Proposition 1.** ***Benchmark.*** *When $\beta_H = \beta_L = 0$, there is a unique equilibrium in which the informed trader always sells in the bad state and always buys in good state (B equilibrium).*

When the informed trader derives no private benefit of the intervention, the trader's orders purely reflect her private information. The trader simply trades as to fully explore her informational advantage about the firm's cash flow: she sells if the fundamental is bad and buys if the fundamental is good. The aggregate order does not reveal the state for some orders of the noise trader, so the informed trader makes positive trading profits in expectation. The trading behavior of the informed trader is as different across states as possible, so the market maker and the policymaker learn as much as possible from market activity given the existence of noise traders. Total orders $X \in \{-2, -1\}$ reveal the state $\theta = L$ and the policymaker does not intervene, while orders $X \in \{1, 2\}$ reveal $\theta = H$ and the policymaker intervenes. For $X = 0$, no information is revealed and the policymaker bases its decision on the prior $\gamma$. Figure 1 illustrates and summarizes.

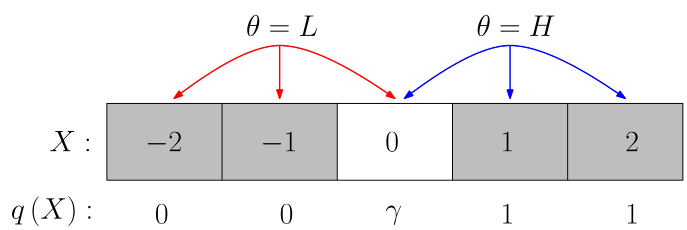


Figure 1: Benchmark without a private benefit of intervention ($\beta_H = \beta_L = 0$). The aggregate order flow $X$ given the equilibrium trading strategy of the informed trader, $l(-1) = 1$ and $h(1) = 1$, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. Order flows with updating (learning) are shaded in grey, while other order flows are not shaded.

We now turn to the general case in which the intervention generates some private benefit for the informed trader. To ease exposition, we focus on the generic case of $\gamma \neq \overline{\gamma}$.[8] Whenever

[8] For $\gamma = \overline{\gamma}$, the Intuitive Criterion fails to rule out some equilibria that depend on unusual off-equilibrium beliefs.

there are multiple equilibria, we restrict attention to the best equilibrium from the perspective of the policymaker in the main text.[9] Figure 2 shows the equilibrium set for $\Delta_\alpha = 0$, where the left panel shows the whole equilibrium set and the right panel shows the best equilibrium.[10] For future reference, we state some bounds on parameters:

$$\begin{gathered}\underline{\beta} = \gamma \left(\Delta_R + \Delta_\alpha\right), \qquad \overline{\beta} = \left(1 + 2\gamma\right)\left(\Delta_R + \Delta_\alpha\right), \qquad \underset{\sim}{\beta} = \gamma \Delta_R, \\ \tilde{\beta} = \frac{\left(1 + \gamma + \overline{\gamma}\right)\Delta_R + \Delta_\alpha + \sqrt{\left[\left(1 + \gamma + \overline{\gamma}\right)\Delta_R + \Delta_\alpha\right]^2 + 4\overline{\gamma}\gamma\Delta_R\Delta_\alpha}}{2}.\end{gathered} \tag{3}$$

**Proposition 2.** ***Equilibrium.*** *For an optimistic prior ($\gamma > \overline{\gamma}$), the negatively informed trader sells ($S$) if $\beta_L \leq \underline{\beta}$, does not trade ($I$) if $\underline{\beta} < \beta_L \leq \overline{\beta}$, and buys ($B$) if $\beta_L > \overline{\beta}$. For a pessimistic prior ($\gamma < \overline{\gamma}$), the negatively informed trader sells ($S$) if $\beta_L \leq \underset{\sim}{\beta}$, randomizes between selling and not trading ($SI$) if $\underset{\sim}{\beta} < \beta_L \leq \tilde{\beta}$, and randomizes between buying and selling ($SB$) if $\beta_L > \tilde{\beta}$.*

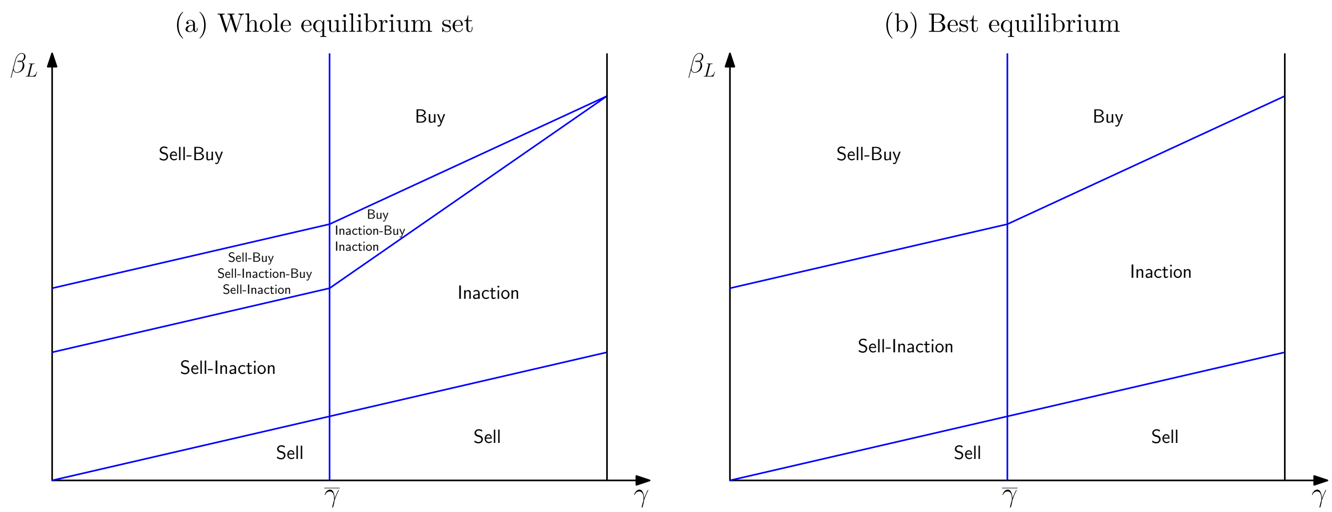


Figure 2: Equilibrium set for different values of the private benefit in the low state, $\beta_L$, and prior probability of the high state, $\gamma$, for $\Delta_\alpha = 0$. When multiple equilibria exist, the best equilibrium is the one preferred by the policymaker.

Proposition 2 shows that the benchmark result of Proposition 1 continues to hold as long the private benefit $\beta_L$ is small enough, that is, below $\underline{\beta}$ for a high prior or below $\underset{\sim}{\beta}$ for a low prior. As $\beta_L$ increases, however, the negatively informed trader gains incentives to deviate from trading on

[9] As discussed in Section 3.1, the policymaker's payoff is the relevant measure of real efficiency in this setting. Focusing on the worst equilibrium would lead to the same qualitative results and Proposition 2 would continue to hold, just with different expressions for $\overline{\beta}$ and $\tilde{\beta}$. See also Appendix A for the entire characterization of equilibrium.

[10] For $\Delta_\alpha \neq 0$, the illustration is qualitatively very similar, just with jumps at $\gamma = \overline{\gamma}$.

her information. In particular, if $\beta_L$ is high enough, the trader always buys the stock with positive probability even upon learning bad news about the firm's fundamentals. Note that the private benefit in the high state is irrelevant for the equilibrium, since the trader always buys upon good news for any $\beta_H$, as discussed in Lemma 1.

To gain some intuition, consider first the case of a high prior, $\gamma > \overline{\gamma}$. For a high private benefit, $\beta_L > \overline{\beta}$, the negatively informed trader buys with probability one, $l(1) = 1$. Since the policymaker is sufficiently optimistic about the fundamental, an intervention takes place if activity in financial markets is absolutely uninformative. Hence, an equilibrium in which the negatively informed trader perfectly mimics the behavior of the positively informed trader can be sustained. If the private benefit of the intervention is sufficiently large, it is profitable for the negatively informed trader to incur a trading loss against the market maker in order not to reveal information that could dissuade the policymaker from intervening. For an intermediate private benefit, $\underline{\beta} < \beta_L \leq \overline{\beta}$, the trader does not incur the losses of buying in the bad state, but she gives up any trading profits from private information in order not to reveal too much information about the state that, in turn, could prevent the policymaker from intervening. Taken together, the informed trader opts for inaction, which is shown in Figure 3.

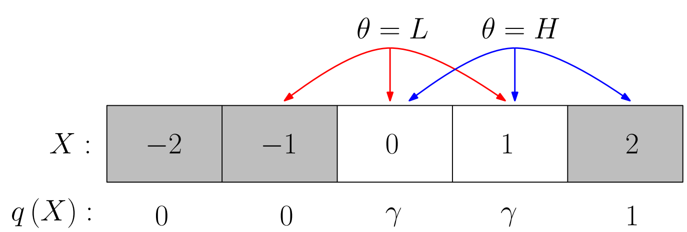


Figure 3: Inaction equilibrium: the aggregate order flow $X$ given the equilibrium trading strategy of the informed trader, $h(1) = 1$ and $l(0) = 1$, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. The order flow $X = -2$ is off the equilibrium path, but $q(-2)$ is uniquely pinned down by the Intuitive Criterion.

We turn to the low prior, $\gamma < \overline{\gamma}$. Since the policymaker is unwilling to intervene under this prior, the information from market activity must be compelling enough to revert the policymaker's prior for an intervention to occur. Thus, there is no equilibrium in pure strategies for high enough $\beta_L$. The incentives of the negatively informed trader to deviate from trading on her information are high, but if she is expected to always do so, this behavior is ineffective in affecting beliefs. The equilibrium emerges from this balance. In the Sell-Inaction equilibrium ($SI$), for instance, both the negatively informed trader and the policymaker play mixed strategies. The latter intervenes with some probability when observing an order flow of $X = 1$ such that the trader is indifferent between selling and not trading. Given the trader's randomization, the policymaker is indifferent between intervening and not intervening upon observing $X = 1$. Figure 4 shows this case.

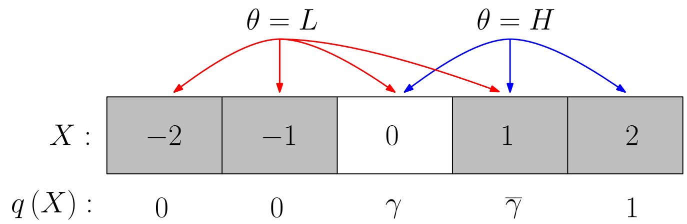


Figure 4: Sell-Inaction equilibrium: the aggregate order flow $X$ given the equilibrium trading strategy of the informed trader, and the belief of the market maker and policymaker about the good state inferred from the aggregate order flow, $q(X)$. At $X = 1$, beliefs are such that the policymaker is indifferent between intervening and not intervening.

For even higher values of $\beta_L$, the equilibrium similarly features mixed strategies. The policymaker randomizes between intervening or not upon observing aggregate orders $X = 1, 2$, and the negatively informed trader randomizes between buying and selling the stock.

It is worth highlighting that when agents are ex ante very pessimistic about the firm (low $\gamma$), even a small private benefit can trigger the trading for bailouts behavior. This comes from the fact that the Sell equilibrium region vanishes as $\gamma$ approaches zero, as shown in Figure 2. The intuition for this result is that when $\gamma$ is low, the informational advantage of the informed trader when receiving bad news is small. With low potential for trading profits, even a trader with small private benefit has incentives to refrain from selling the stock, as to influence the policymaker's beliefs. In fact, in the special case analyzed below, even with $\gamma$ bounded away from zero an arbitrarily small private benefit can trigger the trading for bailouts behavior.

**Special case: a bailout is critical to avoid bankruptcy.** Sometimes a bailout is crucial to avoid firm bankruptcy, and hence in the absence of assistance, there is almost nothing left for shareholders. Such scenario can be easily captured in our model by assuming that $R_L$ and $R_H$ (the firm's cash flows that accrue to shareholders without intervention) are close to zero. In that case, $\alpha_H$ and $\alpha_L$ can be interpreted as the firm's cash flows that accrue to shareholders conditional on each state and on the firm not going bankrupt (which can only happen with an intervention), and thus it is assumed that $\alpha_H > \alpha_L$.

Under such parametrization, $\Delta_R \approx 0$, which implies that the bound $\underaccent{\tilde}{\beta}$ used in Proposition 2 is close to zero. Hence, when the policymaker is ex ante not willing to intervene ($\gamma < \overline{\gamma}$), the Sell equilibrium cannot be sustained, even if the private benefit is arbitrarily small. The intuition for this is that without a bailout there is no room to make trading profits, since the cash flow that accrues to stock owners will be close to zero regardless of the state. In other words, the firm's stock returns are not very information-sensitive when it is about to go bankrupt. Negatively informed traders then have little incentives to trade on their information (selling), since that would make the policymaker not intervene and remove the potential for trading profits anyway.

When, instead, the policymaker is ex ante willing to intervene ($\gamma > \overline{\gamma}$), the Sell equilibrium can be sustained even when $\Delta_R \to 0$: When the negatively informed trader sells, the intervention still happens if her order is offset by a buy order of noise traders, and hence she can make trading profits in expectation by trading on her information. Thus, if the private benefit of intervention is small enough, it is still optimal to sell after bad news, as it does not completely eliminate the possibility of a bailout, and trading profits are positive in states where a bailout does take place.

## 3.1 Market informativeness and the efficiency of interventions

The ex-ante expected government payoff is

$$U_G = \gamma \Pr\left(G = 1|\theta = H\right)(b - c) + (1 - \gamma)\Pr\left(G = 1|\theta = L\right)(-c)\,, \tag{4}$$

where $\Pr(G = 1|\theta)$ denotes the probability of intervention conditional on the state. Since the intervention is the only real decision in our setting and trade in financial markets are pure transfers, we refer to $U_G$ as a measure of real efficiency.

In models where real decision makers learn from the market, price efficiency (the extent to which the price of a security accurately predicts its future value) does not necessarily translate into real efficiency (the extent to which market information improves real decisions), as emphasized by

Bond, Edmans and Goldstein (2012). To analyze the informativeness of market activity in our model, we use the following measure.

**Definition 1.** *The informativeness of market activity is the expected learning rate about the state:*[11]

$$\iota \equiv \gamma \left( \frac{\mathbb{E}\left[q(X)|\theta = H\right] - \gamma}{\gamma} \right) + (1-\gamma) \left( \frac{1 - \mathbb{E}\left[q(X)|\theta = L\right] - (1-\gamma)}{1-\gamma} \right) \tag{5}$$

$$= \frac{\mathbb{E}\left[q\left(X\right)|\theta = H\right] - \gamma}{1-\gamma}. \tag{6}$$

Before discussing our informativeness measure, it is useful to introduce some notation. Since informativeness in this paper is a feature of the equilibrium—it is the result of traders' strategies—we often use the notation $\iota^E$ to refer to the level of informativeness achieved under parameters that lead to a certain equilibrium $E$ (and we apply the same notation for the government payoff $U_G$, writing $U_G^E$). Also, we are interested in understanding how changes in traders' strategies affect the government payoff. Hence, when comparing equilibrium outcomes for different parameters (e.g., different private benefits), we often hold fixed the parameters $\Psi_G \equiv (b, c, \gamma)$, which mechanically affect the government payoff even holding constant traders' strategies (see Equation (4)).

**Lemma 2.** ***Market informativeness*** *$\iota$ has the following desirable properties:*

- *$\iota$ increases in the correctness of beliefs, $\mathbb{E}\left[q(X)|\theta = H\right]$ and $1 - \mathbb{E}\left[q(X)|\theta = L\right]$;*
- *$\iota = 1$ if the state is perfectly learned (i.e., if $\mathbb{E}\left[q(X)|\theta = H\right] = 1$ and $\mathbb{E}\left[q(X)|\theta = L\right] = 0$);*
- *$\iota = 0$ if nothing is learned (i.e., if $\mathbb{E}\left[q(X)|\theta = H\right] = \gamma$ and $1 - \mathbb{E}\left[q(X)|\theta = L\right] = 1 - \gamma$);*
- *For any given $\Psi_G$, $\iota^{E'} > \iota^E$ if and only if equilibrium $E'$ is strictly more informative than $E$ in the sense of Blackwell (hence, any decision maker observing market outcomes in $E'$ would be better off than in $E$).*

As a consequence of the last property in Lemma 2, there is a clear mapping between market informativeness and the efficiency of real decisions in our model.

[11] The fact that the learning rate in (5) can be written as in (6) follows from Bayes rule and the law of iterated expectations, as shown in Appendix B.1.

**Proposition 3.** ***Real efficiency.*** *Given any $\Psi_G$, the ranking of real efficiency equals the ranking of market informativeness:*

$$U_G^{E'} > U_G^E \iff \iota^{E'} > \iota^E. \tag{7}$$

*Market informativeness (and thus real efficiency) are ranked across equilibrium classes according to $\iota^S > \iota^I > \iota^B$ for $\gamma > \overline{\gamma}$, and $\iota^S > \iota^{SI} > \iota^{SB}$ for $\gamma < \overline{\gamma}$.*

Given the government payoff structure and its prior, any change in parameters that induces an equilibrium with higher informativeness increases the efficiency of real decisions in our setting. For instance, an increase in $\beta_L$ associated with moving from the Sell equilibrium to the Inaction equilibrium reduces both market informativeness and the government's expected payoff.

Higher market informativeness may increase real efficiency due to a reduction in the probability of two types of mistakes that the policymaker can make. A type-I error refers to the government not intervening when it should (when $\theta = H$), and a type-II error refers to intervening when it should not ($\theta = L$). The probability of those errors are $\Pr(\text{Type I}) = \gamma \Pr(G = 0|\theta = H)$ and $\Pr(\text{Type II}) = (1-\gamma)\Pr(G = 1|\theta = L)$, so the government payoff in (4) can be rewritten as

$$U_G = \gamma(b-c) - (b-c)\Pr(\text{Type I}) - c\Pr(\text{Type II}). \tag{8}$$

Equation (8) decomposes the expected payoff of the government in three terms. The first term captures the first-best payoff that would be obtained if the intervention were undertaken if and only if the good state arises, $\theta = H$. The second term captures the expected loss due to a type-I error, when the state is good but the government does not intervene, forgoing the net benefit of $(b-c)$. The third term captures the expected loss due to a type-II error, when the state is low but the government still intervenes, incurring the cost $c$.

Proposition 2 implies that type-I errors never occur in equilibrium for an optimistic policymaker ($\gamma > \overline{\gamma}$), since interventions always occur when the state is good. In contrast, both types of errors may occur in equilibrium for a pessimistic policymaker ($\gamma < \overline{\gamma}$).

## 3.2 Comparative statics on private benefits

Our model suggests that the size of private benefits of interventions accruing to informed traders has important implications for the ability of policymakers to learn from market activity, the probability of interventions and, ultimately, for real efficiency. In this section, we inspect in detail how changes in private benefits affect those outcomes.

As previously discussed, one interpretation of such private benefits is that the informed trader previously owns a block of stocks of size $\mu$, in which case private benefits scale with block size ($\beta_\theta = \mu\alpha_\theta$). Hence, increases in the private benefit can be interpreted as increases in the block size $\mu$. Blockholder sizes vary significantly across firms. In Holderness (2009), for example, 96% of the firms have at least one blockholder, with block sizes ranging from 5.4% to 85.5% of ownership. (The usual definition of a blockholder is an ownership share of at least 5%.) If the trader in our model is a blockholder, when submitting an order, she anticipates how it will affect not only its trading profits but also the value of her existing block, via its effect on the policy outcome.

**Proposition 4.** ***Private benefit.*** *The larger the private benefit $\beta_L$: (i) the lower are both market informativeness and real efficiency; and (ii) the higher the ex-ante probability of intervention,* $\Pr(G = 1)$*, if $\gamma > \overline{\gamma}$. For $\gamma < \overline{\gamma}$, however, the probability of intervention is non-monotonic in $\beta_L$.*

Since positively informed traders always trade according to their information in equilibrium (Lemma 1), the size of the private benefit in the high state ($\beta_H$) does not affect equilibrium outcomes. However, the first part of Proposition 4 states that the higher the stakes of the negatively informed trader in the intervention ($\beta_L$), the less able is the policymaker to learn from market activity and, hence, the less efficient is the intervention or bailout. The larger $\beta_L$, the larger the trader's incentives not to trade on her information after bad news, since low aggregate orders would push beliefs closer to the true state $\theta = L$, reducing the chances of an intervention.

The second part of Proposition 4 states that the effect of $\beta_L$ on the ex-ante probability of intervention is positive for an optimistic government but ambiguous for a pessimistic government, as shown in Figure 5. To gain intuition, note that an increase in $\beta_L$ has two effects. First, the negatively informed trader has more incentives to trade strategically and manipulate the belief of the policymaker. Ceteris paribus, this increases the probability of intervention. Second, and in response to the first channel, the policymaker reduces the weight given to market activity.

An optimistic policymaker, $\gamma > \overline{\gamma}$, is willing to intervene even without additional information from the market. Hence, the *trading for bailouts* behavior is effective in increasing the probability of intervention. Both effects stated above push in the same direction: for a larger private benefit, incentives to trade strategically are higher and market activity is less informative, resulting in a higher overall probability of intervention. Manipulation is quite effective in this case: as $\beta_L$ increases, the ex-ante probability of intervention eventually reaches 1 (see top line in Figure 5).

In contrast, a pessimistic policymaker, $\gamma < \overline{\gamma}$, requires some positive updating for an intervention to occur. Hence, no intervention occurs for uninformative market activity. As before, a marginal increase in $\beta_L$ can increase the probability of intervention because it encourages strategic trading to affect the policymaker's beliefs. In contrast to the previous case, the second effect opposes the first effect. For a higher private benefit, the policymaker is also more skeptical about the informativeness of market activity, reducing its reliance on it and ultimately reducing the probability of intervention. Taken together, the probability of intervention can be non-monotonic in $\beta_L$ (see Figure 5). This result shows that manipulation can be ineffective: the presence of traders with high stakes in the intervention can reduce market informativeness and result in a lower probability of intervention. For a large enough private benefit, the policymaker disregards any information from market activity and the probability of intervention approaches zero.

In short, our model implies that private benefits can mitigate an effective channel of communication between the market and the policymaker: the information of the informed trader is not conveyed via market activity, and policy interventions are less efficient.

The mechanism leading to lower real efficiency as $\beta_L$ increases is different for pessimistic and optimistic priors. As previously discussed, efficiency losses can arise from type-I and type-II errors. For $\gamma > \overline{\gamma}$, the policymaker always intervenes in the good state, so the probability of a type-I error is zero. The efficiency loss of larger private benefits is entirely due to an increase in type-II errors, because the policymaker often intervenes when it should not. In contrast, for a pessimistic prior, $\gamma < \overline{\gamma}$, both types of errors occur in equilibrium. As $\beta_L$ increases, the policymaker eventually

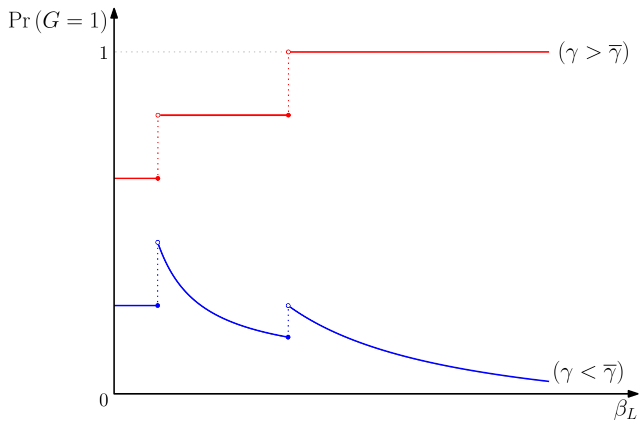


Figure 5: Expected probability of intervention as a function of private benefit $\beta_L$: numerical example with $\gamma$ slightly below or above $\overline{\gamma}$. Jumps occur at the switches between equilibrium classes—see also Figure 2.

intervenes with very low probability, and the main source of inefficiency is type-I errors: the policymaker forgoes desirable interventions too often. The probability of type-I errors increases substantially, while the probability of type-II errors vanishes.

Looking at those results through the lens of our blockholder interpretation, are blockholders good or bad for market informativeness? In our model with government intervention, larger block sizes are related to lower informativeness. In practice, there are many reasons why having large blockholders may be beneficial in general. Companies with some large stockholders tend to have more informative prices (Brockman and Yan, 2009; Boehmer and Kelley, 2009; Gallagher, Gardner and Swan, 2013; Gorton, Huang and Kang, 2016), possibly due to their larger incentives to acquire information (absent in our model). Large stockholders also exert an important role in corporate governance (for an extensive review, see Edmans and Holderness, 2017). However, our focus is not on average informativeness but on *informativeness around government interventions*. Our model suggests that the strategic behavior of large informed blockholders can lower market informativeness around such interventions and, hence, the efficiency of government policies.

Our model also has testable implications on how the concentration of ownership, proxied by block size, affects the probability of a government intervention. However, empirical counterparts for this measure may be harder to obtain than measures for market informativeness.

As a final comment on our positive analysis, note that for simplicity we assume that the large trader has superior information only regarding the firm's fundamental (unknown to the policymaker), but not regarding the ex-ante probability of a bailout. If the large trader had also superior information about, for instance, the policymaker's cost of intervening, she could not only "trade *for* bailouts", but also "trade bailouts": conditional on fundamentals, incentives to buy stocks of a firm increase if it is more likely to be the target of a bailout—or to receive a cash inflow of any sort. Note that the trader would hold this additional informational advantage over noise traders, but not over the policymaker (who knows its intervention cost well), and hence there should not be any deep implications of this second layer of information asymmetry for our conclusions.

# 4 Normative implications

In this section we discuss how institutional features affecting the costs of policy implementation and how the design of bailouts can improve market informativeness around interventions, and consequently, increase welfare.

## 4.1 Intervention costs and welfare

First, we study how intervention costs affect market informativeness and welfare in our model.

**Proposition 5.** ***Sand in the wheels.*** *A higher intervention cost* ($c$) *can increase market informativeness and welfare. In particular, for a low prior* $\gamma < \overline{\gamma}$ *and a high private benefit* $\beta_L > \tilde{\beta}$*, a higher intervention cost unambiguously increases informativeness and weakly increases welfare.*

Common wisdom might suggest that welfare is reduced when implementing policy interventions becomes more costly. If we take as given the information set of the policymaker, this is naturally true in our setting as well. However, we show that an increase in intervention costs $c$ may improve market informativeness—in fact, it always does so when the prior probability of the good state is low, and private benefits of intervention for traders are high. This beneficial effect operates through the policymaker's ex-ante willingness to intervene: higher values of $c$ mean that the posterior probability the policymaker must assign to the good state for it to intervene is larger ($\overline{\gamma}$ increases in $c$). If the intervention cost is large, the policymaker is more reluctant to intervene, which facilitates learning for two reasons: (i) the negatively informed trader may give up distorting its trading behavior (for instance, buying stocks) to convince the policymaker to intervene; (ii) if the trader is still willing to do so, for her to have any chance of affecting the policymaker's decision, she must sell the stock with larger probability so that the change in beliefs after observing high aggregate orders is more substantial.

Proposition 5 suggests that throwing some *sand in the wheels* of the policymaker can actually be good for overall welfare. The direct effect of a higher intervention cost is to destroy value when the bailout takes place, reducing welfare. However, the indirect effect of a higher $c$—changes in market informativeness—is often positive and may overcome the direct effect. Taken together, welfare can be higher overall.

## 4.2 Gradual bailout implementation

We now consider the possibility of the policymaker giving out an assistance package $\underline{A} > 0$ *before* observing market activity. The policymaker still reacts to market activity and chooses ex post whether to complement its assistance (providing additional funds). Such policy can be easily implemented by offering an unconditional credit line to banks (such as the Federal Reserve discount window), or by implementing bailouts gradually (providing a smaller assistance in a first step). Hence, the size of the intervention $G \in [0,1]$ is now given by $G = \underline{A} + a$, where $a \in [0, 1-\overline{A}]$ denotes the size of the intervention undertaken after observing market activity. All the other assumptions of the model remain the same. As will be shown, early implementation ($\underline{A} > 0$) can be beneficial.[12]

As a tie-breaking convention, we assume that when indifferent between implementing some $\underline{A} > 0$ and $\underline{A} = 0$, the policymaker chooses the latter. Also, throughout this section, we make the additional parametric assumption that $\Delta_\alpha \geq 0$, which arises naturally in applications (see Section

[12] It is often argued that committing *not to provide* too much assistance to financial institutions could be beneficial (for instance, if there are moral hazard concerns). The main issue is that if the policymaker believes a bailout is socially desirable ex post, it has incentives to deviate and increase assistance. In our setting, limiting the size of maximum assistance can also improve welfare, but we focus on a minimum assistance that is easier to implement.

5.1, for instance).[13] We can now state a formal result.

**Proposition 6.** ***Gradual bailout implementation.*** *Offering some $\underline{A} > 0$ ex ante can increase market informativeness and welfare (despite the potential cost of early assistance). Let $\underline{A}^*$ denote the optimal initial assistance. There exist $\underline{\beta}^\dagger \in \left(\underline{\beta}, \overline{\beta}\right)$, $\underset{\sim}{\beta}^\dagger \in \left(\underset{\sim}{\beta}, \tilde{\beta}\right)$ and $\tilde{\beta}^\dagger > \tilde{\beta}$ such that:*

1. *For a high prior, $\gamma > \overline{\gamma}$, we have:*

$$\underline{A}^* = \begin{cases} 1 - \frac{\gamma \Delta_R}{\beta_L - \gamma \Delta_\alpha} & \text{for } \underline{\beta} < \beta_L < \underline{\beta}^\dagger, \\ 1 - \frac{(1+2\gamma)\Delta_R}{\beta_L - (1+2\gamma)\Delta_\alpha} & \text{for } \beta_L > \overline{\beta}, \\ 0 & \text{otherwise}; \end{cases}$$

2. *For a low prior, $\gamma < \overline{\gamma}$, we have:*

$$\underline{A}^* = \begin{cases} 1 - \frac{\gamma \Delta_R}{\beta_L} & \text{for } \underset{\sim}{\beta} < \beta_L < \underset{\sim}{\beta}^\dagger, \\ 1 - \frac{\beta_L(1+\gamma+\overline{\gamma})\Delta_R + \gamma\overline{\gamma}\Delta_R\Delta_\alpha}{\beta_L(\beta_L - \Delta_\alpha)} & \text{for } \tilde{\beta} < \beta_L < \tilde{\beta}^\dagger, \\ 0 & \text{otherwise.} \end{cases}$$

*The bounds $\underline{\beta}, \overline{\beta}, \underset{\sim}{\beta}$ and $\tilde{\beta}$ are as defined in* (3)*, and $\underline{\beta}^\dagger$, $\underset{\sim}{\beta}^\dagger$ and $\tilde{\beta}^\dagger$ are defined in* (B.9)*,* (B.13)*, and* (B.16)*, respectively.*

Figure 6 illustrates the optimal gradual implementation results in Proposition 6. If the information the government could obtain from the market were exogenous to the policy, there would be no gains from making a policy decision earlier, with less information. However, the result is different when the informational content of market activity is *endogenous* to the policy. Despite the first stage of the policy ($\underline{A}$) being undertaken with little information, this early decision boosts the informativeness of market activity on which the policymaker can rely. As a result, the uncertainty regarding the desirability of additional support is reduced.

The intuition is that providing some initial assistance early on reduces the residual benefit

[13] If this assumption were violated, it would mean that firms obtain a higher private gain from an intervention in the low state than in the high state, while the social benefit of intervention is larger in the high state.

of an (ex-post) additional intervention. That is, offering $\underline{A} > 0$ *ex ante* reduces the stakes of the trader on the policy decision to provide additional assistance *ex post*. Therefore, incentives to trade for bailouts are reduced, allowing the policymaker to learn more from market activity and to implement additional assistance (beyond $\underline{A}$) more efficiently.

However, the early implementation of a positive $\underline{A}$ can be costly ex post. For instance, even if later market activity perfectly reveals the bad state ($\theta = L$), the government has to incur the cost of the early support ($c\underline{A}$), which in this case is smaller than the social benefit. Still, some level of early assistance is often welfare-improving ex ante, with gains in market informativeness more than compensating for the additional implementation cost.

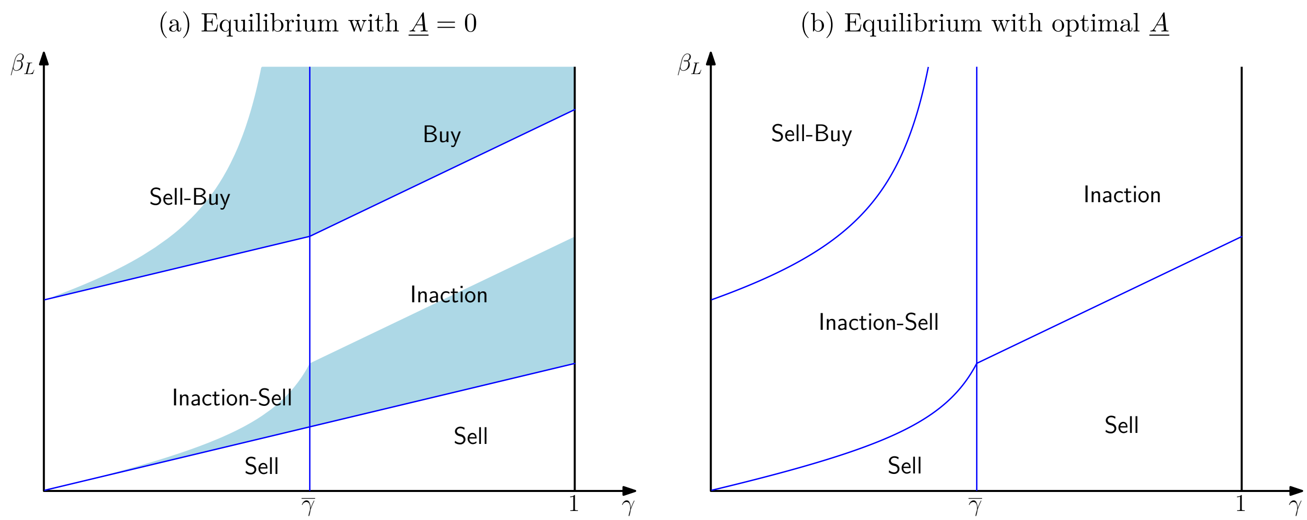


Figure 6: Gradual bailout implementation: an illustration for $\Delta_\alpha = 0$. On the left, the equilibrium set for different values of private benefit $\beta_L$ and prior probability of high state $\gamma$ when $\underline{A} = 0$; the shaded areas show the regions where gradual implementation is welfare-improving, $\underline{A}^* > 0$. On the right, the equilibrium set under the optimal initial assistance, $\underline{A}^*$ (see Proposition 6).

**Corollary 1.** *Offering an early assistance $\underline{A} > 0$ may reduce the total expected bailout size.*

Corollary 1 helps explain the benefits of a gradual bailout implementation. Consider, for instance, the case of $\gamma > \overline{\gamma}$. With the aid of Figure 6, one can see that if a Buy equilibrium is played for $\underline{A} = 0$, the policymaker always benefits from implementing early on the lowest $\underline{A} > 0$ that triggers the Inaction equilibrium instead. The reason is that giving out early assistance in the Buy equilibrium is cheap. In its absence, the policymaker cannot learn from the market and ends up giving a full bailout $G = 1$, so assisting with at least $\underline{A} < 1$ implies no additional cost, regardless of the information that can be obtained later on. Giving out a large enough early assistance $\underline{A}$ actually *reduces* the total expected bailout size. By triggering the Inaction equilibrium, the policymaker ensures higher market informativeness and no longer implements the full bailout with certainty.

In contrast, if for $\underline{A} = 0$ the Inaction equilibrium is played, early assistance is not always beneficial. In the Inaction equilibrium, market activity reveals the bad state with some probability, in which case the policymaker (correctly) refrains from providing any assistance. Giving out some $\underline{A} > 0$ early on then implies additional costs. As a result, the policymaker only does so if the amount $\underline{A}$ needed to trigger the Sell equilibrium is low enough (shaded area in Figure 6), in which case the gain in informativeness achieved with a switch to the Sell equilibrium more than compensates the additional implementation costs.

As a result, for $\gamma > \overline{\gamma}$, under the optimal initial assistance $\underline{A}^*$ the equilibrium set features an enlarged Sell equilibrium region, and the least informative equilibrium (Buy equilibrium) disappears altogether (see right panel of Figure 6). Similarly, for $\gamma < \overline{\gamma}$, the region of parameters in which the least informative equilibrium ($SB$) is played is reduced (although it does not completely disappear), and the most informative equilibrium area ($S$) is enlarged.

Our results on the optimal timing of government interventions are broadly related to Cong, Grenadier and Hu (2020), although our setting is very different. In a rollover game where policymakers want to avoid runs at two dates, they find that optimal interventions should often prioritize avoiding runs at the first date, since successful rollover reveals good news about fundamentals and mitigates runs in the second date as well. Hence, their result concerns the optimal dynamic implementation of two separate policies with different objectives when there are coordination failures. In our setting, instead, we show that even when a policymaker has a single policy instrument and objective, it can be optimal to implement policies gradually. Our results also relate to Nosal and Ordonez (2016). In their setting, government uncertainty about the need of a bailout resembles a commitment device not to intervene too early, which attenuates banks' moral hazard. Here, we show that early but partial interventions can have a novel benefit, as it allows the policymaker to learn more from the market and make more efficient decisions overall.

We conclude this section with some comparative statics regarding the optimal size of early intervention.

**Proposition 7.** *Whenever* $\underline{A}^* > 0$, $\partial \underline{A}^*/\partial \beta_L > 0$, $\partial \underline{A}^*/\partial \gamma < 0$, $\partial \underline{A}^*/\partial \Delta_R < 0$ *and* $\partial \underline{A}^*/\partial \Delta_\alpha \leq 0$.

The first result arises because the larger the private benefit of intervention for the informed trader, $\beta_L$, the larger her incentives for trading for bailouts, and thus the larger the initial assistance needed to cause a switch to a more informative equilibrium. Likewise, the lower $\Delta_R$, $\Delta_\alpha$, and $\gamma$, the lower the negatively informed trader's informational advantage, so the lower her potential for trading profits and thus the larger the incentives for trading for bailouts.

# 5 Alternative settings

In this section, we line out a simple model of liquidity support to a distressed bank to illustrate how our main model can map into applications, and finally, present an extension where orders are subject to mandatory disclosure.

## 5.1 An application to bank liquidity support

The model presented below is isomorphic to the main setting, but with the payoffs of the policy-maker and the large trader linked to the market conditions of a bank in need of liquidity support.

At the beginning of $t = 0$, a bank has an exogenous amount $D > 0$ of short-term debt not rolled over by its creditors, per unit of outstanding stock. Denote by $\nu$ the solvency status of the bank, where $\nu = 1$ represents that the bank is solvent and $\nu = 0$, insolvent. At $t = 1$, the bank's assets (per outstanding stock) are worth $\tilde{V}_\theta = V$ if the bank is solvent and $\tilde{V}_\theta = 0$ if insolvent. Denote by $\omega_\theta$ the probability of solvency of the bank in state $\theta$, with $\Delta_\omega \equiv \omega_H - \omega_L > 0$. If liquidated prematurely, those assets are worth only $(1 - \psi)\,\tilde{V}_\theta$, where $\psi \in (0, 1)$ represents asset illiquidity (e.g., a fire-sale penalty). Consistent with the interpretation that the bank is solvent when $\nu = 1$, we assume that $(1 - \psi)\,V > D$.[14] In the absence of any government assistance

[14]That is, if $\nu = 1$, the bank is still illiquid in the sense that it does not have liquid assets to cover its short-term

(explained below), the bank must sell a fraction $\tilde{y}(\nu) = \frac{D}{(1-\psi)V}\mathbb{I}_{\{\nu=1\}}$ of its assets to meet creditor withdrawals, where $\mathbb{I}$ is the indicator function. If $\nu = 0$, the bank simply defaults on its debt.

A policymaker may want to offer liquidity assistance to reduce the deadweight loss caused by the fire sale. The policymaker can purchase a fraction of the firm's debt and roll it over. When the government buys (and rolls over) a dollar amount $A \leq D$ per outstanding stock, the bank only needs to liquidate a reduced fraction

$$y(\nu, A) = \frac{D - A}{(1-\psi)V}\mathbb{I}_{\{\nu=1\}}$$

of assets. However, raising funds has a social cost $\tau$ per dollar. One (classical) interpretation is that taxation is distortionary. Another interpretation is related to the bailout funds for banks that governments set up in several jurisdictions after the Great Financial Crisis. Thus, larger pre-funded resources correspond to lower values of $\tau$. To focus on our key channel, we abstract from moral hazard issues throughout.

The expected stock dividend conditional on the state $\theta$ is thus

$$\begin{aligned} v(\theta, A) &= \omega_\theta \left\{[1 - y(1, A)]V + y(1, A)(1-\psi)V - D\right\} \\ &= \omega_\theta \left(V - A - \frac{D-A}{1-\psi}\right). \end{aligned} \tag{9}$$

If the bank turns out to be insolvent, any assistance $A > 0$ represents a mere transfer from the policymaker to creditors, so it is a waste of resources because $\tau > 0$. If the bank is solvent, any $A > 0$ is paid back to the government and helps avoid fire-sale costs. This simple setting captures a key concern faced by policymakers during the 2008 crisis, namely, the risk of injecting taxpayer dollars into insolvent institutions (see also the introduction).

After observing financial market activity, the (utilitarian) policymaker forms the belief $q(X)$ and chooses the size of assistance $A$ in order to maximize welfare. Conditional on the aggregate

obligations (it must incur liquidation costs), but it is solvent in the sense that it has more than enough assets to cover those obligations, so it does not default.

order, expected wealth in this economy is proportional to

$$W = \mathbb{E}\left[\left(1 - y\left(\nu, A\right)\right)\tilde{V}_\theta + y\left(\nu, A\right)\left(1 - \psi\right)\tilde{V}_\theta | X\right] - \tau A$$
$$= \Omega + A\kappa\left[\omega_L + q\left(X\right)\Delta_\omega\right] - \tau A, \tag{10}$$

where $\kappa \equiv \frac{\psi}{1-\psi}$ and $\Omega \equiv (V - \kappa D)\left[\omega_L + q\left(X\right)\Delta_\omega\right]$.[15] Henceforth, we refer to $\kappa$ as the liquidation cost instead of $\psi$. The term $\Omega$ in (10) represents the expected wealth in the absence of intervention; the second term captures the expected fire-sale costs that an intervention of size $A$ avoids; the last term is the intervention cost. Welfare is the ex-ante expectation $\mathbb{E}[W]$, formed using the prior $\gamma$.

If $\tau$ is large enough, raising funds is too costly and the government does not intervene, regardless of its beliefs $q\left(X\right)$. In contrast, if $\tau$ is low enough, the policymaker purchases all debt $D$ regardless of its beliefs. In either case, traders trivially trade on their information in equilibrium (buying following good news and selling following bad news). We focus on the interesting case of $\tau \in (\kappa\omega_L, \kappa\omega_H)$, in which the policymaker benefits from learning from market activity. In this case, the policymaker implements a full bailout $A = D$ if $q$ is high enough, and does not assist otherwise ($A = 0$). We can thus map those strategies into a binary intervention $G \in \{0, 1\}$.[16] Specifically, the policymaker is willing to intervene (buying all the debt) whenever $q\left(X\right) \geq \frac{\tau - \kappa\omega_L}{\kappa\Delta_\omega}$.

Finally, the informed trader is a blockholder who owns $\mu$ shares of the bank's stock. Hence, her private benefit from an intervention is $\beta_\theta = \mu\omega_\theta\kappa D$, since $\omega_\theta\kappa D$ is the expected gain in the stock dividend caused by an intervention (see Equation (9)). The timing and information structure of the game are the same as in the main model, and the equilibrium preferred by the policymaker is selected when multiple equilibria exist. For the purpose of computing the equilibrium, the application is isomorphic to the main model, which can be seen by letting $R_\theta = \omega_\theta\left[V - (1+\kappa)D\right]$, $\alpha_\theta = \omega_\theta\kappa D$, $b_\theta = \omega_\theta\kappa D$, $c = \tau D$, and $\overline{\gamma} = \frac{\tau - \kappa\omega_L}{\kappa\Delta_\omega}$. We can then build on the results of the main model to understand how balance sheet and market conditions affect the ability of policymakers

[15] Note that we can ignore $\Omega$ when computing the equilibrium as it does not depend on $A$ (although it matters for comparative statics). Also note that $\mathbb{E}\left[\Omega\right] = (V - \kappa D)\left[\omega_L + \gamma\Delta_\omega\right]$ since $\mathbb{E}\left[q\left(X\right)\right] = \gamma$, and therefore $\mathbb{E}\left[\Omega\right]$ only depends on exogenous parameters, and not on players' strategies. Finally, note that the aggregate expected wealth conditional on the state is $W$ times the quantity of outstanding shares.

[16] When the policymaker is indifferent between any level of intervention, we assume that it chooses $A = D$ as a tie-break rule. This is without loss of generality because we allow for mixed strategies: choosing some $A \in (0, D)$ is analogous to choosing $A = D$ with some interior probability and $A = 0$ with the complementary probability.

to learn from the market.

For instance, consider an increase in the size of the liquidity need $D$. One can see that it increases the private benefit $\beta_L$, while at the same time reducing the bounds that limit the Sell equilibrium region, $\underline{\beta}$ and $\underset{\sim}{\beta}$ (see Proposition 2 and Figure 2). Hence, it is harder to sustain the most informative equilibrium when $D$ is larger, which implies that the direct negative effect of a larger $D$ on welfare can be amplified by an endogenous deterioration of market informativeness.

Now, suppose there is an equal increase in the probability of solvency of the firm in both states. Such increase has a direct positive effect on welfare. However, it can worsen market informativeness, since it increases the private benefit $\beta_L$ and it does not affect the equilibrium bounds in (3). The overall effect on welfare can be negative. An increase in asset value conditional on solvency, $V$, however, has an unambiguous positive effect on both informativeness and welfare, since it increases the bounds in (3), via an increase in $\Delta_R$.

## 5.2 Mandatory disclosure of large orders

Our main setting can also capture situations in which large informed traders have to disclose their orders after they have been executed, as long as the policymaker is unsure about whether such orders are due to some liquidity shock faced by speculators. Still, even if the policymaker can perfectly observe the large trader's orders and knows that the informed trader has no liquidity motives to trade, our results remain, as discussed in this subsection.

We consider an extension in which the policymaker observes the order of the large informed trader before deciding on the intervention. This is meant to capture more explicitly situations where such large orders become public news, or are executed by insiders subject to regulations such as the SEC Form 4 filing requirement, for instance.[17] Suppose that the timing of the game changes as follows: (i) traders place orders, (ii) the marker maker observes aggregate orders and execute them at the fair price, (iii) the policymaker observes the large trader's order and makes the intervention decision. The next proposition shows that the trading for bailouts behavior of the

[17] In short, every director, officer or owner of a stake larger than 10% of a firm equity is required to report changes in stock ownership, through the filing of SEC Form 4, two business days after order execution.

trader and our main results extend to this alternative setting.

**Proposition 8.** *Consider the setting in which the policymaker observes the informed trader's individual order $s$ before deciding on the intervention. For $\beta_L$ and $\beta_H$ sufficiently large, we have:*

1. *The negatively informed trader selling the stock and the positively informed trader buying the stock is not an equilibrium outcome;*

2. *Both the positively and the negatively informed traders buying the stock is an equilibrium outcome for $\gamma > \overline{\gamma}$;*

3. *The positively informed trader buying the stock and the negatively informed trader randomizing between buying and selling is an equilibrium outcome for $\gamma < \overline{\gamma}$.*

Proposition 8 shows that, even if the policymaker is perfectly aware of the orders placed by the large informed trader, incentives to manipulate the policymaker's belief to trigger an intervention remain. The intuition behind the results are analogous to those in our main setting.

# 6 Conclusion

We study the extent to which policymakers can learn from market activity when large informed traders have high stakes in the outcome of an intervention. Such stakes naturally arise when the trader is an insider, a blockholder, or a creditor of the firm targeted by a government intervention.

We show that the strategic motives of traders with high stakes in the intervention reduce market informativeness and the efficiency of bailouts. One key testable implication is that, around government interventions, market activity is less informative for firms with large blockholders. We characterize conditions under which the trading for bailouts behavior of traders effectively alters policy outcomes.

In terms of normative implications, we show that a larger cost of implementing bailouts can actually boost informativeness and increase welfare. We also discuss implications for bailout design, offering a rationale for the gradual implementation of assistance. Implementing partial bailouts early on (with little information) can reduce the expected total disbursement of government money due to its positive effects on the policymaker's ability to learn from the market, which helps to avoid wasteful interventions and can raise welfare.

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# A Equilibrium characterization

Before characterizing the equilibrium, we introduce some notation and establish some basic results. In the appendix, we often refer to the positively (negatively) informed trader as the high (low) type. The price of the asset will be given by

$$p(X) = R_L + q(X)\Delta_R + g(X)\left[\alpha_L + q(X)\Delta_\alpha\right]. \tag{A.1}$$

Using (A.1), the payoff of the high type can be written as $\pi^H(s) = \Pi_T^H(s) + \Pi_G^H(s)$, where $\Pi_T^H(s) = \mathbb{E}_s\left[s\left(1 - q(X)\right)\left(\Delta_R + g(X)\Delta_\alpha\right)\right]$ is the expect trading profit and $\Pi_G^H(s) = \mathbb{E}_s\left[g(X)\beta_H\right]$ is the expected benefit of the intervention. Similarly, the payoff of the low type can be written as $\pi^L(s) = \Pi_T^L(s) + \Pi_G^L(s)$, where $\Pi_T^L(s) = \mathbb{E}_s\left[-sq(X)\left(\Delta_R + g(X)\Delta_\alpha\right)\right]$ and $\Pi_G^L(s) = \mathbb{E}_s\left[g(X)\beta_L\right]$. Also note that $\Pi_G^L(s) = \frac{\beta_L}{\beta_H}\Pi_G^H(s)$. Finally, note that the market maker and the policymaker never learn the true state when observing $X = 0$, regardless of the traders' strategies. Therefore, we assume $q(0) \in (0, 1)$ hereafter. This implies that $\Pi_T^L(-1) > 0$, $\Pi_T^L(1) < 0$, $\Pi_T^H(-1) < 0$ and $\Pi_T^H(1) > 0$ in any possible equilibrium. In words: the high (low) type always makes a trading profit when she buys (sells), and a trading loss when she sells (buys).

## A.1 Equilibrium strategies for the high type

In this section we prove Lemma 1. We begin by showing that there can be no equilibrium with $h(1) < 1$ and $l(1) > 0$. First, suppose $h(1) < 1$ and $h(0) > 0$. We must then have $\Pi_G^H(0) \geq \Pi_T^H(1) + \Pi_G^H(1)$ and therefore $\Pi_G^H(0) > \Pi_G^H(1)$. But then $\pi^L(0) = \frac{\beta_L}{\beta_H}\Pi_G^H(0) > \Pi_T^L(1) + \frac{\beta_L}{\beta_H}\Pi_G^H(1) = \pi^L(1)$. Hence, $l(1) = 0$. Second, suppose $h(1) < 1$ and $h(-1) > 0$. We must then have $\Pi_T^H(-1) + \Pi_G^H(-1) \geq \Pi_T^H(1) + \Pi_G^H(1)$, which implies $\Pi_G^H(-1) > \Pi_G^H(1)$. But then $\pi^L(-1) = \Pi_T^L(-1) + \frac{\beta_L}{\beta_H}\Pi_G^H(-1) > \Pi_T^L(1) + \frac{\beta_L}{\beta_H}\Pi_G^H(1) = \pi^L(1)$. Hence, $l(1) = 0$. To summarize, we have shown that if there exists an equilibrium with $h(1) < 1$ we must have $l(1) = 0$. In what follows, we look for an equilibrium with $l(1) = 0$ and $h(1) < 1$ and show that it cannot be constructed. We divide the remainder of this proof in three cases.

**Case 1: $h(1) \in (0, 1)$.** Suppose $h(1) \in (0, 1)$. Since $l(1) = 0$, we have $g(2) = 1$. First, assume $h(0) > 0$. Since $\pi^H(1) - \pi^H(0) = \Pi_T^H(1) + \frac{1}{3}\beta_H\left[g(2) - g(-1)\right] > 0$, there can be no equilibrium with $h(0) > 0$ and $h(1) \in (0, 1)$. Second, assume $h(0) = 0$. Then $h(-1) > 0$ and $g(-2) + g(-1) > g(2) + g(1)$ (otherwise the high type would not play $s = -1$ with positive probability). Since $g(2) = 1$, $g(-2) - g(1) > 1 - g(-1) \geq 0$, and therefore

$$\pi^L(-1) - \pi^L(0) = \Pi_T^L(-1) + \frac{1}{3}\beta_H\left[g(-2) - g\left(1\right)\right] > 0. \tag{A.2}$$

Hence, the low type plays $l(-1) = 1$. But this implies that $g(1) = 1$, which together with $g(2) = 1$ shows that $g(-2) + g(-1) > g(2) + g(1)$ cannot be satisfied, a contradiction.

**Case 2: $h(1) = 0$ and $h(-1) > 0$.** Suppose $h(1) = 0$ and $h\left(-1\right) > 0$. We must then have $\pi^H(-1) - \pi^H(0) = \Pi_T^H(-1) + \frac{1}{3}\beta_H\left[g(-2) - g\left(1\right)\right] \geq 0$, and since $\Pi_T^H(-1) < 0$, $g(-2) > g\left(1\right)$. But then $l(-1) = 1$ (see (A.2)). First consider $h\left(-1\right), h\left(0\right) \in (0, 1)$. Then $q\left(1\right) = g(1) = 1$, which implies that $g(-2) > g(1) = 1$ is violated, a contradiction. Now consider $h(-1) = 1$. Then $q\left(-2\right) = q\left(-1\right) = q\left(0\right) = \gamma$. If

$\gamma < \overline{\gamma}$, $g(-2) = 0 \leq g(1)$, a contradiction. If $\gamma > \overline{\gamma}$, $g(-2) = g(-1) = g(0) = 1$, and the intuitive criterion implies that $q(1) = q(2) = 1$. To see that, note that even under the best-case scenario that an intervention occurs following off-equilibrium orders $X = 1, 2$, the low type cannot gain from deviating to $s = 0$ or $s = 1$ (it would forego trading profits and would get the same intervention benefit at best). The high type, on the other hand, could have a profitable deviation to $s = 0$ or $s = 1$ (since $\Pi_T^H(-1) < 0$). But then $g(1) = g(2) = 1$, contradicting $g(-2) > g(1)$. Therefore, there can be no such equilibrium.

**Case 3: $\boldsymbol{h(0) = 1}$.** Suppose $h(0) = 1$. Then, $\pi^H(0) - \pi^H(1) = -\Pi_T^H(1) + \frac{1}{3}\beta_H [g(-1) - g(2)] \geq 0$, and since $\Pi_T^H(1) > 0$, $g(-1) > g(2)$. Since $l(1) = 0$, $q(-1) = \gamma$. But if $\gamma < \overline{\gamma}$, we have $g(-1) = 0$ and hence $g(-1) > g(2)$ cannot be satisfied. Thus, that cannot be an equilibrium when $\gamma < \overline{\gamma}$. Hence, in the remainder of this proof we assume $\gamma > \overline{\gamma}$.

First, suppose $l(-1) = 1$. Then, $g(-1) = g(0) = g(1) = 1$ and $g(-2) = 0$. Moreover, $q(1) = 1$, $q(-2) = 0$ and $q(0) = q(1) = \gamma$. For the low type not to deviate to $s = 0$ we must have $\pi^L(-1) - \pi^L(0) = \Pi_T^L(-1) + \frac{1}{3}\beta_L [g(-2) - g(1)] \geq 0$, which using $g(-2) = 0$ and $g(1) = 1$ becomes $\beta_L \leq 3\Pi_T^L(-1)$. Note that $\Pi_T^L(-1) = \frac{2}{3}\gamma(\Delta_R + \Delta_\alpha)$, and therefore the low type will not deviate to zero if

$$\beta_L \leq 2\gamma(\Delta_R + \Delta_\alpha). \tag{A.3}$$

Hence, when the condition above is violated that cannot be an equilibrium. Suppose now (A.3) is satisfied. In that case, the intuitive criterion imposes $q(2) = 1$. To see that, note that a strict upper bound on the low type gain in deviating from $s = -1$ to $s = 1$ is $\overline{\Delta}_{DEV} = \beta_L - \pi^L(-1) = \beta_L - \frac{2}{3}[\gamma(\Delta_R + \Delta_\alpha) + \beta_L]$ (the gain if an intervention happens for sure and she does not incur any trading losses). The actual payoff of deviating (under the best case scenario) is strictly smaller than $\overline{\Delta}_{DEV}$ since the low type incurs a trading loss when $X = 0$. One can verify that $\overline{\Delta}_{DEV} \leq 0$ when $\beta_L \leq 2\gamma(\Delta_R + \Delta_\alpha)$, and therefore the deviation is strictly dominated for the low type. The high type clearly has incentives to deviate to $s = 1$ if she believes $g(2) = 1$ (under that scenario she still gets the intervention with probability one, but could make trading profits when $X = 0$). Hence, the intuitive criterion imposes $q(2) = 1$, which implies $g(2) = 1$. But then the high type deviates to 1 and that cannot be an equilibrium.

Second, suppose $l(0), l(-1) > 0$. Beliefs on the equilibrium path are $q(0) = q(-1) = \gamma$, $q(-2) = 0$ and $q(1) > \gamma$. Thus, $g(-1) = g(0) = g(1) = 1$ and $g(-2) = 0$. Indifference between 0 and $-1$ for the low type implies $\pi^L(-1) - \pi^L(0) = \frac{2}{3}[\gamma(\Delta_R + \Delta_\alpha) + \beta_L] - \beta_L = 0$. Thus, unless $\beta_L = 2\gamma(\Delta_R + \Delta_\alpha)$, that cannot be an equilibrium. If $\beta_L = 2\gamma(\Delta_R + \Delta_\alpha)$, the low type's payoff in such an equilibrium would be $\beta_L$. If the low type deviates to 1 she is worse off in any scenario: she gets at most $\beta_L$ from the government intervention but incurs a trading loss when $X = 0$. The high type could have a profitable deviation in a scenario where $g(2) = 1$. Hence, when $\beta_L = 2\gamma(\Delta_R + \Delta_\alpha)$, the intuitive criterion requires $q(2) = 1$, which implies $g(2) = 1$. But then the high type has incentives to deviate to $s = 1$ and that cannot be an equilibrium.

Third, suppose $l(0) = 1$. Then $q(-1) = q(0) = q(1) = \gamma$, which implies $g(-1) = g(0) = g(1) = 1$. The intuitive criterion requires $q(2) = 0$. To see that, note that the payoff of the low type under the presumed equilibrium is $\beta_L$. If she deviates to $s = 1$ she incurs a trading loss when $X = 0$ and, at best, still gets the intervention with probability one, so the deviation cannot be profitable. The high type can have a profitable deviation if $g(2) = 1$. Hence, the intuitive criterion imposes $q(2) = 1$, which implies $g(2) = 1$. But then the high type has incentives to deviate. We have then ruled out any possibility other than $h(1) = 1$ in equilibrium. $\square$

## A.2 Equilibrium characterization when $\gamma > \overline{\gamma}$

### A.2.1 Beliefs and equilibrium strategies for policymaker

The next lemma reduces the set of possible strategies for the policymaker and beliefs.

**Lemma A.1.** *Assume $\gamma > \overline{\gamma}$. Then, in any equilibrium: $q(2) \geq \gamma$, $q(1) \geq \gamma$, $q(0) = \gamma$, $q(-1) = q(-2) = 0$, $g(2) = g(1) = g(0) = 1$ and $g(-1) = g(-2) = 0$.*

*Proof.* Notice that conditional on any state and trader's strategies, $X = 0$ with probability $1/3$. Hence $q(0) = \gamma > \overline{\gamma}$ and $g(0) = 1$ in any equilibrium. From Lemma 1, $h(1) = 1$ in any equilibrium. Hence, for $x \in \{2, 1\}$, $\Pr(X = x|\theta = L) \leq \Pr(X = x|\theta = H)$. Bayes rule then implies that the policymaker never updates the probability of $\theta = H$ downwards upon observing $X \in \{2, 1\}$. Hence, $q(2), q(1) \geq \gamma$, and consequently $g(2) = g(1) = 1$. In the remaining of this proof we then assume that in any equilibrium $g(2) = g(1) = g(0) = 1$, $q(2) \geq \gamma$, $q(1) \geq \gamma$, $q(0) = \gamma$ and $h(1) = 1$. It remains to show that $g(-1) = g(-2) = 0$ and $q(-1) = q(-2) = 0$ in any equilibrium.

First, suppose that $l(1) = 1$. In that case, $X = -1$ and $X = -2$ are off the equilibrium path. The high type has no incentives to deviate, since under her equilibrium strategy $s = 1$ the intervention happens for sure and she makes trading profits (any deviation implies she does not make a trading profit). For the low type, a deviation to either $s = 0$ or $s = -1$ could be profitable, for instance, if the policymaker would intervene with probability 1 after observing $X \in \{-2, -1\}$ (she would eliminate the trading loss and still get the intervention for sure). Thus, by the intuitive criterion, agents must believe any deviation came from the low type and therefore $q(-1) = q(-2) = 0$, implying $g(-1) = g(-2) = 0$.

Second, suppose $l(0) > 0$. Since $h(1) = 1$, observing $X = -1$ reveals that $\theta = L$, so $q(-1) = 0$ and $g(-1) = 0$. When $X = -2$, by the same reasons as in the previous paragraph, the intuitive criterion implies $q(-2) = 0$ and therefore $g(-2) = 0$.

Third, suppose $l(-1) > 0$. Since $h(1) = 1$, observing $X = -1, -2$ reveals that $\theta = L$, and thus $q(-1) = q(-2) = 0$ and $g(-1) = g(-2) = 0$. $\square$

In the remainder of Section A.2 we assume that $h(1) = 1$ (Lemma 1) and that the functions $q(\cdot)$ and $g(\cdot)$ satisfy the conditions in Lemma A.1. Note that, in any equilibrium candidate satisfying those lemmas, the high type has no incentives to deviate from $h(1) = 1$. Hence, to verify if a strategy profile and beliefs are an equilibrium we only need to check if the low type has no incentives to deviate.

### A.2.2 Low type equilibrium payoffs

Since $g(-1) = g(-2) = 0$ and $q(-1) = q(-2) = 0$ in equilibrium, the low type obtains zero payoff when $X = -1, -2$. Moreover, using $q(0) = \gamma$, her expected payoffs of playing $s = -1, 0, 1$, respectively, are $\pi^L(-1) = \frac{1}{3}\gamma(\Delta_R + \Delta_\alpha) + \frac{1}{3}\beta_L$, $\pi^L(0) = \frac{2}{3}\beta_L$, and $\pi^L(1) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)[\gamma + q(1) + q(2)] + \beta_L$. We define the following upper bounds on the low type payoff of buying: $\pi^L_{UB}(1) = -\gamma(\Delta_R + \Delta_\alpha) + \beta_L$. It is constructed assuming that when playing $s = 1$ the low type gets the intervention for sure and is faced with a market maker as pessimistic as possible, taking into account the restrictions of $q(\cdot)$ imposed by Lemma A.1 (that is, assuming beliefs $q(2) = q(1) = q(0) = \gamma$ for the market maker).

### A.2.3 Sell equilibrium ($S$)

Here we look for an equilibrium with $l(-1) = 1$. A necessary condition to sustain such an equilibrium is

$$\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma(\Delta_R + \Delta_\alpha) - \frac{1}{3}\beta_L \geq 0, \tag{A.4}$$

which is equivalent to $\beta_L \leq \gamma(\Delta_R + \Delta_\alpha)$. It remains to check that the low type has no incentive to deviate to $s = -1$. A sufficient condition for that is $\pi^L(-1) - \pi^L_{UB}(1) = \frac{4}{3}\gamma(\Delta_R + \Delta_\alpha) - \frac{2}{3}\beta_L \geq 0$, which is automatically satisfied if (A.4) holds. Therefore, an $S$ equilibrium exists if and only if $\beta_L \leq \gamma(\Delta_R + \Delta_\alpha)$.

**About uniqueness of the sell equilibrium.** When $\beta_L < \gamma(\Delta_R + \Delta_\alpha)$, we have $\pi^l(-1) - \pi^l(0) > 0$ and $\pi^l(-1) - \pi^l(1) > 0$, and therefore the $S$ equilibrium is the unique equilibrium. When $\beta_L = \gamma(\Delta_R + \Delta_\alpha)$, $\pi^l(-1) - \pi^l(1) > 0$, but $\pi^l(-1) = \pi^l(0)$. Therefore, in this case there are also $SI$ equilibria in which $l(-1) = m \in [0, 1)$ and $l(0) = 1 - m$. Given that we have fully characterized the equilibrium set for $\beta_L \leq \gamma(\Delta_R + \Delta_\alpha)$ and $\gamma > \overline{\gamma}$, in the remainder of section A.2 we assume $\beta_L > \gamma(\Delta_R + \Delta_\alpha)$.

### A.2.4 Buy equilibrium ($B$)

Since $\beta_L > \gamma(\Delta_R + \Delta_\alpha)$, we have that $\pi^L(0) > \pi^L(-1)$ (see equation A.4). Hence, a $B$ equilibrium can exist if and only if $\pi^L(1) - \pi^L(0) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)[\gamma + q(1) + (2)] + \frac{1}{3}\beta_L \geq 0$, which is equivalent to $\beta_L \geq 3\gamma(\Delta_R + \Delta_\alpha)$, since in a $B$ equilibrium $q(2) = q(1) = \gamma$. Therefore, a $B$ equilibrium exists if and only if $\beta_L \geq 3\gamma(\Delta_R + \Delta_\alpha)$.

### A.2.5 Inaction equilibrium ($I$)

Since $\beta_L > \gamma(\Delta_R + \Delta_\alpha)$, we have that $\pi^L(0) > \pi^L(-1)$. Therefore, an $I$ equilibrium requires $\pi^L(0) - \pi^L(1) = \frac{1}{3}(\Delta_R + \Delta_\alpha)[\gamma + q(1) + q(2)] - \frac{1}{3}\beta_L \geq 0$. Note that in an $I$ equilibrium $q(2) = 1$ and $q(1) = \gamma$. Therefore the previous inequality becomes $\beta_L \leq (\Delta_R + \Delta_\alpha)(2\gamma + 1)$. Hence, an $I$ equilibrium exists if and only if $\gamma(\Delta_R + \Delta_\alpha) \leq \beta_L \leq (2\gamma + 1)(\Delta_R + \Delta_\alpha)$.[18]

[18]Recall that the inaction equilibrium exists for $\beta_L = \gamma(\Delta_R + \Delta_\alpha)$ (see Section A.2.3).

### A.2.6 Mixed strategies equilibria

We now check if there are other mixed strategies equilibria besides the one when $\beta_L = \gamma(\Delta_R + \Delta_\alpha)$ (see Section A.2.3). Since we are focusing on the case with $\beta_L > \gamma(\Delta_R + \Delta_\alpha)$, we already know that $\pi^L(0) > \pi^L(-1)$ (see equation A.4) and therefore $l(-1) = 0$. Hence, we only need to look for equilibria in which only $l(0)$ and $l(1)$ are interior. Indifference between $s = 0$ and $s = 1$ implies

$$\pi^L(1) - \pi^L(0) = -\frac{1}{3}(\Delta_R + \Delta_\alpha)[\gamma + q(1) + q(2)] + \frac{1}{3}\beta_L = 0. \tag{A.5}$$

By Bayesian updating, $q(1) = \gamma$ and $q(2) = \frac{\gamma}{\gamma+(1-\gamma)l(1)}$, which can be plugged into (A.5) to get

$$l(1) = \frac{\gamma}{1-\gamma}\left(\frac{(\Delta_R + \Delta_\alpha)(1+2\gamma) - \beta_L}{\beta_L - 2\gamma(\Delta_R + \Delta_\alpha)}\right) \equiv \xi. \tag{A.6}$$

If $\xi \in (0,1)$ we have found an $IB$ equilibrium with $l(1) = \xi$ and $l(0) = 1 - \xi$. For $\xi > 0$ we need $(1 + 2\gamma)(\Delta_R + \Delta_\alpha) - \beta_L > 0$ and $\beta_L - 2\gamma(\Delta_R + \Delta_\alpha) > 0$, which requires $2\gamma(\Delta + \Delta_\alpha) < \beta_L < (2\gamma+1)(\Delta_R + \Delta_\alpha)$. Given this condition, one can verify that $\xi < 1$ if and only if $\beta_L > 3\gamma(\Delta_R + \Delta_\alpha)$. Therefore, there is an $IB$ equilibrium if and only if $3\gamma(\Delta_R + \Delta_\alpha) < \beta_L < (2\gamma + 1)(\Delta_R + \Delta_\alpha)$. In that case, $l(1)$ is given by (A.6).

### A.2.7 Summary of equilibrium set when $\gamma > \overline{\gamma}$

Next proposition summarizes the results of Section A.2. Define the following boundaries $\delta_1 \equiv \gamma(\Delta_R + \Delta_\alpha)$, $\delta_2 \equiv 3\gamma(\Delta_R + \Delta_\alpha)$, $\delta_3 \equiv (1 + 2\gamma)(\Delta_R + \Delta_\alpha)$. Note that $\delta_3 > \delta_2 > \delta_1$.

**Proposition A.1.** *Suppose $\gamma > \overline{\gamma}$. In any equilibrium, $h(1) = 1$, $g(2) = g(1) = g(0) = 1$, $g(-1) = g(-2) = 0$, $q(0) = \gamma$ and $q(-1) = q(-2) = 0$. The negatively informed trader's strategy is as follows:*

1. *If $\beta_L < \delta_1$ there is an $S$ equilibrium and it is the unique equilibrium.*
2. *If $\beta_L = \delta_1$ the equilibrium set consists of an $S$ equilibrium, an $I$ equilibrium, and a continuum of $SI$ equilibria with any $l(0) \in (0,1)$ and $l(-1) = 1 - l(0)$.*
3. *If $\delta_1 < \beta_L < \delta_2$ there is an $I$ equilibrium and it is the unique equilibrium.*
4. *If $\beta_L = \delta_2$ the equilibrium set consists of an $I$ equilibrium and a $B$ equilibrium.*
5. *If $\delta_2 < \beta_L < \delta_3$ the equilibrium set consists of an $I$ equilibrium, a $B$ equilibrium, and an $IB$ equilibrium in which $l(1)$ is given by (A.6).*
6. *When $\beta_L = \delta_3$ the equilibrium set consists of an $I$ equilibrium and a $B$ equilibrium.*
7. *When $\beta_L > \delta_3$ there is a $B$ equilibrium and it is the unique equilibrium.*

Beliefs $q(2)$ and $q(1)$ are omitted in Proposition A.1, but can be easily computed by Bayes rule.

## A.3 Equilibrium characterization when $\gamma < \overline{\gamma}$

We start establishing the following result that reduces the set of possible equilibria for $\gamma < \overline{\gamma}$.

**Lemma A.2.** *Assume $\gamma < \overline{\gamma}$. Then, there is no $B$, $I$ nor $BI$ equilibrium.*

*Proof.* By Lemma 1, $h(1) = 1$ in any equilibrium. First, suppose $l(1) = 1$. Then, $q(2) = q(1) = q(0) = \gamma$ and there is no intervention on the equilibrium path. But then, since the low type is making a trading loss, she would deviate to $s = 0$ or $s = -1$. Second, suppose $l(0) = 1$. Then, $q(2) = 1$, $q(1) = q(0) = \gamma$ and $q(-1) = 0$, and there is no intervention when $X \in \{-1, 0, 1\}$, so the low type has zero payoff. Therefore, she would deviate to $s = -1$ and make a trading profit when $X = 0$ at least. Third, suppose $l(0) = 1 - l(1) > 0$. In this case, $q(1) = q(0) = \gamma < \overline{\gamma}$ and $q(-1) = 0$, which implies that $g(-1) = g(0) = g(1) = 0$. But then the low type would deviate to a strategy with $l(0) = 0$ and $l(-1) > 0$, since $s = 0$ yields zero payoff, while $s = -1$ yields some trading profit when $X = 0$. $\square$

### A.3.1 Beliefs and equilibrium strategies for policymaker

The next lemma is analogous to Lemma A.1 for the case with $\gamma < \overline{\gamma}$.

**Lemma A.3.** *Assume $\gamma < \overline{\gamma}$. In any equilibrium, $q(2) \geq \gamma$, $q(1) \geq \gamma$, $q(0) = \gamma$, $q(-1) = q(-2) = 0$, and $g(0) = g(-1) = g(-2) = 0$.*

*Proof.* The proof that $q(2) \geq \gamma$, $q(1) \geq \gamma$, $q(0) = \gamma$ is identical to the proof in Lemma A.1, since we only use $h(1)$ to show those relations. The fact that $g(0) = 0$ follows from $q(0) = \gamma < \overline{\gamma}$. Finally, by Lemma A.2, $l(-1) > 0$ in any equilibrium. Hence, since $h(1) = 1$ (Lemma 1), $q(-1) = q(-2) = 0$ and $g(-1) = g(-2) = 0$. $\square$

In the remaining of Section A.3, we assume that $h(1) = 1$ (by Lemma 1) and that $q(\cdot)$ and $g(\cdot)$ satisfy the conditions in Lemma A.3. Note that in any equilibrium candidate that satisfies the conditions in Lemma A.3 the high type has no incentives to deviate from $h(1) = 1$. Hence, to verify if a strategy profile and beliefs satisfying Lemma A.3 constitute an equilibrium we only need to check if the low type has no incentives to deviate.

### A.3.2 Sell equilibrium ($S$)

Suppose $l(-1) = 1$. Then, since $h(1) = 1$, $q(2) = q(1) = 1$ and $g(2) = g(1) = 1$. Hence, $\pi^L(-1) = \frac{1}{3}\gamma\Delta_R$, $\pi^L(0) = \frac{1}{3}\beta_L$ and $\pi^L(1) = -\frac{2}{3}(\Delta_R + \Delta_\alpha) - \frac{1}{3}\gamma\Delta_R + \frac{2}{3}\beta_L$. For the low type not to deviate to $s = 0$ we need $\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma\Delta_R - \frac{1}{3}\beta_L \geq 0$, which is equivalent to $\beta_L \leq \gamma\Delta_R$. A deviation to $s = 1$ is not profitable if $\pi^L(-1) - \pi^L(1) = \frac{1}{3}\gamma\Delta_R + \frac{2}{3}(\Delta_R + \Delta_\alpha) + \frac{1}{3}\gamma\Delta_R - \frac{2}{3}\beta_L \geq 0$, which is equivalent to $\beta_L \leq (\Delta_\alpha + \Delta_R) + \gamma\Delta_R$. Hence, since $\Delta_\alpha + \Delta_R > 0$, an $S$ equilibrium exists if and only if $\beta_L \leq \gamma\Delta_R$.

### A.3.3 Sell-Inaction equilibrium ($SI$)

Suppose $l(0) = 1 - l(-1) > 0$. Since $h(1) = 1$, $q(2) = 1$, $g(2) = 1$ and

$$q(1) = \frac{\gamma}{\gamma + (1-\gamma)\, l(0)}. \tag{A.7}$$

Indifference between $s = 0$ and $s = -1$ for the low type requires $\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma\Delta_R - \frac{1}{3}g(1)\beta_L = 0$, which implies that

$$g\,(1) = \frac{\gamma\Delta_R}{\beta_L}. \tag{A.8}$$

Therefore, a necessary condition for such an equilibrium to exist is $\beta_L \geq \gamma\Delta_R$.

First, consider $\beta_L > \gamma\Delta_R$, in which case $g(1) \in (0,1)$. Indifference for the policy maker requires $q(1) = \frac{\gamma}{\gamma+(1-\gamma)l(0)} = \overline{\gamma}$, and therefore

$$l(0) = \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma} < 1. \tag{A.9}$$

Therefore, the payoff for the low type of deviating to $s = 1$ is

$$\begin{aligned}
\pi^L(1) &= -\frac{1}{3}\left(\Delta_R + \Delta_\alpha\right) - \frac{1}{3}\gamma\Delta_R - \frac{1}{3}\overline{\gamma}\left[\Delta_R + g\,(1)\,\Delta_\alpha\right] + \frac{1}{3}\left[1 + g\,(1)\right]\beta_L \\
&= -\frac{1}{3}\left(\Delta_R + \Delta_\alpha\right) - \frac{1}{3}\gamma\Delta_R - \frac{1}{3}\overline{\gamma}\left[\Delta_R + \frac{\gamma\Delta_R}{\beta_L}\Delta_\alpha\right] + \frac{1}{3}\left(1 + \frac{\gamma\Delta_R}{\beta_L}\right)\beta_L.
\end{aligned}$$

Also note that $\pi^L(0) = \pi^L(-1) = \frac{1}{3}\gamma\Delta_R$ . For the low type not to deviate to $s = 1$ we need $\pi^L(-1) \geq \pi^L(1)$, which yields

$$\beta_L^2 - \beta_L\left[\Delta_\alpha + \Delta_R\left(1 + \gamma + \overline{\gamma}\right)\right] - \gamma\overline{\gamma}\Delta_R\Delta_\alpha \leq 0. \tag{A.10}$$

Using $\Delta_R + \Delta_\alpha > 0$ and doing some algebra, one can verify that (A.10) is satisfied with strict inequality for $\beta_L = \gamma\Delta_R$. Hence, we only need to ensure that $\beta_L \leq r_1$, where $r_1$ is the largest root of the LHS of (A.10), given by

$$r_1 = \frac{\left(1+\gamma+\overline{\gamma}\right)\Delta_R + \Delta_\alpha + \sqrt{\left[\left(1+\gamma+\overline{\gamma}\right)\Delta_R + \Delta_\alpha\right]^2 + 4\overline{\gamma}\gamma\Delta_R\Delta_\alpha}}{2}. \tag{A.11}$$

Note that $r_1$ is a positive real number and $r_1 > \gamma\Delta_R$, since (A.10) is satisfied with inequality for $\beta_L = \gamma\Delta_R$. Hence, when $\gamma\Delta_R < \beta_L \leq r_1$ there is an $SI$ equilibrium, with $l(0)$ given by (A.9), $g(1)$ given by (A.8), $q(1) = \overline{\gamma}$, $q(2) = 1$ and $g(2) = 1$.

Now consider $\beta_L = \gamma\Delta_R$. In an $SI$ equilibrium we must have $g(1) = 1$ (see (A.8)) and hence $q(1) = \frac{\gamma}{\gamma+(1-\gamma)l(0)} \geq \overline{\gamma}$, which implies that $l(0) \leq \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}$. Therefore, if $\beta_L = \gamma\Delta_R$, any $l(0) \in \left(0, \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}\right]$ and $l(-1) = 1 - l(0)$ constitute an $SI$ equilibrium. To conclude, we have shown that an $SI$ equilibrium exists if and only if $\gamma\Delta_R \leq \beta_L \leq r_1$.

### A.3.4 Sell-Buy equilibrium ($SB$)

Suppose $l(-1) = 1 - l(1) > 0$. Since $h(1) = 1$, we have

$$q(1) = q(2) = \frac{\gamma}{\gamma + (1-\gamma)\, l(1)}. \tag{A.12}$$

Note we cannot have $g(1) = g(2) = 0$, since that would be inconsistent with the choice of $l(1) > 0$. Hence, it must be that $q(1) = q(2) \geq \overline{\gamma}$ (so that $g(2) > 0$ and/or $g(1) > 0$). In what follows we first search for $SB$ equilibria with $q(1) = q(2) = \overline{\gamma}$ and then we consider equilibria with $q(1) = q(2) > \gamma$.

**Case 1.** Suppose $q(1) = q(2) = \overline{\gamma}$. Then, (A.12) implies $l(1) = \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma} < 1$. The indifference condition between $s = 1$ and $s = -1$ for the low type, after some rearranging, implies

$$g\,(1) + g\,(2) = \frac{2(\gamma + \overline{\gamma})\Delta_R}{\beta_L - \overline{\gamma}\Delta_\alpha}. \tag{A.13}$$

For the low type not to be willing to deviate to $s = 0$ we need $\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma\Delta_R - \frac{1}{3}g(1)\beta_L \geq 0$, implying

$$g(1) \leq \frac{\gamma\Delta_R}{\beta_L} \equiv \mathcal{U}_1. \tag{A.14}$$

We need to guarantee that there exists a $g(2) \in [0,1]$ implied by (A.13), for a given $g(1) \leq \mathcal{U}_1$. Using (A.13), we need to check that $g\,(2) = \frac{2(\gamma+\overline{\gamma})\Delta_R}{\beta_L - \overline{\gamma}\Delta_\alpha} - g(1) \in [0,1]$. This imposes the following additional bounds on $g(1)$:

$$g(1) \leq \frac{2(\gamma + \overline{\gamma})\Delta_R}{\beta_L - \overline{\gamma}\Delta_\alpha} \equiv \mathcal{U}_2 \quad \text{and} \quad g(1) \geq \mathcal{U}_2 - 1 \equiv \mathcal{L}_1. \tag{A.15}$$

For a $g(1) \in [0,1]$ satisfying (A.14) and (A.15) to exist we need: $\mathcal{U}_1 \geq 0$, $\mathcal{U}_2 \geq 0$, $\mathcal{L}_1 \leq 1$ and $\mathcal{L}_1 \leq \min\{\mathcal{U}_1, \mathcal{U}_2\}$. $\mathcal{U}_1$ is clearly larger than zero. $\mathcal{U}_2 \geq 0$ whenever $\beta_L \geq \overline{\gamma}\Delta_\alpha$, so we assume that is the case from now on in Case 1. One can verify that $\mathcal{L}_1 \leq 1$ whenever

$$\beta_L \geq \gamma\Delta_R + \overline{\gamma}\left(\Delta_R + \Delta_\alpha\right). \tag{A.16}$$

Note that $\mathcal{U}_2 \geq \mathcal{U}_1$ whenever (A.16) holds. Thus, it remains to check whether $\mathcal{L}_1 \leq \mathcal{U}_1$, which is equivalent to

$$\beta_L^2 - \left[(2\overline{\gamma} + \gamma)\Delta_R + \overline{\gamma}\Delta_\alpha\right]\beta_L - \gamma\overline{\gamma}\Delta_\alpha\Delta_R \geq 0 \tag{A.17}$$

Using $\Delta_R + \Delta_\alpha > 0$, with some algebra one can see that (A.17) is violated for $\beta_L = \gamma\Delta_R + \overline{\gamma}\left(\Delta_R + \Delta_\alpha\right)$. This implies that the equilibria we have been looking for exists if and only if $\beta_L \geq r_2$, where $r_2$ is the largest root of the LHS of (A.17), which is given by

$$r_2 = \frac{(2\overline{\gamma} + \gamma)\Delta_R + \overline{\gamma}\Delta_\alpha + \sqrt{\left[(2\overline{\gamma} + \gamma)\Delta_R + \overline{\gamma}\Delta_\alpha\right]^2 + 4\gamma\overline{\gamma}\Delta_R\Delta_\alpha}}{2}. \tag{A.18}$$

Note that $r_2$ is a positive real number. We have then shown that an $SB$ equilibrium with $q(1) = q(2) = \overline{\gamma}$ exists if and only if $\beta_L \geq r_2$ . In such an equilibrium, $l(1) = \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}$. Any combination of $g(2)$ and $g(1)$

satisfying $g(1) \in [\mathcal{L}_1, \mathcal{U}_1]$ and (A.13) is consistent with such an equilibrium.

**Case 2.** Now suppose $q(1) = q(2) > \overline{\gamma}$. Then, $g(1) = g(2) = 1$. Using $q(1) = q(2)$, the indifference condition between $s = -1$ and $s = 1$ for the low type, after some rearranging, yields $q(1) = \frac{\beta_L - \gamma\Delta_R}{\Delta_R + \Delta_\alpha}$. A necessary condition for $\overline{\gamma} < q(1) < 1$ is $\gamma\Delta_R + \overline{\gamma}(\Delta_R + \Delta_\alpha) < \beta_L < \gamma\Delta_R + (\Delta_R + \Delta_\alpha)$. For the low type not to deviate to $s = 0$, it must be that $\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma\Delta_R - \frac{1}{3}\beta_L \geq 0$, which is equivalent to $\beta_L \leq \gamma\Delta_R$, contradicting $\overline{\gamma}(\Delta_R + \Delta_\alpha) + \gamma\Delta_R < \beta_L < (\Delta_R + \Delta_\alpha) + \gamma\Delta_R$. Hence, we have shown that an $SB$ equilibrium with $q(1) = q(2) > \overline{\gamma}$ does not exist. Hence, an $SB$ equilibrium exists if and only if $\beta_L \geq r_2$.

### A.3.5 Sell-Inaction-Buy equilibrium ($SIB$)

Suppose that $l(-1), l(0), l(1) > 0$. Since $h(1) = 1$, we have

$$q(1) = \frac{\gamma}{\gamma + (1-\gamma)\left[l(0) + l(1)\right]} \quad \text{and} \quad q(2) = \frac{\gamma}{\gamma + (1-\gamma)\, l(1)}. \tag{A.19}$$

After some rearranging, indifference between $s = 1$ and $s = 0$ for the low type implies

$$\left[q(2) + q(1) + \gamma\right]\Delta_R + \left[q(2)g(2) + q(1)g(1)\right]\Delta_\alpha = g(2)\beta_L \tag{A.20}$$

If $\Delta_\alpha \geq 0$ the LHS of (A.20) is clearly strictly larger than zero. If $\Delta_\alpha < 0$ one can see that it is also strictly larger than zero since $\Delta_R + \Delta_\alpha \geq 0$. Hence, if $g(2) = 0$, (A.20) cannot be satisfied and therefore if there is an $SIB$ equilibrium it must be that $g(2) > 0$ and $q(2) \geq \overline{\gamma}$. Indifference between $s = 0$ and $s = -1$ for the low type implies $\pi^L(-1) - \pi^L(0) = \frac{1}{3}\gamma\Delta_R - \frac{1}{3}g(1)\beta_L = 0$. Therefore:

$$g(1) = \frac{\gamma\Delta_R}{\beta_L}. \tag{A.21}$$

Hence, a necessary condition for such an equilibrium to exist is $\beta_L \geq \gamma\Delta_R$. Suppose such an $SIB$ equilibrium exists for $\beta_L = \gamma\Delta_R$. Then $g(1) = 1$, $q(1) \geq \overline{\gamma}$, $q(2) > \overline{\gamma}$ (since by (A.19) $q(2) > q(1)$), and $g(2) = 1$. But then indifference condition (A.20) gives us the following contradiction: $\beta_L = \gamma\Delta_R + \left[q(2) + q(1)\right](\Delta_\alpha + \Delta_R) > \gamma\Delta_R$. Hence, another necessary condition for an $SIB$ equilibrium to exist is $\beta_L > \gamma\Delta_R$.

Suppose then $\beta_L > \gamma\Delta_R$. In this case, (A.21) implies $g(1)$ is interior and therefore $q(1) = \overline{\gamma}$. Since $q(2) > q(1) = \overline{\gamma}$, we must have $g(2) = 1$. Replacing those equalities, (A.19) and (A.21) in (A.20) we get

$$\beta_L \frac{\gamma}{\gamma + (1-\gamma)\, l(1)} (\Delta_R + \Delta_\alpha) = \beta_L^2 - (\overline{\gamma} + \gamma)\,\Delta_R\beta_L - \overline{\gamma}\gamma\Delta_R\Delta_\alpha. \tag{A.22}$$

Note that the LHS is strictly positive. Hence, we need the RHS the be positive. One can verify that when $\beta_L = \gamma\Delta_R$, the RHS is negative. Hence, we assume in what follows that $\beta_L > r_3$ where $r_3$ is the largest root of the RHS of (A.22), given by

$$r_3 = \frac{(\overline{\gamma} + \gamma)\,\Delta_R + \sqrt{[(\overline{\gamma} + \gamma)\,\Delta_R]^2 + 4\overline{\gamma}\gamma\Delta_R\Delta_\alpha}}{2}.$$

Solving (A.22) for $l(1)$:

$$l(1) = -\frac{\gamma}{1-\gamma}\left\{\frac{\beta_L^2 - \beta_L\left[\Delta_R + \Delta_\alpha + (\gamma + \overline{\gamma})\,\Delta_R\right] - \overline{\gamma}\gamma\Delta_R\Delta_\alpha}{\beta_L^2 - (\gamma + \overline{\gamma})\,\Delta_R\beta_L - \overline{\gamma}\gamma\Delta_R\Delta_\alpha}\right\}. \tag{A.23}$$

The denominator of the term in braces is positive since $\beta_L > r_3$. Using $q(1) = \overline{\gamma}$ and (A.19) we can solve for $l(0)$ as a function of $l(1)$:

$$l(0) = \frac{\gamma}{1-\gamma}\frac{1-\overline{\gamma}}{\overline{\gamma}} - l(1). \tag{A.24}$$

Note that (A.24) implies $l(0) + l(1) < 1$ and then $l(-1) > 0$. Since we need $l(0) > 0$, (A.24) requires that $l(1) < \frac{\gamma}{1-\gamma}\frac{1-\overline{\gamma}}{\overline{\gamma}}$. Therefore, we need to check under which parameters $l(1)$ given by (A.23) is contained in $\left(0, \frac{\gamma}{1-\gamma}\frac{1-\overline{\gamma}}{\overline{\gamma}}\right)$. After some rearranging $l(1) < \frac{\gamma}{1-\gamma}\frac{1-\overline{\gamma}}{\overline{\gamma}}$ implies

$$\beta_L^2 - \left[(\overline{\gamma} + \gamma)\,\Delta_R + \overline{\gamma}\,(\Delta_R + \Delta_\alpha)\right]\beta_L - \overline{\gamma}\gamma\Delta_R\Delta_\alpha > 0 \tag{A.25}$$

Notice that the LHS of (A.25) is the same as the LHS of (A.17). Also note that when $\beta_L = r_3$, (A.25) is violated. Then, we only need to check if $\beta_L > r_2$, where $r_2$ is the largest root of the LHS of (A.25), given by (A.18). Thus, we further limit $\beta_L$, assuming $\beta_L > r_2$ in what follows. Since $\beta_L > r_2 > r_3$, $l(1) > 0$ if and only if the numerator of the term in braces in (A.23) is strictly smaller than zero:

$$\beta_L^2 - \left[(\gamma + \overline{\gamma})\,\Delta_R + \Delta_R + \Delta_\alpha\right]\beta_L - \overline{\gamma}\gamma\Delta_R\Delta_\alpha < 0. \tag{A.26}$$

Note that the LHS of (A.26) is the same as the LHS of (A.10). Also, notice that when $\beta_L = r_2$, (A.26) is satisfied. Hence, we need to ensure that $\beta_L < r_1$, where $r_1$ is the largest root of (A.26), given by (A.11). Note that $r_1 > r_2$, since (A.26) is satisfied for $\beta_L = r_2$.

Therefore, we have shown that there is an $SIB$ equilibrium if and only if $r_2 < \beta_L < r_1$ (where $r_2$ and $r_1$ are given by (A.18) and (A.11)). In this equilibrium, $l(1)$ is given by (A.23) and $l(0)$ is given by (A.24). The government plays $g(2) = 1$, $g(1)$ is interior and given by (A.21). Moreover, $q(1) = \overline{\gamma}$ and $q(2)$ is obtained by combining (A.19) and (A.23).

### A.3.6 Summary of equilibrium set when $\gamma < \overline{\gamma}$

Before we summarize the results, it useful to define: $\varphi_1 = \gamma\Delta_R$, $\varphi_2 = r_2$ (where $r_2$ is given by (A.18)), and $\varphi_3 = r_1$ (where $r_1$ is given by (A.11)). The next lemma establishes some relations between those variables.

**Lemma A.4.** *For any parameters we have $\varphi_3 > \varphi_2 > \varphi_1$.*

*Proof.* That $\varphi_3 > \varphi_2$ was established in Section A.3.5. To check that $\varphi_2 > \varphi_1$, we replace $\beta_L = \varphi_1$ in

(A.17), which after some rearranging yields $\Delta_R + \Delta_\alpha \leq 0$, which is violated by assumption. Since $\varphi_2$ is by the definition the largest root of the LHS of (A.17), it must be that $\varphi_2 > \varphi_1$. $\square$

Note that the lemma above implies the $S$ equilibrium is unique when $\beta_L < \varphi_1$. Next proposition summarizes the characterization for $\gamma < \overline{\gamma}$.

**Proposition A.2.** *Suppose $\gamma < \overline{\gamma}$. Then, in any equilibrium we have $h(1) = 1$, $g(0) = g(-1) = g(-2) = 0$, $q(0) = \gamma$, and $q(-1) = q(-2) = 0$. Moreover:*

- *An $S$ equilibrium exists if and only if $\beta_L \leq \varphi_1$, and it is the unique equilibrium when $\beta_L < \varphi_1$. In that equilibrium, $g(2) = g(1) = 1$ and $q(2) = q(1) = 1$.*

- *An $SI$ equilibrium exists if and only if $\varphi_1 \leq \beta_L \leq \varphi_3$. In those equilibria, $g(2) = 1$, $g(1) = \varphi_1/\beta_L$, $q(2) = 1$. If $\beta_L > \varphi_1$ then we have $l(0)$ given by (A.9) and $q(1) = \overline{\gamma}$. If $\beta_L = \varphi_1$ then any $l(0) \in \left(0, \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}\right]$ is consistent with such an equilibrium and $q(1)$ is given by (A.7).*

- *An $SB$ equilibrium exists if and only if $\beta_L > \varphi_2$. In those equilibria, $q(1) = q(2) = \overline{\gamma}$, $l(1) = \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}$ and any combination of $g(1)$ and $g(2)$ satisfying $g(1) \in [\mathcal{L}_1, \mathcal{U}_1]$ and (A.13) are consistent with equilibrium (with $\mathcal{L}_1$ and $\mathcal{U}_1$ given by (A.14) and (A.15)).*

- *An $SIB$ equilibrium exists if and only if $\varphi_2 < \beta_L < \varphi_3$. In this equilibrium, $l(1)$ is given by (A.23) and $l(0)$ is given by (A.24). Moreover, $q(1) = \overline{\gamma}$, $q(2)$ is given by (A.19), $g(2) = 1$ and $g(1) = \varphi_1/\beta_L$.*

## A.4 Efficiency of intervention

Next proposition characterizes the ranking of equilibrium according to the government payoff.

**Proposition A.3.** *Fix $b$, $c$ and $\gamma$. For a given equilibrium $E \in \{B, I, S, IB, SB, SI, SIB\}$ described in Propositions A.1 and A.2, let $U_G^E$ denote the ex-ante government payoff in equilibrium $E$ for any arbitrary set of parameters such that equilibrium $E$ exists. Suppose $\gamma > \overline{\gamma}$. Then,*

$$U_G^S > U_G^{SI} > U_G^I > U_G^{IB} > U_G^B.$$

*Now suppose $\gamma < \overline{\gamma}$. Then,*

$$U_G^S > U_G^{SI} > U_G^{SIB} > U_G^{SB}.$$

*Proof.* Let $\Pr(\cdot)$ denote the probability of a given event taking as given some (equilibrium) strategy profile and agents' prior belief about the state. To ease the notation, we omit the strategy profile as an argument of $\Pr(\cdot)$. The ex-ante government payoff is given by (4).

**Part 1.** We start by analyzing the case with $\gamma > \overline{\gamma}$. In that case, from Proposition A.1, in any equilibrium $\Pr(G = 1|\theta = H) = 1$. Hence, the government expected payoff is larger the lower $\Pr(G = 1|\theta = L)$ in equilibrium. Since $g(-1) = g(-2) = 0$ and $g(1) = g(2) = g(0) = 1$ in any equilibrium, we have that $\Pr(G=1|\theta=L) = \Pr(X=2|\theta=L) + \Pr(X=1|\theta=L) + \Pr(X=0|\theta=L)$. Therefore, under an $I$ equilibrium, $\Pr(G=1|\theta=L) = 2/3$. Under a $B$ equilibrium, $\Pr(G=1|\theta=L) = 1$. Under an $S$ equilibrium, $\Pr(G=1|\theta=L) = 1/3$. Under an $IB$ equilibrium, $\Pr(G=1|\theta=L) \in (2/3, 1)$, and under an $SI$ equilibrium, $\Pr(G=1|\theta=L) \in (1/3, 2/3)$. This yields the desired result.

**Part 2.** Now assume $\gamma < \overline{\gamma}$. From Proposition A.2, in any equilibrium $g(0) = g(-1) = g(-2) = 0$. Hence, under any equilibrium strategy profile we can write:

$$U_G = \Pr(X=2)\{g(2)[q(2)b - c]\} + \Pr(X=1)\{g(1)[q(1)b - c]\}. \tag{A.27}$$

Note that the first (second) term in braces denote the government expected payoff conditional on observing $X=2$ ($X=1$). Hence, whenever the government is indifferent between intervening or not for a given $X \in \{1,2\}$, the associated term in braces is equal to zero.

First, we show the government always prefers the $S$ equilibrium over any other equilibria. Under the $S$ equilibrium $\Pr(G=1|\theta=L) = 0$ and $\Pr(G=1|\theta=H) = 2/3$ (which is its maximum possible value given that in all equilibria $g(0)=0$). Since in all equilibria with $l(0) > 0$ we have $g(1) > 0$, those equilibria have $\Pr(G=1|\theta=L) > 0$. In any $SB$ equilibrium we have $g(1) + g(2) > 0$ and therefore $\Pr(G=1|\theta=L) > 0$ as well. Using (4), we have then shown that the government *strictly* prefers the $S$ equilibrium to any other equilibria. Therefore, in what follows we focus on the case of parameters where the $S$ equilibrium does not exist, assuming $\beta_L > \varphi_1$ hereafter.

Second, we show that the $SI$ equilibrium is preferred over the $SIB$ equilibrium. Note that under the $SI$ equilibrium with $\beta_L > \varphi_1$ the government is indifferent between intervening or not when $X=1$. Hence, using (A.27) we can write $U_G^{SI} = \frac{1}{3}\gamma(b-c) > 0$. In the $SIB$ equilibrium, the government is indifferent between intervening or not when $X=1$. Also, $g(2)=1$ and $q(2) = \frac{\gamma}{\gamma+(1-\gamma)l(1)}$, which implies that $U_G^{SIB} = \frac{1}{3}[\gamma(b-c) - (1-\gamma)l(1)c] < \frac{1}{3}\gamma(b-c) = U_G^{SI}$.

Finally, as shown in Section A.3.5, in the $SIB$ equilibrium $l(1) < \frac{\gamma}{1-\gamma}\frac{1-\overline{\gamma}}{\overline{\gamma}}$, which implies that $U_G^{SIB} > 0$. From Proposition A.1, whenever a $SB$ equilibrium exists, the government is indifferent between intervening or not when $X \in \{1,2\}$, which yields a payoff $U_G^{SB} = 0 < U_G^{SIB}$. □

# B Proofs and technical details

Proposition 1 follows immediately from Propositions A.1 and A.2, and Proposition 2 follows from Propositions A.1, A.2, and A.3 in Appendix A if one defines $\underline{\beta} = \delta_1$, $\overline{\beta} = \delta_3$, $\underset{\sim}{\beta} = \varphi_1$, and $\tilde{\beta} = \varphi_3$. Also, Lemma 1 is proved in Appendix A.1. The remaining results are proved in this section.

## B.1 Informativeness measure

Here we show that the expected learning rate in (5) can be written as in (6). Fix any strategy profile. Notice that $\mathbb{E}[q(X)|\theta=L] = \sum_{X\in\mathcal{X}} \Pr(X|\theta=L)q(X)$ and $\mathbb{E}[q(X)|\theta=H] = \sum_{X\in\mathcal{X}} \Pr(X|\theta=H)q(X)$. Using Bayes' rule, we have that $\Pr(X|\theta=L) = \frac{\Pr(\theta=L|X)\Pr(X)}{\Pr(\theta=L)}$ and $\Pr(X|\theta=H) = \frac{\Pr(\theta=H|X)\Pr(X)}{\Pr(\theta=H)}$.

Moreover, we know that $\Pr(\theta = L) = 1-\gamma$ , $\Pr(\theta = H) = \gamma$ and $\Pr(\theta = L|X) = 1 - \Pr(\theta = H|X)$. Hence, $\mathbb{E}\left[q(X)|\theta = L\right] = \sum_{X\in\mathcal{X}} \frac{[1-\Pr(\theta=H|X)]\Pr(X)}{1-\gamma} q(X)$, which can be written as

$$\mathbb{E}\left[q(X)|\theta = L\right] = \frac{1}{1-\gamma}\sum_{X\in\mathcal{X}} \Pr(X)q(X) - \frac{1}{1-\gamma}\sum_{X\in\mathcal{X}} \Pr(\theta = H|X)\Pr(X)q(X),$$

where the last sum is equal to $\gamma\mathbb{E}\left[q(X)|\theta = H\right]$. Finally, using the law of iterated expectations, $\mathbb{E}\left[q(X)\right] = \sum_{X\in\mathcal{X}} \Pr(X)q(X) = \gamma$. Thus, $\mathbb{E}\left[q(X)|\theta = L\right] = \frac{\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma}\mathbb{E}\left[q(X)|\theta = H\right]$, which after some rearranging yields

$$\frac{\mathbb{E}\left[q(X)|\theta = H\right] - \gamma}{\gamma} = \rho\frac{1 - \mathbb{E}\left[q(X)|\theta = L\right] - (1-\gamma)}{1-\gamma}, \tag{B.1}$$

where $\rho = (1-\gamma)^2/\gamma^2 > 0$. The definition of informativeness in (5) plus (B.1) yield $\iota = \frac{\mathbb{E}(q(X)|\theta=H)-\gamma}{1-\gamma}$.

## B.2 Proof of Lemma 2

The first three properties follow immediately from the definition of $\iota$. We now prove the last claim. In what follows, we refer to the aggregate orders observed under the strategy profile of some equilibrium $E$ by $X^E$. Whenever a signal $x$ dominates a signal $x'$ in the sense of Blackwell and is not dominated by $x'$, we write $x \succ_B x'$. Using Blackwell's (1951) theorem, we can verify whether $X^E$ Blackwell-dominates $X^{E'}$ by checking whether $X^{E'}$ is a garbling of $X^E$ (see also de Oliveira, 2018). Also, we denote by $\mathbb{E}\left[q\left(X\right)|\theta = H\right]^E$ and $\iota^E$ the expectation of $q\left(X\right)$ conditional on $\theta = H$ and informativeness when $E$ is played, respectively. Throughout the proof, we make use of the equilibrium characterization in Proposition 2 and Appendix A.

Consider first $\gamma > \overline{\gamma}$. In the $S$ equilibrium, the joint pdf of $X$ and $\theta$ can be represented by the following table:

| | $X=-2$ | $X=-1$ | $X=0$ | $X=1$ | $X=2$ |
|---|---|---|---|---|---|
| $\theta=L$ | $(1-\gamma)/3$ | $(1-\gamma)/3$ | $(1-\gamma)/3$ | 0 | 0 |
| $\theta=H$ | 0 | 0 | $\gamma/3$ | $\gamma/3$ | $\gamma/3$ |

In the $I$ equilibrium the joint pdf is represented by:

| | $X=-2$ | $X=-1$ | $X=0$ | $X=1$ | $X=2$ |
|---|---|---|---|---|---|
| $\theta=L$ | 0 | $(1-\gamma)/3$ | $(1-\gamma)/3$ | $(1-\gamma)/3$ | 0 |
| $\theta=H$ | 0 | 0 | $\gamma/3$ | $\gamma/3$ | $\gamma/3$ |

Hence, a decision maker observing $X^S$ can recover a signal with distribution identical to $X^I$ by applying the following map:

$$X \mapsto \begin{cases} X & \text{if } X \in \{-1,0,1,2\}, \\ 1 & \text{if } X = -2. \end{cases}$$

Therefore $X^I$ is a garbling of $X^S$ (and the opposite is not true since, by Proposition A.3, the decision maker in our model strictly prefers $S$ to $I$, which would contradict the Blackwell theorem if $X^S$ were a garbling of $X^I$). Now consider the joint pdf in the $B$ equilibrium:

| | $X=-2$ | $X=-1$ | $X=0$ | $X=1$ | $X=2$ |
|---|---|---|---|---|---|
| $\theta=L$ | 0 | 0 | $(1-\gamma)/3$ | $(1-\gamma)/3$ | $(1-\gamma)/3$ |
| $\theta=H$ | 0 | 0 | $\gamma/3$ | $\gamma/3$ | $\gamma/3$ |

A decision maker observing $X^I$ can recover a signal with distribution identical to $X^B$ by applying the following map:

$$X \mapsto \begin{cases} X & \text{if } X \in \{-2,0,1,2\}, \\ 2 & \text{if } X=-1. \end{cases}$$

Hence, $X^B$ is a garbling of $X^I$ (and the opposite is not true, again by Proposition A.3). We have then shown that $X^S \succ_B X^I \succ_B X^B$. It remains to show that $\iota^S > \iota^I > \iota^B$.

Given the equilibrium characterization, one can easily compute $\mathbb{E}\left[q(X)\,|\theta=H\right]^S = \frac{2+\gamma}{3}$, $\mathbb{E}\left[q(X)\,|\theta=H\right]^I = \frac{1+2\gamma}{3}$, $\mathbb{E}\left[q(X)\,|\theta=H\right]^B = \gamma$. Using the expression for $\iota$ in (6),

$$\iota^S = \frac{2}{3} > \iota^I = \frac{1}{3} > \iota^B = 0. \tag{B.2}$$

This concludes the proof for $\gamma > \overline{\gamma}$.

Now assume $\gamma < \overline{\gamma}$. The joint pdf for the $S$ equilibrium is the same as in the case with $\gamma > \overline{\gamma}$. The joint pdf for the $SI$ equilibrium is:

| | $X=-2$ | $X=-1$ | $X=0$ | $X=1$ | $X=2$ |
|---|---|---|---|---|---|
| $\theta=L$ | $l_{SI}(-1)\frac{(1-\gamma)}{3}$ | $(1-\gamma)/3$ | $(1-\gamma)/3$ | $l_{SI}(0)\frac{(1-\gamma)}{3}$ | 0 |
| $\theta=H$ | 0 | 0 | $\gamma/3$ | $\gamma/3$ | $\gamma/3$ |

where $l_{SI}(0) = \frac{\gamma}{\overline{\gamma}}\frac{1-\overline{\gamma}}{1-\gamma}$ and $l_{SI}(-1) = 1 - l_{SI}(0)$ (see Proposition A.2). Hence, a decision maker observing $X^S$ can recover a signal with distribution identical to $X^{SI}$ by applying the following stochastic map:

$$X \mapsto \begin{cases} X & \text{if } X \in \{-1,0,1,2\}, \\ M & \text{if } X=-2, \end{cases}$$

where $M$ is a random variable that equals $-2$ with probability $l_{SI}(-1)$ and 1 with probability $l_{SI}(0)$. Hence $X^{SI}$ is a garbling of $X^S$ (and the opposite is not true by Proposition A.3). Finally, the joint pdf for the $SB$ equilibrium is

| | $X=-2$ | $X=-1$ | $X=0$ | $X=1$ | $X=2$ |
|---|---|---|---|---|---|
| $\theta=L$ | $l_{SB}(-1)\frac{(1-\gamma)}{3}$ | $l_{SB}(-1)\frac{(1-\gamma)}{3}$ | $(1-\gamma)/3$ | $l_{SB}(1)\frac{(1-\gamma)}{3}$ | $l_{SB}(1)\frac{(1-\gamma)}{3}$ |
| $\theta=H$ | 0 | 0 | $\gamma/3$ | $\gamma/3$ | $\gamma/3$ |

where $l_{SB}(1) = \frac{\gamma}{\overline{\gamma}} \frac{1-\overline{\gamma}}{1-\gamma}$ and $l_{SB}(-1) = 1 - l_{SB}(1)$ (see Proposition A.2). Hence, a decision maker observing $X^{SI}$ can recover a signal with distribution identical to $X^{SB}$ by applying the following stochastic map:

$$X \mapsto \begin{cases} X & \text{if } X \in \{-2, 0, 1, 2\}, \\ N & \text{if } X = -1, \end{cases}$$

where $N$ is a random variable that equals $-1$ with probability $l_{SB}(-1)$ and 2 with probability $l_{SB}(1)$. Hence $X^{SB}$ is a garbling of $X^{SI}$ (and the opposite is not true by Proposition A.3). We have then shown that $X^S \succ_B X^{SI} \succ_B X^{SB}$. It remains to show that $\iota^S > \iota^{SI} > \iota^{SB}$.

Given the equilibrium characterization, one can easily compute $\mathbb{E}[q(X)|\theta = H]^S = \frac{2+\gamma}{3}$, $\mathbb{E}[q(X)|\theta = H]^{SI} = \frac{1+\gamma+\overline{\gamma}}{3}$ and $\mathbb{E}[q(X)|\theta = H]^{SB} = \frac{\gamma+2\overline{\gamma}}{3}$. Using the definition of $\iota$ in (6),

$$\iota^S = \frac{2}{3}, \quad \iota^{SI} = \frac{2}{3} - \frac{1-\overline{\gamma}}{3(1-\gamma)}, \quad \text{and} \quad \iota^{SB} = \frac{2}{3} - \frac{2(1-\overline{\gamma})}{3(1-\gamma)}. \tag{B.3}$$

Hence, $\iota^S > \iota^{SI} > \iota^{SB}$. □

## B.3 Proof of Proposition 3

Using (4), the equilibrium characterization in Propositions A.1 and A.2, and Proposition A.3, we have that the government payoffs in the best equilibria are:

$$U_G^S = \gamma(b-c) - \frac{(1-\gamma)}{3}c > U_G^I = \gamma(b-c) - \frac{2}{3}(1-\gamma)c > U_G^B = \gamma b - c \tag{B.4}$$

if $\gamma > \overline{\gamma}$, and

$$U_G^S = \frac{2}{3}\gamma(b-c) > U_G^{SI} = \frac{\gamma}{3}(b-c) > U_G^{SB} = 0 \tag{B.5}$$

if $\gamma < \overline{\gamma}$. Proposition 3 then follows directly from Lemma 2, (B.2), (B.3) and the fact that, in any equilibrium $E$, the government payoffs $U_G^E$ in (B.4) and (B.5) only directly depend on parameters $\gamma$, $b$ and $c$. □

## B.4 Proof of Proposition 4

Inspection of (B.4) and (B.5) shows that $\beta_L$ only affects the government payoff (in the best equilibrium) through the determination of which equilibrium will be played—that is, through the relative position of $\beta_L$ with respect to $\underline{\beta}$, $\overline{\beta}$, $\underset{\sim}{\beta}$ and $\tilde{\beta}$ as defined in (3)—but within each equilibrium class government payoffs do not depend on $\beta_L$ (and thus on $\mu$). Inspection of B.2 and B.3 shows that the same holds for informativeness in the best equilibria. Denote with the superscript $E$ the value of each variable under equilibrium $E$.

Consider $\gamma > \overline{\gamma}$. Given our focus on the best equilibrium, as $\mu$ increases, the equilibrium eventually switches from a Sell equilibrium ($l(-1) = 1$) to an Inaction equilibrium ($l(0) = 1$), and then to a Buy equilibrium ($l(1) = 1$). Also, from Proposition 3 we know that $\iota^S > \iota^I > \iota^B$ and $U_G^S > U_G^I > U_G^B$. Hence, informativeness and government payoffs are decreasing in $\mu$. Regarding the ex-ante probability of intervention, we have that $\Pr(G=1)^B = 1 > \Pr(G=1)^I = \gamma + \frac{2}{3}(1-\gamma) > \Pr(G=1)^S = \gamma + \frac{1}{3}(1-\gamma)$.

Now consider $\gamma < \overline{\gamma}$. As $\mu$ (and $\beta_L$) increases, we move from an $S$ equilibrium to an $SI$ and then to an $SB$ equilibrium. Hence, informativeness and the government payoff are decreasing in $\mu$ since, from Proposition 3, $\iota^S > \iota^{SI} > \iota^{SB}$ and $U_G^S > U_G^{SI} > U_G^{SB}$. Regarding the ex-ante probability of intervention, one can verify that $\Pr(G=1)^S = \gamma\frac{2}{3}$,

$$\Pr(G=1)^{SI} = \frac{\gamma}{3}\left[1 + \frac{\gamma\Delta_R}{\overline{\gamma}\beta_L}\right], \quad \text{and} \quad \Pr(G=1)^{SB} = \frac{\gamma}{3\overline{\gamma}}\frac{2(\gamma+\overline{\gamma})\Delta_R}{\beta_L - \overline{\gamma}\Delta_\alpha}.$$

Notice that as $\beta_L \to \underset{\sim}{\beta} = \gamma\Delta_R$, $\Pr(G=1)^{SI} > \Pr(G=1)^S$, but $\Pr(G=1)^{SI}$ is strictly decreasing in $\beta_L$ (and thus in $\mu$). This shows the non-monotonicity of the probability of intervention with respect to $\beta_L$: For some $\varepsilon > 0$, when $\beta_L$ increases from $\underset{\sim}{\beta} - \varepsilon$ to $\underset{\sim}{\beta} + \varepsilon$ there is an increase in the expected probability of intervention, but as $\beta_L$ continues to grow in the range where the equilibrium is the $SI$ equilibrium, $\Pr(G=1)^{SI}$ decreases. Moreover, $\Pr(G=1)^{SB}$ can be larger or smaller than $\Pr(G=1)^{SI}$ depending on parameters, but whenever we are in the parameter range where the $SB$ equilibrium is played, the probability of intervention decreases in $\beta_L$ (and thus in $\mu$). $\square$

## B.5 Proof of Proposition 5

Suppose that initially $\gamma < \overline{\gamma}$ and $\beta_L > \tilde{\beta}$, so that the economy is in the $SB$ equilibrium, with welfare equal to $U_G^{SB} = 0$ (see (B.5)). Suppose $c$ increases from $c$ to $c' > c$. Using $\overline{\gamma} = c/b$, $\gamma < \overline{\gamma}$ and $\Delta_R + \Delta_\alpha > 0$, one can verify that the increase in $c$ strictly increases the bound $\tilde{\beta}$, not affecting $\underset{\sim}{\beta}$. Hence, such an increase either maintains the economy in the $SB$ equilibrium, or moves it to the $SI$ equilibrium. In the former case, welfare remains at zero, while in the later it increases to $U_G^{SI} = \frac{\gamma}{3}(b - c') > 0$ (see (B.5)). Informativeness strictly increases in both cases, given (B.3). $\square$

## B.6 Proof of Proposition 6

Consider the setting in which the policymaker offers an initial assistance of $\underline{A}$ dollars before trading takes place. After observing market activity $X$, it can then decide on an additional intervention $a \in [0, 1-\underline{A}]$. We can write the firm's equity value (given $\underline{A}$) as

$$v(\theta, a) = R_\theta + \alpha_\theta a + \alpha_\theta \underline{A},$$

the trader's payoff as

$$\pi = s(v - p) + \beta_\theta a + \beta_\theta \underline{A},$$

and the policymaker's payoff given beliefs $q$ as $(qb - c)(a + \underline{A})$. Hence, after observing $X$, the policymaker wishes to set $a = 1 - \underline{A}$ if $q(X) > \overline{\gamma} = \frac{c}{b}$ and $a = 0$ if $q(X) < \overline{\gamma}$. We can thus map this into a binary intervention $G \in \{0, 1-\underline{A}\}$. Therefore, for the purpose of computing the equilibrium, the model is the same as our main setting up to a parametric change: we can keep considering $G \in \{0,1\}$ and multiply $\alpha_\theta$ and $\beta_\theta$ by $(1-\underline{A})$. Therefore, we only need to consider that setting $\underline{A} = \underline{A}'$ leads to a new private benefit $\beta_L' = \beta_L (1-\underline{A}')$ (recall that $\beta_H$ does not affect the equilibrium), and a new distance between the increase in the firm cash flow caused by the intervention across states, $\Delta_\alpha' \equiv (1-\underline{A})\Delta_\alpha$.

Suppose $\gamma > \overline{\gamma}$. Consider the equilibrium characterization in Proposition 2. Consider $\beta_L \le \underline{\beta}$. Any $\underline{A} \in [0,1]$ keeps the economy in the $S$ equilibrium since

$$\beta_L (1-\underline{A}) \le \gamma \left[\Delta_R + \Delta_\alpha (1-\underline{A})\right] \iff \beta_L \le \gamma \left(\frac{\Delta_R}{1-\underline{A}} + \Delta_\alpha\right),$$

which is satisfied for $\beta_L \le \underline{\beta} = \gamma(\Delta_R + \Delta_\alpha)$. The policymaker's payoff in that equilibrium is

$$U_\dagger^S(\underline{A}) = \left[\gamma(b-c) - \frac{1}{3}(1-\gamma)c\right](1-\underline{A}) + (\gamma b - c)\underline{A}, \tag{B.6}$$

which is strictly decreasing in $\underline{A}$. The policymaker then sets $\underline{A} = 0$. Now suppose $\beta_L \in \left(\underline{\beta}, \overline{\beta}\right]$. If $\underline{A} = 0$, the $I$ equilibrium is played, and the regulator's expected payoff is $U_G^I$, as given by (B.4). If the policymaker sets some

$$\underline{A} \ge 1 - \frac{\gamma \Delta_R}{\beta_L - \gamma\Delta_\alpha} \equiv \underline{A}^S, \tag{B.7}$$

it triggers the Sell equilibrium and obtains $U_\dagger^S(\underline{A})$ in (B.6). Now notice that for any $\underline{A}$ the Buy equilibrium is never played, since

$$(1-\underline{A})\beta_L > (1+2\gamma)(\Delta_R + (1-\underline{A})\Delta_\alpha) \iff \beta_L > \overline{\beta} + \left(\frac{\underline{A}}{1-\underline{A}}\right)(1+2\gamma)\Delta_R$$

is never satisfied. Hence, if the policymaker sets any $0 < \underline{A} < \underline{A}^S$, the economy remains in the $I$ equilibrium, and the policymaker obtains

$$U_\dagger^I(\underline{A}) = \left[\gamma(b-c) - \frac{2}{3}(1-\gamma)c\right](1-\underline{A}) + (\gamma b - c)\underline{A} < U_G^I. \tag{B.8}$$

Hence, since $U_\dagger^S(\underline{A})$ is strictly decreasing, the policymaker either sets $\underline{A} = 0$ or $\underline{A} = \underline{A}^S$. The latter is strictly preferable whenever $U_\dagger^S\left(\underline{A}^S\right) > U_G^I$, which using the fact that $c = \overline{\gamma} b$ simplifies to

$$\beta_L < \gamma(2\Delta_R + \Delta_\alpha) \equiv \underline{\beta}^\dagger < \overline{\beta}. \tag{B.9}$$

For $\beta_L = \underline{\beta}^\dagger$, the policymaker is indifferent between $\underline{A} = 0$ and $\underline{A} = \underline{A}^S$, and for $\beta_L \in \left(\underline{\beta}^\dagger, \overline{\beta}\right]$, the policymaker chooses $\underline{A} = 0$.

Now suppose $\beta_L > \overline{\beta}$. If $\underline{A} = 0$, the $B$ equilibrium is played , and the regulator's expected payoff is $U_G^B$ (see (B.4)). If the policymaker sets $\underline{A} \ge \underline{A}^S$, it triggers the $S$ equilibrium and obtains $U_\dagger^S(\underline{A})$ (which is strictly decreasing in $\underline{A}$). If it sets

$$\underline{A}^{I} \equiv 1 - \frac{(1+2\gamma)\,\Delta_R}{\beta_L - (1+2\gamma)\,\Delta_\alpha} \leq \underline{A} < \underline{A}^{S}, \tag{B.10}$$

it triggers the $I$ equilibrium and obtains $U^{I}_{\dagger}(\underline{A})$, defined above in (B.8). Using $c = \overline{\gamma}b$, one can verify that $U^{I}_{\dagger}(\underline{A})$ is also decreasing, and $U^{I}_{\dagger}\left(\underline{A}^{I}\right) > U^{S}_{\dagger}\left(\underline{A}^{S}\right)$ for any $\beta_L > \overline{\beta}$ and $\Delta_\alpha \geq 0$. Hence, triggering the $S$ equilibrium never pays off in that region of parameters, and the policymaker either sets $\underline{A} = 0$ or $\underline{A} = \underline{A}^{I}$. The latter is strictly preferable if $U^{I}_{\dagger}\left(\underline{A}^{I}\right) > U^{B}_{G}$, which is always satisfied for $\beta_L > \overline{\beta}$:

$$U^{I}_{\dagger}\left(\underline{A}^{I}\right) - U^{B}_{G} = \frac{1}{3}\frac{\overline{\gamma}b\Delta_R\left(1+\gamma-2\gamma^2\right)}{\beta_L - (1+2\gamma)\,\Delta_\alpha}$$

is larger than zero since $1+\gamma-2\gamma^2 > 0$ for all $\gamma \in (0,1)$. Hence, for all $\beta_L > \overline{\beta}$, the optimal early intervention is $\underline{A}^{I}$.

Hereafter, assume $\gamma < \overline{\gamma}$. Consider the equilibrium characterization in Proposition 2. Suppose $\beta_L \leq \underset{\sim}{\beta}$. Setting any $\underline{A} \in [0,1]$ keeps the economy at the $S$ equilibrium (since $\beta'_L = (1-\underline{A})\,\beta_L \leq \gamma\Delta_R = \underset{\sim}{\beta}$) and leads to a payoff of

$$U^{S}_{\dagger,2}(\underline{A}) = \frac{2\gamma}{3}(b-c)(1-\underline{A}) + (\gamma b - c)\,\underline{A}. \tag{B.11}$$

Using $c = \overline{\gamma}b$, one can verify that the expression above is strictly decreasing in $\underline{A}$. The policymaker then sets $\underline{A} = 0$. Now suppose $\beta_L \in \left(\underset{\sim}{\beta}, \tilde{\beta}\right]$. If the policymaker sets

$$\underline{A} \geq 1 - \frac{\gamma\Delta_R}{\beta_L} \equiv \underline{A}^{S}_{2}, \tag{B.12}$$

it triggers the Sell equilibrium and obtains $U^{S}_{\dagger,2}(\underline{A})$. We have not ruled out the possibility that when the policymaker sets some $\underline{A} \in \left(0, \underline{A}^{S}_{2}\right)$, the equilibrium switches to $SB$. Suppose first that for some $\underline{A} \in \left(0, \underline{A}^{S}_{2}\right)$ the economy stays in the $SI$ equilibrium. In that case, using (B.5), the policymaker obtains

$$U^{SI}_{\dagger}(\underline{A}) = \frac{\gamma}{3}(b-c)(1-\underline{A}) + (\gamma b - c)\,\underline{A}.$$

Using $c = \overline{\gamma}b$ and $\gamma < \overline{\gamma}$, one can verify that $U^{SI}_{\dagger}(\underline{A})$ is strictly decreasing in $\underline{A}$. Hence, whenever the policymaker chooses $\underline{A}$ to remain in the $SI$ equilibrium, it chooses $\underline{A} = 0$. Now suppose there is some $\underline{A} \in \left(0, \underline{A}^{S}_{2}\right)$ that leads to the $SB$ equilibrium. Such $\underline{A}$ is dominated since the payoff of the $SB$ equilibrium is $U^{SB}_{\dagger}(\underline{A}) = (\gamma b - c)\,\underline{A} < U^{SI}_{\dagger}(0)$ (see (B.5)). Hence, the policymaker sets either $\underline{A} = 0$ or $\underline{A} = \underline{A}^{S}_{2}$. The latter is strictly preferable whenever $U^{S}_{\dagger,2}\left(\underline{A}^{S}_{2}\right) > U^{SI}_{\dagger}(0)$, which is equivalent to

$$\beta_L < \frac{\gamma\Delta_R\left(3\overline{\gamma} - 2\gamma\overline{\gamma} - \gamma\right)}{3\overline{\gamma} - \gamma\overline{\gamma} - 2\gamma} \equiv \underset{\sim}{\beta}^{\dagger} > \underset{\sim}{\beta}. \tag{B.13}$$

For $\beta_L = \underset{\sim}{\beta}^{\dagger}$, the policymaker is indifferent between $\underline{A} = 0$ and $\underline{A} = \underline{A}^{S}_{2}$. One can verify that $\underset{\sim}{\beta}^{\dagger} < \tilde{\beta}$ (this can be seen by evaluating $\tilde{\beta}$ at $\Delta_\alpha = 0$, since $\tilde{\beta}$ is increasing in $\Delta_\alpha$).

Now consider $\beta_L > \tilde{\beta}$. If $\underline{A} = 0$, the $SB$ equilibrium is played and the policymaker's expected payoff is $U^{SB} = 0$ (see (B.5)). Any $\underline{A} > 0$ that does not cause an equilibrium switch leads to a payoff of $(\gamma b - c)\,\underline{A} < 0$. If the policymaker sets $\underline{A} \geq \underline{A}_2^S$, it triggers the $S$ equilibrium and obtains $U_{\dagger,2}^S(\underline{A})$. If it sets $\underline{A} \in \left[\underline{A}^{SI}, \underline{A}_2^S\right)$, where $\underline{A}^{SI}$ solves

$$\beta_L = \frac{(1+\gamma+\overline{\gamma})\frac{\Delta_R}{1-\underline{A}^{SI}} + \Delta_\alpha + \sqrt{\left[(1+\gamma+\overline{\gamma})\frac{\Delta_R}{1-\underline{A}^{SI}} + \Delta_\alpha\right]^2 + 4\overline{\gamma}\gamma\Delta_\alpha\frac{\Delta_R}{1-\underline{A}^{SI}}}}{2}, \tag{B.14}$$

it triggers the $SI$ equilibrium and obtains $U_\dagger^{SI}(\underline{A})$. Recall that both $U_{\dagger,2}^S(\underline{A})$ and $U_\dagger^{SI}(\underline{A})$ are strictly decreasing, so the candidates for optimal $\underline{A}$ are 0, $\underline{A}_2^S$ and $\underline{A}^{SI}$. One can verify that (B.14) simplifies to

$$\underline{A}^{SI} = 1 - \frac{\beta_L(1+\gamma+\overline{\gamma})\Delta_R + \gamma\overline{\gamma}\Delta_R\Delta_\alpha}{\beta_L(\beta_L - \Delta_\alpha)}. \tag{B.15}$$

We now show that $U_\dagger^{SI}\left(\underline{A}^{SI}\right) > U_{\dagger,2}^S\left(\underline{A}_2^S\right)$, which makes $\underline{A}_2^S$ dominated. Since $U_\dagger^{SI}(\underline{A})$ is decreasing in $\underline{A}$ and $\underline{A}^{SI}$ is decreasing in $\Delta_\alpha$, $U_\dagger^{SI}\left(\underline{A}^{SI}\right)$ increases with $\Delta_\alpha$. Given that $U_{\dagger,2}^S\left(\underline{A}_2^S\right)$ is independent of $\Delta_\alpha$, it then suffices to check that $\nabla \equiv U_\dagger^{SI}\left(\underline{A}^{SI}\right) - U_{\dagger,2}^S\left(\underline{A}_2^S\right)\Big|_{\Delta_\alpha=0} > 0$. We have that

$$\nabla = \frac{b\Delta_R}{3\beta_L}\left[(3-\gamma)\overline{\gamma}^2 + \left(3 - 3\gamma + \gamma^2\right)\overline{\gamma} - \gamma^2 - 2\gamma\right],$$

which is decreasing in $\gamma$. Evaluating that expression at $\gamma = \overline{\gamma}$, it simplifies to $\frac{b\Delta_R}{3\beta_L}\overline{\gamma}(1-\overline{\gamma}) > 0$. Therefore, $\nabla$ is positive for all $\gamma \in [0, \overline{\gamma}]$. We have then shown that $U_\dagger^{SI}\left(\underline{A}^{SI}\right) > U_{\dagger,2}^S\left(\underline{A}_2^S\right)$ for all $\gamma \in [0, \overline{\gamma})$. The policymaker then chooses among $\underline{A} = 0$ and $\underline{A} = \underline{A}^{SI}$. The latter is strictly preferable whenever $U_\dagger^{SI}\left(\underline{A}^{SI}\right) > 0 = U_G^{SB}$. It is easy to see that $U_\dagger^{SI}\left(\underline{A}^{SI}\right)$ decreases with $\beta_L$, since $\underline{A}^{SI}$ is increasing in $\beta_L$. For $\beta_L \to \tilde{\beta}$, $U_\dagger^{SI}\left(\underline{A}^{SI}\right) \to (1-\overline{\gamma})\gamma b/3 > 0$. For $\beta_L \to \infty$, $U_\dagger^{SI}\left(\underline{A}^{SI}\right) \to -b(\overline{\gamma} - \gamma) < 0$. Hence, there exists $\tilde{\beta}^\dagger > \tilde{\beta}$ such that $U_\dagger^{SI}\left(\underline{A}^{SI}\right) = 0$ when $\beta_L = \tilde{\beta}^\dagger$, $U_\dagger^{SI}\left(\underline{A}^{SI}\right) > 0$ for $\tilde{\beta} < \beta_L < \tilde{\beta}^\dagger$, and $U_\dagger^{SI}\left(\underline{A}^{SI}\right) < 0$ for $\beta_L > \tilde{\beta}^\dagger$. Then, for $\beta_L > \tilde{\beta}^\dagger$, the policymaker sets $\underline{A} = 0$, for $\tilde{\beta} < \beta_L < \tilde{\beta}^\dagger$ it sets $\underline{A} = \underline{A}^{SI}$, and for $\beta_L = \tilde{\beta}^\dagger$ it is indifferent between the two. Therefore, $\tilde{\beta}^\dagger$ is the largest of the two roots of $U_\dagger^{SI}\left(\underline{A}^{SI}\right) = 0$:

$$\tilde{\beta}^\dagger = \frac{(1+\gamma+\overline{\gamma})\Delta_R}{2}\frac{3\overline{\gamma} - \gamma\overline{\gamma} - 2\gamma}{3(\overline{\gamma} - \gamma)} + \frac{\Delta_\alpha}{2} + \frac{\sqrt{\Upsilon}}{6(\overline{\gamma} - \gamma)}, \tag{B.16}$$

where

$$\Upsilon \equiv (1+\gamma+\overline{\gamma})^2(3\overline{\gamma} - \gamma\overline{\gamma} - 2\gamma)^2\Delta_R^2 + 6\Delta_R\Delta_\alpha(\overline{\gamma} - \gamma)(3\overline{\gamma} - \gamma\overline{\gamma} - 2\gamma)(1+\gamma+\overline{\gamma}+2\gamma\overline{\gamma}) + 9\Delta_\alpha^2(\overline{\gamma} - \gamma)^2.$$

□

## B.7 Proof of Corollary 1

We prove this result with an example. Consider $\gamma > \overline{\gamma}$ and $\beta_L > \overline{\beta}$. If $\underline{A} = 0$, it follows from Propositions 2 and A.1 that the $B$ equilibium is played and the probability of intervention is one, so the expected bailout size is one. Under the optimal initial assistance $\underline{A}^* \in (0,1)$ of Proposition 6, the $I$ equilibrium is played (see the proof of Proposition 6). Using Proposition A.1, the probability of (additional) intervention in equilibrium is 2/3, and the total expected bailout size is then $\underline{A}^* + \frac{2}{3}(1 - \underline{A}^*) < 1$. □

## B.8 Proof of Proposition 7

The result follows directly from taking derivatives of the expressions for $\underline{A}^*$ in Proposition 6 whenever it is strictly larger than zero. □

## B.9 Proof of Proposition 8

We must first introduce some additional notation. Denote the market maker's beliefs by $\mu_{G'}^{\theta'}(x) \equiv \Pr(\theta = \theta' \cap G = G' | X = x)$, where $\theta' \in \{L, H\}$ and $G' \in \{0,1\}$, and the policymaker's beliefs by $\eta(s') \equiv \Pr(\theta = H | s = s')$. To save on notation, let $g(s)$ denote the policymaker's strategy (note that it is no longer a function of $X$ as in the baseline model).

Suppose that in equilibrium $l(-1) = h(1) = 1$. Then $g(1) = 1$, $g(-1) = 0$, and prices are given by $p(-2) = p(-1) = R_L$, $p(0) = R_L + \gamma(\alpha_L + \Delta_R + \Delta_\alpha)$, and $p(1) = p(2) = R_L + \Delta_R + \alpha_L + \Delta_\alpha$. Equilibrium requires $\pi^L(-1) \geq \pi^L(1)$, which is equivalent to

$$\frac{1}{3}[p(0) - R_L] \geq R_L + \alpha_L + \beta_L - \frac{1}{3}[p(0) + p(1) + p(2)]$$

$$\iff \beta_L \leq \frac{2}{3}(1+\gamma)(\Delta_R + \Delta_\alpha) + \frac{1}{3}(2\gamma - 1)\alpha_L.$$

Therefore, if $\beta_L$ is large enough, the condition above is violated and there is no such equilibrium. This proves the first statement.

Now consider $\gamma > \overline{\gamma}$ and suppose that in equilibrium $h(1) = l(1) = 1$. Then $\eta(1) = \gamma$ and $g(1) = 1$. Also,

$$p(X) = R_L + \alpha_L + \gamma(\Delta_R + \Delta_\alpha) \text{ for } X = 0, 1, 2,$$

$$\pi^H(1) = (1-\gamma)(\Delta_R + \Delta_\alpha) + \beta_H \quad \text{and} \quad \pi^L(1) = -\gamma(\Delta_R + \Delta_\alpha) + \beta_L.$$

Since $s = 0$ is off equilibrium, Bayes rule does not pin down $\eta(0)$. The maximum possible payoffs for the high and low types if they deviate to $s = 0$ are:

$$\pi_{\max}^H(0) = \beta_H < \pi^H(1) \quad \text{and} \quad \pi_{\max}^L(0) = \beta_L > \pi^L(1)$$

(which obtains if $g(0) = 1$). Hence, the high type has no incentives to deviate to $s = 0$, so the intuitive criterion implies $\eta(0) = g(0) = 0$, and therefore a deviation of the low type to $s = 0$ would yield a payoff

$\pi^L(0) = 0$. Then, if $\pi^L(1) = -\gamma(\Delta_R + \Delta_\alpha) + \beta_L \geq 0 \Leftrightarrow \beta_L \geq \gamma(\Delta_R + \Delta_\alpha)$ no one has incentives to deviate to $s = 0$. Now consider deviations to $s = -1$: For the low type, in the best case scenario there is an intervention and positive trading profits, so, in particular, $\eta(-1) = 0$ and $\mu_0^L(X) = 1$ for $X = -2, -1$ are compatible with the intuitive criterion. Note that, under those beliefs, the high type has no incentives to deviate to $s = -1$ as long as:

$$\pi^H(-1) = \frac{2}{3}R_L + \frac{1}{3}\left[R_L + \alpha_L + \gamma(\Delta_R + \Delta_\alpha)\right] - R_H \leq \pi^H(1)$$

$$\iff \beta_H \geq \frac{1}{3}\alpha_L + \left(\frac{4}{3}\gamma - 1\right)(\Delta_R + \Delta_\alpha) - \Delta_R \equiv \beta_H^B.$$

For the low type not to deviate to $s = -1$ it must be that

$$\pi^L(1) \geq \frac{1}{3}\left[R_L + \alpha_L + \gamma(\Delta_R + \Delta_\alpha) - R_L\right] = \pi^L(-1)$$

$$\iff \beta_L \geq \frac{1}{3}\alpha_L + \frac{4\gamma}{3}(\Delta_R + \Delta_\alpha) \equiv \beta_L^B > \gamma(\Delta_R + \Delta_\alpha).$$

Hence, there is an equilibrium with $h(1) = l(1) = 1$, $g(1) = 1$ and $g(-1) = g(0) = 0$, as long $\beta_L \geq \beta_L^B$ and $\beta_H \geq \beta_H^B$. This concludes the proof of the second statement.

We now prove the third statement. Consider $\gamma < \overline{\gamma}$ and suppose that in equilibrium $h(1) = 1$, $l(1) = 1 - l(-1) \in (0,1)$, $g(-1) = g(0) = 0$, and $g(1) \in (0,1)$. Indifference for the policymaker at $s = 1$ implies

$$\Pr(\theta = H|s = 1) = \eta(1) = \frac{\gamma}{\gamma + (1-\gamma)\,l(1)} = \overline{\gamma} \Rightarrow l(1) = \frac{\gamma(1-\overline{\gamma})}{\overline{\gamma}(1-\gamma)} \in (0,1). \tag{B.17}$$

One can also compute the following prices:

$$p(-2) = p(-1) = R_L,$$

$$p(0) = R_L + \Pr(s = 1|X = 0)\left\{g(1)\,\alpha_L + \Pr(\theta = H|s = 1)\left[\Delta_R + g(1)\,\Delta_\alpha\right]\right\}.$$

Note from (B.17) that $\Pr(\theta = H|s = 1) = \overline{\gamma}$, and Bayes rule yields $\Pr(s = 1|X = 0) = \frac{\gamma}{\overline{\gamma}}$, so

$$p(0) = R_L + \gamma\left[\Delta_R + g(1)\,\Delta_\alpha\right] + \frac{\gamma}{\overline{\gamma}}g(1)\,\alpha_L. \tag{B.18}$$

Also, since $\Pr(s = 1|X = 1) = \Pr(s = 1|X = 2) = 1$,

$$\begin{aligned} p(1) = p(2) &= R_L + g(1)\,\alpha_L + \Pr(\theta = H|s = 1)\left[\Delta_R + g(1)\,\Delta_\alpha\right] \\ &= R_L + g(1)\,\alpha_L + \overline{\gamma}\left[\Delta_R + g(1)\,\Delta_\alpha\right]. \end{aligned} \tag{B.19}$$

Indifference for the low type implies $\pi^L(1) = \pi^L(-1)$, which after some algebra yields

$$g(1) = \frac{2\Delta_R(\gamma + \overline{\gamma})}{3\beta_L - 2\Delta_\alpha(\gamma + \overline{\gamma}) - \alpha_L\frac{(2\gamma - \overline{\gamma})}{\overline{\gamma}}}. \tag{B.20}$$

For $g(1)$ to be in $(0,1)$, we need

$$\beta_L > \frac{2}{3}\Delta_\alpha (\gamma + \overline{\gamma}) + \frac{1}{3}\alpha_L \frac{(2\gamma - \overline{\gamma})}{\overline{\gamma}} \equiv \underline{\beta}^{SB}$$

and

$$\beta_L > \frac{2}{3}(\Delta_R + \Delta_\alpha)(\gamma + \overline{\gamma}) + \frac{1}{3}\alpha_L \frac{(2\gamma - \overline{\gamma})}{\overline{\gamma}} \equiv \beta^{SB} > \underline{\beta}^{SB}.$$

Equilibrium also requires that

$$\pi^L(-1) = \frac{\gamma}{3}[\Delta_R + g(1)\Delta_\alpha] + \frac{\gamma}{3\overline{\gamma}} g(1)\alpha_L(\text{B.20}) \geq \pi^L(0) = 0, \tag{B.21}$$

which is satisfied since $\Delta_R + \Delta_\alpha > 0$. For the high type not to deviate to $s = 0$ we need

$$\pi^H(1) = R_L + \Delta_R + g(1)(\alpha_L + \Delta_\alpha + \beta_H) - \frac{1}{3}[p(0) + p(1) + p(2)] \geq \pi^H(0) = 0,$$

By (B.18) and (B.19), note that $p(0), p(1), p(2) < R_L + \Delta_R + g(1)(\alpha_L + \Delta_\alpha)$ and hence the inequality above holds. For the high type not to deviate to $s = -1$ it suffices to check that

$$\pi^H(-1) = \frac{1}{3}[p(-2) + p(-1) + p(0)] - (R_L + \Delta_R) \leq 0,$$

which replacing the obtained expressions for prices yields

$$\pi^H(-1) = \frac{2}{3}R_L + \frac{1}{3}\left\{R_L + \gamma[\Delta_R + g(1)\Delta_\alpha] + \frac{\gamma}{\overline{\gamma}} g(1)\alpha_L\right\} - (R_L + \Delta_R) \leq 0.$$

Now, using (B.20), note that when $\beta_L \to \infty$, $g(1) \to 0$ and $\pi^H(-1) \to -\Delta_R(1 - \gamma/3) < 0$. Therefore, the inequality above holds for $\beta_L$ sufficiently large.

It remains to check that there are beliefs $\eta(-1), \eta(0) \leq \overline{\gamma}$ consistent with Bayes rule and the intuitive criterion. Given the equilibrium strategies, $\eta(-1) = 0$. As for $\eta(0)$, note that the maximum possible payoff for the low type when deviating to $s = 0$ is $\beta_L$, and as argued above, as $\beta_L \to \infty$, $g(1) \to 0$, and hence $\pi^L(1) = \pi^L(-1) \to \gamma\Delta_R/3$ (see (B.21)). Therefore, for $\beta_L$ large enough, a deviation to $s = 0$ is not dominated by the equilibrium strategy for the low type, and hence $\eta(0) = 0$ is consistent with the intuitive criterion. We have then shown that, for $\beta_L$ sufficiently large, the proposed strategies and beliefs constitute an equilibrium. □