Safety First: A Theory of Banking *

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Abstract

We study the portfolio choice of households with a structural preference for safety and heterogeneous self-insurance options. To ensure a minimum consumption level, households directly control personal assets and sometimes liquidate risky assets. Private intermediaries arise endogenously, where households with high self-insurance returns become equity holders and indirectly secure households with low self-insurance returns who hold debt. The conflict over interim risk choices is solved by demandable debt, forcing early liquidation. Our theory rationalizes an empirical finding in household finance: the endogenous returns on debt and equity in our model can account for documented large differences in risky returns across the wealth distribution. Finally, we show that public provision of safety can *crowd in* the private provision of safety: deposit insurance can boost both intermediation and productive investment.

Keywords: safe assets, demandable debt, intermediation.

JEL classifications: G21, G28, G11, G51

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1 Introduction

Since the Great Financial Crisis, new evidence emerged on an inelastic demand for safe assets (Gorton et al., 2012). In addition, there is a documented spike in demand and the underlying return when moving from almost zero risk and exactly zero risk (absolute safety), which cannot be fully explained by liquidity and transaction needs, the predominant explanations in the literature (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990).[1][2] In fact, this *safety premium* is substantial, ranging from 20 to 70 basis points (Gorton et al., 2012; Krishnamurthy and Vissing-Jorgensen, 2012; Kacperczyk et al., 2021; Christensen and Mirkov, 2022). Private intermediaries have responded to the increasing safety demand by expanding short-term debt, raising both maturity transformation and, potentially, financial instability (Krishnamurthy and Vissing-Jorgensen, 2015).

Existing work has treated safety premium as reflecting either common linear or concave preference for safe assets because of their safe liquidity value (Stein, 2012) or safety demand arising from a subset of agents with extreme preferences or beliefs (Gennaioli et al., 2013; Caballero and Farhi, 2018). Our work complements these views by allowing for explicit safety preferences by all households, where safe asset demand reflects a portfolio choice.

Our contribution is three-fold. First, we provide a safety-driven theory of financial intermediation and demandable debt. Second, we study two forms of the public provision of safety, public debt and deposit insurance, and show that they have opposite predictions once we allow for the ex-ante effects of safety provision. Third, in a general equilibrium setting, we i) capture the crowding in / crowding out effects from public to private provision; and ii) shed light on recent empirical evidence in household finance.

We study a model in which households have safety needs given by classic Stone-Geary

[1] Exploring the 2014 SEC reform of Money Market Funds (MMF) targeting at eliminating any promise of full protection of capital for all MMF not fully invested in government paper, Cipriani and La Spada (2021) show that investor demand shifted massively to funds ensuring absolute safety from very liquid and better-remunerated prime mutual funds, even though prime MMF offered very safe liquidity with minimum price fluctuations (total outflows exceeded 1 trillion dollars, with an implied premium for money-likeness between 20 and 30 bps).

[2] Figure 7 shows that the ratio of U.S. safe assets to GDP has grown dramatically, as opposed to a fairly stable share to total wealth. If the demand for safe assets were driven mainly by transaction and liquidity needs, one would expect those safe assets to be a constant fraction of GDP, commonly used as an aggregate measure of transaction/liquidity needs.

preferences. These preferences imply a huge utility loss below a minimum level of consumption, which is in line with recent evidence.[3] An innovation of our model is that apart from a productive investment, we allow for non-financial (real) personal assets, e.g., land, house, or human capital. Personal assets are privately controlled, so they ensure a higher downside value but promise a lower expected return.[4] Unlike productive investment returns, personal asset returns vary as they reflect personal skills or circumstances and cannot be safely transferred by contracts.[5] Households allocate their endowments among personal assets, a productive investment, and securities issued by (endogenously arising) intermediaries.

We first establish two benchmarks. In autarky, all households seek to achieve safety via personal assets (i.e., self-insurance) and, possibly, the liquidation of productive investment. Two strategies arise: partial and full self-insurance. Partial self-insurance, chosen by investors with low self-insurance returns, involves higher productive investment but value-destroying liquidation when safety is at risk. Full self-insurance, chosen by investors with high self-insurance returns, involves a lower productive investment (forgoing a higher expected return) but does not require any value-destroying liquidation. Autarky is inefficient. A social planner able to transfer productive returns can reduce inefficient self-insurance and boost productive investment by reallocating safe proceeds to ensure the safety of all investors.

Our setting gives rise to a safety-driven theory of financial intermediation and demandable debt. Competitive intermediaries achieve the efficient allocation by tranching investment payoffs and issuing senior debt backed by adequate loss-absorbing equity. Investors with low self-insurance returns rely on debt for their safety. Equity is provided by investors with high self-insurance returns, who can bear risk in exchange for a large leveraged payoff in good times. Interestingly, the senior claim is not safe in states where the expected present value of continuation is positive but there is a chance of debt default. In those states, equity holders

[3]These preferences are consistent with the established psychological notion of Maslow's pyramid of needs (Maslow, 1943). Evidence on minimum safety preferences has been documented by Choi and Robertson (2017), who show that 36% of respondents describe consumption commitments as very or extremely important for their portfolio allocation decision. Moreover, this specification is consistent with "habit preferences" widely used in asset pricing and macro to model strong responses around a reference point (Campbell and Cochrane, 1999; King and Rebelo, 1993; Fuhrer, 2000).

[4]Ownership and residual control rights enhance value in non-verifiable states (Grossman and Hart, 1986).

[5]In particular human capital is inalienable, so its reward cannot be committed to others (Hart and Moore, 1994). Direct control over an asset enables the diversion of returns so they cannot be reliably transferred.

prefer continuation, while safety-seeking debt holders prefer liquidation. As the interim state is non-verifiable, intermediaries cannot commit to a contingent liquidation or control shift (Aghion and Bolton, 1992; Dewatripont and Tirole, 1994).

This conflict is resolved by demandable debt, which implements the efficient allocation. Specifically, the option to withdraw empowers safety-seeking debt holders to trigger liquidation of risky assets, making intermediaries' debt safe. This novel view of demandable debt is driven by direct safety provision (rather than liquidity cross-insurance, as in Diamond and Dybvig (1983). It also supports the view that short-term debt is the foremost safe private claim, consistent with recent evidence (Kacperczyk et al., 2021).

As debt and equity are determined endogenously as a solution to investors' asset allocation problem, our model generates a set of implications. When bank equity is abundant, competitive intermediaries offer safety at the efficient safe rate, while under scarcity, the safe rate is lower (implying a higher safety premium, consistent with Krishnamurthy and Vissing-Jorgensen (2012). Factors contributing to more scarce safety capacity of intermediaries decrease the safe rate and aggregate investment and increase return on equity.

Our findings shed light on recent empirical evidence in household finance. We show that investors with high self-insurance returns achieve a higher expected return on their optimal risky portfolio, which contrasts the prediction of modern portfolio theory that the optimal risky portfolio and its return are the same for everyone. Thus, to the extent that wealthier investors have better self-insurance returns (due to, for example, better education), our results can account for large differences in risky returns across the wealth distribution (Fagereng et al., 2016). This insight is also in line with Bach et al. (2020), who find that the higher return earned by wealthier investors on risky assets appears to reflect "compensations for high systematic risk that the rest of the population seem unwilling or unable to take".

Next, we consider public provision of safety. Critically, we study the case where governments do not have any superior technology for creating safety, but they can redistribute it. Specifically, we consider transfers of financial safety by (i) public debt and (ii) deposit insurance, each of which is backed by the fiscal capacity to tax safe personal returns. Assuming balanced-budget policies allows us to endogenize an important cost of public provision of safety: the negative ex-ante implications of taxation. In particular, since safe public debt or deposit insurance can only be credibly funded by taxes on safe income, public safety induces more self-insurance (to cover prospective taxes), reducing the supply of loss-absorbing equity. In line with the literature, the introduction of public debt crowds out private provision of safety and decreases the equilibrium safe rate and aggregate investment.

Deposit insurance, by contrast, can *crowd in* private provision of safety and increase the equilibrium safe rate and aggregate investment. The underlying channel depends on safety capacity of intermediaries. When safety capacity is abundant, deposit insurance reduces inefficient liquidation. When safety capacity is scarce, deposit insurance can relax intermediaries' safety capacity constraint and increase the equilibrium safe rate, shifting self-insurance from less efficient to more efficient investors that boosts aggregate investment.

Literature. A growing literature studies the implications of safety demand. For instance, Caballero and Krishnamurthy (2009) and Stein (2012) highlight the impact of safety demand on refinancing risk, while Gorton and Ordonez (2014, 2022) and Moreira and Savov (2017) focus on boom and bust episodes. We contribute by showing that safety demand (i) gives rise to intermediaries as safety providers, and (ii) can explain empirical patterns in household finance (Calvet and Sodini, 2014; Fagereng et al., 2016; Bach et al., 2020).

Our view of demandable debt is due to competing preferences over risk, closely related to a conflict over bank asset value and its capture (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). In Calomiris and Kahn (1991), depositor runs prevent bankers from absconding with funds in bad states, while in Diamond and Rajan (2001), they serve as a threat to enforce loan collection and avoid ex-post renegotiation. In our safety setup, all parties agree in high and low states, while differences between safety-driven debt holders and return-driven equity holders arise in residual risk states. Critically, withdrawals occur in equilibrium, implementing a safe outcome in a conflict over risk choices. In addition, withdrawals by patient investors that force liquidation are undesirable in a liquidity insurance setup (Diamond and Dybvig, 1983), while they are essential for safety in our setup.

Our work provides new insights both in terms of how safety demand is modeled and in terms of how safety is achieved. Regarding the former, in the current literature, safety demand is usually the result of either a homogeneous convenience yield (Stein, 2012; Jackson and Pennacchi, 2021) or a subset of extreme risk-averse agents (Gennaioli et al., 2013; Caballero and Farhi, 2018). In contrast, our model relies on similar agents sharing a structural need for minimum safety and heterogeneous self-insurance capacity. Structural preferences for safety find empirical support (De Martino and Adolphs, 2010; Choi and Robertson, 2017) and can help interpret some distinct empirical facts. Furthermore, our work provides novel channels on how safety can be achieved. Unlike the literature focusing on diversification as a source of safety creation (Gennaioli et al., 2013; Diamond, 2020), in our setting, safety comes from conditional liquidation of risky assets and debt demandability. We also emphasize the role of direct control of personal assets in achieving safety, which is consistent with the idea that the need for safety existed long before financial intermediaries or safe public debt emerged. Thus, we provide a global view of safety provision, including self-insurance, private provision via intermediaries, and public provision via public debt and/or deposit insurance.

We identify a novel channel through which deposit insurance can be beneficial. On top of alleviating inefficient liquidation, which is closely related to the underlying channel in Diamond (1984), deposit insurance can relax intermediaries' safety capacity constraint, which can improve the allocation of resources. However, we also internalize the negative ex-ante implications of deposit insurance related to additional taxation, contributing to the literature exploring the potential costs of deposit insurance (Acharya and Dreyfus, 1989; Cooperstein et al., 1995).

Finally, our work sheds light on the interaction between private and public provision of safety. First, in a general equilibrium setting where the ex-ante effects of public debt (related to additional taxation) are internalized, we verify the findings of existing work that private provision of safety and public debt are substitutes (Gorton et al., 2012; Sunderam, 2015; Lenel et al., 2017; Jackson and Pennacchi, 2021; Gorton and Ordonez, 2022).[6] By contrast, once we focus on a different form of public provision (deposit insurance), public provision of safety can crowd in its private provision and investment.

[6] Gorton and Ordonez (2022) show that although government bonds crowd out the creation of private safe assets, they do crowd in their safety.

2 Model

Environment. There are three dates $t = 0, 1, 2$ and a single divisible good for consumption and investment. Investors of unit mass are endowed with one unit at $t = 0$ and consume c_t at date t. All investors are risk-neutral once they consume a reference level S at either $t = 1$ or $t = 2$ but suffer a large disutility if their total consumption falls below this level:[7]

$$u(c_1, c_2) = \begin{cases} c_1 + c_2 & \text{if } c_1 + c_2 \geq S \\ -\infty & \text{if } c_1 + c_2 < S. \end{cases} \tag{1}$$

These preferences imply *safety-seeking*, as investors prioritize achieving their safety reference level S in all states (defined below). To highlight the impact of safety-seeking, our specification abstracts from liquidity and transaction motives (e.g., Diamond and Dybvig, 1983, Gorton and Pennacchi, 1990).

Investment technologies. At $t = 0$, each investor has access to two technologies: a personal real asset and a productive investment common for all investors. By personal real assets, we have in mind assets directly controlled by individuals, such as human capital and personal residence. The two technologies differ in four key dimensions, discussed below.

First, personal assets have a higher downside return compared to the productive investment. The rationale is that owners of personal assets, by directly controlling them, can avoid risks arising from moral hazard or uncontracted contingencies, ensuring a high minimum return in all states.[8] To simplify the analysis, we assume that personal assets return r with certainty, whereas productive investment returns R with probability $\gamma \in (0, 1)$ and 0 with probability $1 - \gamma$. The main findings are qualitatively similar if personal assets are risky, as long as their lowest return exceeds the lowest return of the productive investment.[9]

[7] This feature finds empirical support (De Martino and Adolphs, 2010; Choi and Robertson, 2017) and it is consistent with habit preferences in asset pricing models, where risk aversion becomes infinite as consumption approaches the reference point. See also the discussion in the Introduction.

[8] In addition, direct control enables investors to protect asset payoffs also in non-verifiable states (Grossman and Hart, 1986).

[9] In that case, r would represent the minimum realized return.

Second, there is heterogeneity in the return of personal assets among investors: r is distributed according to $F(r)$ over the support $[r_L, r_H]$. This heterogeneity captures the idea that investors might have different skills, access to real assets, or circumstances. Third, in contrast to the productive investment, the return of personal assets is non-contractible.[10] Direct control and the fact that personal assets are inherently linked to their owner imply that returns cannot be reliably transferred to others.[11] Fourth, personal assets are illiquid at $t = 1$. In contrast, liquidating the productive investment at $t = 1$ yields $\alpha > 0$.

Information. At $t = 1$, a non-verifiable signal occurs with probability $\delta \in (0, 1)$, resolving all uncertainty over the return of the productive investment at $t = 2$. With probability $1-\delta$, the return remains unknown; thus, $1-\delta$ can be interpreted as a measure of investment opacity. The occurrence of the signal is independent of the investment return. As a result, there are three interim states, H, L, and RR. The expected return of continuing the productive investment is R in the high state H; 0 in the low state L; and, γR in the residual risk state RR. Figure 1 summarizes.

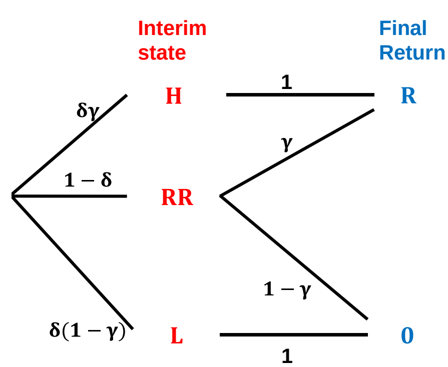

Figure (1) Payoffs and information structure of investment.

Parameter space. We are interested in an environment where personal assets have an advantage in providing safety but promise a lower expected return than the productive

[10] Personal asset returns may also be partially contractible as in Holmström and Tirole (1998), provided their pledgeable amount is below the minimum return.

[11] This is clear in the case of income from human capital (Hart and Moore, 1994). Even if personal returns are verifiable, their value might be subject to moral hazard. This seems to be the case for labor income. Any minimal risk of loss would then rule out their use for safety provision.

investment. To this end, we adopt the following set of restrictions. First, personal asset returns always exceed the liquidation value of the investment, i.e., $r_L > \alpha$. Second, the present value of the productive investment, conditional on being liquidated in state L only (denoted as PV and defined below), exceeds the personal asset's returns, i.e., $PV > r_H$. Third, investors cannot achieve safety by investing all their resources, i.e., $S > \alpha$. Fourth, the return of personal assets is sufficiently large to satisfy investors' safety needs, i.e., $r_L \geq S$. Finally, we focus on the case where liquidating investment in state RR is costly, i.e., $\alpha < \gamma R$. Costly liquidation implies that an investor who has secured safety finds it suboptimal to liquidate unless the interim state is L. Combining all these conditions, we obtain (2), which we assume holds throughout the paper.

$$PV \equiv \gamma R + (1-\gamma)\delta\alpha > r_H > r_L \geq S > \alpha > 0. \tag{2}$$

2.1 Autarky

Our first benchmark is Autarky. In this setting, investors choose the share x allocated to personal assets ("self-insurance"), with the remaining share $1-x$ allocated to the productive investment ("investment"). Second, after observing the signal about investment prospects, investors decide which fraction to liquidate.

Proposition 1. ***Autarky.*** *Investors with return $r < r^A$ self-insure $x = \underline{x} \equiv \frac{S-\alpha}{r-\alpha}$, invest $1-\underline{x}$, and liquidate the entire investment in states RR and L. Investors with return $r > r^A$ self-insure $x = \overline{x} \equiv \frac{S}{r}$, invest $1-\overline{x}$, and liquidate the entire investment in state L. Investors with return $r = r^A$ are indifferent between the two strategies. Aggregate self-insurance is $X^A = \int_{r_L}^{r^A} \underline{x}(r)dF(r) + \int_{r^A}^{r_H} \overline{x}(r)dF(r)$ and aggregate investment is $I^A \equiv 1 - X^A$.*

Proof. See Appendix C.1. □

Full liquidation is always optimal in state L (since $\alpha > 0$), whereas liquidation is always suboptimal in state H (since $R > \alpha$). In state RR, given that liquidation is value-destroying (since $\gamma R > \alpha$), investors liquidate the minimum fraction necessary to secure safety, $\ell \equiv \frac{S-rx}{\alpha(1-x)} \in [0,1]$. Anticipating their future liquidation actions and given that expected returns are linear in x, the optimal portfolio choice is a corner solution, i.e., $x^A = \underline{x}$ or $x^A = \overline{x}$.

Effectively, each investor chooses between two strategies: partial or full self-insurance. Partial self-insurance involves high investment but value-destroying liquidation in state RR ($x^A = \underline{x}, \ell = 1$). In contrast, full self-insurance involves low investment (which forgoes potential high return) but does not require any value-destroying liquidation in state RR ($x^A = \overline{x}, \ell = 0$). The latter strategy is more (less) attractive to investors whose personal assets promise a large (small) return, $r > r^A$ ($r < r^A$). We restrict our attention to parameter values such that r^A is interior, where r^A is given by:

$$r^A \equiv \frac{PV}{1 + (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)}. \tag{3}$$

2.2 Efficient Allocation

Our second benchmark is the allocation chosen by a planner whose objective is to maximize expected aggregate output conditional on providing safety to all investors. As the proceeds from personal assets are non-contractible, the planner can only redistribute investment proceeds. Thus, the resulting planner's allocation is *constrained efficient*. The planner aims to avoid unproductive self-insurance, which releases resources that can be diverted to the productive investment. The efficient allocation depends on safety capacity.

Definition 1 *Safety capacity is* ***abundant*** *when it is feasible to provide full insurance to investors with* $r < r^A$ *by liquidating investment* $I(r^A) = 1 - \int_{r^A}^{r^H} \frac{S}{r} dF(r)$*, i.e.,*

$$S\ F(r^A) \leq \alpha\ I(r^A). \tag{4}$$

Safety capacity is ***scarce*** *when condition* (4) *is violated.*

In the main text, we focus on the general case where safety capacity is scarce. The extreme case of abundant safety capacity is considered in the Online Appendix.

Proposition 2. *Efficient allocation.* *The unique efficient allocation is defined by a threshold* r^E*, such that: (i) investors with* $r > r^E$ *self-insure* $x^E(r) = \frac{S}{r}$ *and invest all residual endowment, (ii) investors with* $r \leq r^E$ *invest their entire endowment. Threshold* r^E *solves equation* (8) *with equality, and the safety for low-return investors is ensured by liquidating the entire investment in states* L *and* RR*.*

Proof. See Appendix C.2. □

The underlying intuition is as follows. First, note that the linear nature of the portfolio problem implies that, for a given investor with a return r', the planner finds it optimal to implement one of two options: (i) instruct her to self-insure fully ($x(r') = \frac{S}{r'}$) and invest the residual endowment; (ii) instruct her to invest her entire endowment ($x(r') = 0$). In strategy (ii), the planner commits to providing safety to the investor. The contribution of each strategy to the expected aggregate output is denoted as w_{full}, and w_{no}, where:

$$w_{full} \equiv \left(\frac{S}{r'}\right) r' + \left(1 - \frac{S}{r'}\right) PV, \tag{5}$$

$$w_{no} = PV - (1-\delta)(\gamma R - \alpha)\left(\frac{S}{\alpha}\right). \tag{6}$$

The planner's preferred strategy depends on the return of personal asset r' as follows:

$$\text{Planner's preferred strategy} = \begin{cases} w_{no} > w_{full} & \text{if } \; r' < r^A, \\ w_{no} = w_{full} & \text{if } \; r' = r^A, \\ w_{full} > w_{no} & \text{if } \; r' > r^A. \end{cases} \tag{7}$$

The latter implies that if safety capacity was abundant, the planner would wish investors with $r' \leq r^A$ to invest their entire resources and investors with $r' > r^A$ to self-insure fully and invest their remaining resources. However, when safety capacity is scarce, the planner's preferred strategy can no longer be implemented because liquidating the entire aggregate investment is insufficient to guarantee safety for investors with $r' \leq r^A$. In this case, the planner requires not only high-return but also intermediate-return investors ($r' \in [r^E, r^A)$) to self-insure fully. Thus, a key insight of our model is that when safety capacity is scarce, in

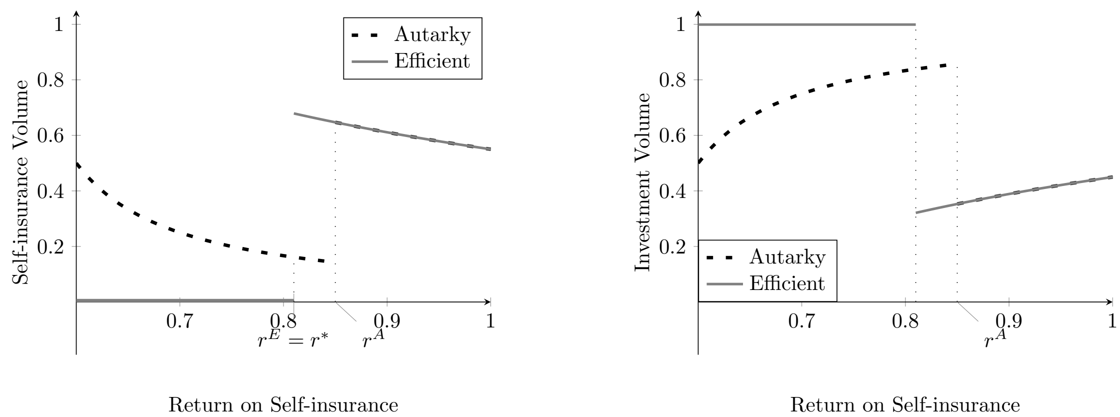

Figure (2) Self-insurance and investment when safety capacity is scarce. The dashed line illustrates the share of endowment allocated to self-insurance (left panel) and investment (right panel) under Autarky. The solid line represents the share allocated to self-insurance (left panel) and investment (right panel) under the efficient allocation. Compared to Autarky, the efficient allocation redistributes self-insurance from less efficient to more efficient investors, which boosts investment. Parameters: $S = 0.55$, $r_L = 0.6$, $r_H = 1$, $\alpha = \gamma = \delta = 0.5$, $R = 4$, $r \sim G[0.6, 1]$ with $G(r) = 12.5r - 6.25r^2 - 5.25$; For these values, $r^A = 0.85$, $r^E = r^* = 0.815$.

the efficient allocation, there is a redistribution of self-insurance from low-return investors to high-return investors. The optimal value of r^E solves the planner's safety capacity constraint (8) with equality:

$$SF(r^E) \leq \alpha \left[1 - \int_{r^E}^{r^H} \frac{S}{r} dF(r) \right]. \tag{8}$$

Figure 2 illustrates the level of self-insurance (left panel) and investment (right panel) in Autarky and in the efficient allocation. The redistribution of self-insurance when safety capacity is scarce is illustrated in the left panel of Figure 2, where compared to Autarky, in the efficient allocation investors with return $r \in [r^E, r^A]$ self-insure more.

Corollary 1. *In the efficient allocation, when safety capacity is scarce: (i) aggregate self-insurance is lower than in Autarky,* $X^E(r^E) < X^A$*; (ii) investment and the expected output are higher than in Autarky,* $I(r^E) > I^A$ *and* $Y(r^E) > Y^A$.

3 Intermediation with demandable debt

This section shows how competitive intermediaries implement the efficient allocation. Intermediation implies a delegation of control over investment, whereby equity holders decide on interim liquidation. We show how investors' participation constraints shape the contractual form required for intermediaries to attract funding.

Table 1 shows the timeline. At $t = 0$, intermediaries attract funding in competitive markets. Next, investors can self-insure, and both investors and intermediaries invest. At $t = 1$, the interim state is realized and withdrawals occur (if a claim is demandable). Direct investors and intermediaries' equity holders choose whether to liquidate part of investment and interim consumption occurs. At $t = 2$, all returns are realized, claims are paid out, and final consumption occurs.

$t = 0$	$t = 1$	$t = 2$
1. Intermediaries issue claims	1. Interim state realized	1. Maturity of investments
2. Portfolio choice	(2. Withdrawals)	2. Intermediaries pay claims
	3. Liquidation choice	3. Consumption
	4. Consumption	

Table (1) Timeline of events.

To offer a safe claim, intermediaries can carve out the senior portion (debt) of the verifiable investment return, backed by an adequate amount of loss-absorbing junior claims (equity). Let e denote equity and d denote debt with face value r^*, the safe rate. Then, when an intermediary's funding is invested fully, $I = d+e$, for debt to be safe, the liquidation value of investment should be large enough to repay all debt claims, $\alpha I \geq r^* d$. This minimum equity level $\underline{e} \equiv \frac{r^*-\alpha}{\alpha} d$ represents a *market-imposed capital ratio* required to attract funding from safety-seeking investors.

Lemma 1. *Seniority is not sufficient to ensure safety; demandability is necessary.*

Note that seniority ensures safety of debt in state L, when debt holders and equity holders agree to liquidate investment. However, seniority per se cannot *secure* safety. Similar to Ahnert and Perotti (2021), a conflict between debt holders and equity holders arises in state RR, since continuation of investment has a higher expected return than liquidation value ($\gamma R > \alpha$) but its realized return can be zero. Equity holders value their payoff as part of their risky portfolio so they prefer continuation, while safety-seeking debt holders prefer to liquidate. Interestingly, since the signal is not verifiable, there can be no contractual solution by a state-contingent allocation of control (Aghion and Bolton, 1992; Dewatripont and Tirole, 1994), nor can be trading of claims as there is no liquidity at $t = 1$. In these circumstances, without demandability, safety-seeking investors would not invest in any long-term debt claim on the intermediary.

The solution is to offer *demandable* debt. The option to withdraw upon demand at $t = 1$ forces partial or full liquidation of investment and guarantees a safe payoff in state RR. Effectively, demandability shifts liquidation decision from equity holders to debt holders. Thus, demandability eliminates the possibility that equity holders will refrain from liquidating, jeopardizing debt holders' safety. Finally, debt is demandable at face value at $t = 1$ because there is no need for a liquidity premium.

Compared to investing directly, holding equity implies a loss due to value-destroying liquidation in state RR. To be compensated for that loss, equity receives a large payoff in state H, when debt holders and equity holders agree to continue investment. Without loss of generality, we assume that if an investor is indifferent between investing directly and acquiring equity at the intermediary, she will prefer the latter.

3.1 Private provision of safety

Intermediaries choose the safe rate r, the *excess* return on equity ϵ, the level of debt d and equity e to solve the following optimization problem:

$$\max_{r,\epsilon,d,e} V(r,\epsilon,d,e) \equiv [PV - (1-\delta)\ell(\gamma R - \alpha)](d+e) - rd \tag{9}$$

$$V(r,\epsilon,d,e) \geq (PV+\epsilon)e, \tag{10}$$

$$e \geq \frac{r-\alpha}{\alpha} d, \tag{11}$$

$$d \leq \hat{d}, \tag{12}$$

$$e \leq \hat{e}. \tag{13}$$

where $V(r,\epsilon,d,e)$ is the intermediary's profit, given by the value of investment ($I = d + e$) net of expected liquidation losses in state RR and debt payments. The objective function internalizes that intermediaries find it optimal to continue investment in state H, liquidate fully in state L and liquidate a fraction ℓ in state RR. Constraint (10) guarantees that offering a return on equity $PV + \epsilon$ is feasible, where excess return ϵ is the return above and beyond the return of investing directly, PV. Constraint (11) guarantees that intermediaries are sufficiently capitalized, such that debt is indeed safe. Finally, conditions (12) and (13) imply that the level of debt and equity cannot exceed their demand, as they are determined from the asset allocation problem of investors, summarized in Lemma 3 in Appendix B.

Similar to Bertrand competition on prices, in equilibrium, competing intermediaries offer the highest feasible safe rate, defined by feasibility constraint (11), which binds in equilibrium: otherwise, the intermediary has excessive equity; thus, it could lower ϵ and increase the safe rate r. We show in the proof of Proposition 3 that constraint (10) is satisfied when (11) is satisfied. Solving (11) with respect to e and substituting it into (9) we obtain that $V(r,\epsilon,d,e)$ is increasing in debt; therefore, constraint (12) binds. Combining the latter with the observation that debt capacity increases with the level of loss-absorbing equity, implies that, for given ϵ, the intermediary fully absorbs the supply of equity, i.e., (13) binds. Hence, the optimal safe rate r^* solves (11) with equality (given $d = \hat{d}$ and $e = \hat{e}$). The equilibrium excess return on equity allowing the intermediary to afford to offer r^* is

$$\epsilon^* = \frac{\alpha\delta[\alpha(1-\gamma) - r^*] + \gamma R[\alpha - (1-\delta)r^*]}{r^* - \alpha}. \tag{14}$$

Intuitively, excess return on equity ϵ^* is positive, as scarcity of safety forces intermediaries to promise a return above investors' outside option PV to attract loss-absorbing equity. This differs from the case where safety is abundant (see Online Appendix), where intermediaries are sufficiently capitalized; therefore, they do not need to offer an excess return. Finally, for r^* and ϵ^*, by Lemma 3, $\hat{d}$ and $\hat{e}$ are given by

$$\hat{d} = \int_{r_L}^{r^*} \frac{S - \alpha}{r^* - \alpha} dF(r), \tag{15}$$

$$\hat{e} = \int_{r^*}^{r_H} \left(1 - \left(\frac{S}{r}\right)\right) dF(r). \tag{16}$$

As a result, the equilibrium safe rate equates demand and supply of safety:

$$SF(r^*) = \alpha \left[F(r^*) + \int_{r^*}^{r^H} \left(1 - \frac{S}{r}\right) dF(r)\right]. \tag{17}$$

Intermediaries attain efficiency. An interesting implication is that intermediaries attain efficiency: Note that (17) coincides with the planner's safety capacity constraint (8); thus, intermediaries implement the efficient allocation. The main difference is that in the case of intermediation, low-return (high-return) investors do not invest directly, but indirectly by holding debt (equity) in an intermediary that, in turn, allocates this amount to investment. Resources are transferred from high-return to low-return investors in state L via seniority and in state RR via demandability. In state H, the equity claim receives a high payoff, such that high-return investors are compensated for losses in other states. Proposition 3 summarizes. We illustrate Proposition 3 in Figure 3.

Proposition 3. ***Private provision of safety.*** *Competitive intermediaries attain efficiency. Each intermediary issues safe demandable debt d^* given by* (15) *with face value r^* solving* (17)*, backed by equity e given by* (16) *promised a return $PV + \epsilon^*$ where ϵ^* is given by* (14)*. In states L and RR, depositors withdraw, forcing intermediaries to liquidate their entire investment. Aggregate investment is $I = 1 - \int_{r^*}^{r_H} \frac{S}{r} dF(r)$, which is equal to aggregate investment under efficient allocation.*

Proof. See Appendix C.3. □

3.2 Comparative statics and household finance implications

Proposition 4 presents the comparative statics with respect to the fundamentals of the economy. In general, changes in the safe rate, r^*, can have an ambiguous effect on demand for safe debt. At the intensive margin, low-return investors need less debt as its return r^* rises (similar to an income effect), while at the extensive margin, a subset of investors that would

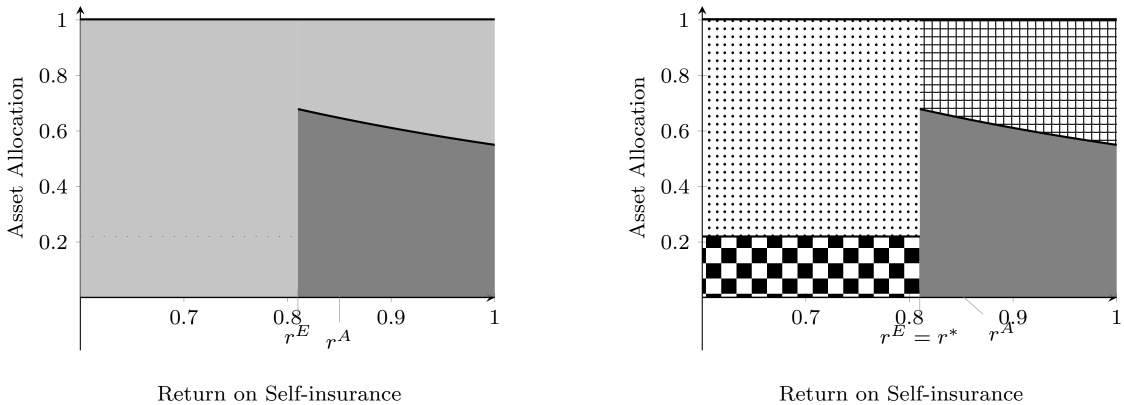

Figure (3) Allocation of resources when safety capacity is scarce. The left panel illustrates the efficient allocation: darkly shaded area - self-insurance; lightly shaded area - investment. The right panel illustrates the allocation of resources under intermediation: darkly shaded area - self-insurance; gridded area - equity; checkerboard area - debt; dotted area - direct investment. The level of debt plus equity plus direct investment is equivalent to the investment under the efficient allocation. Parameters: $S = 0.55$, $r_L = 0.6$, $r_H = 1$, $\alpha = \gamma = \delta = 0.5$, $R = 4$, $r \sim G[0.6, 1]$ with $G(r) = 12.5r - 6.25r^2 - 5.25$; For these values, $r^A = 0.85$, $r^E = r^* = 0.815$.

self-insure switches to safe debt (similar to a substitution effect). We focus on the case where the substitution effect dominates the income effect; thus, the volume of safety-seeking debt increases with the safe rate.[12]

Proposition 4. ***Comparative statics.***

(i) Better investment prospects (higher R, γ, δ) do not affect the safe rate, the volume of safe debt and equity, and the level of aggregate investment. Excess return on equity increases in γ, δ. For high transparency, i.e., $\delta > \hat{\delta}$, excess return increases in R.[13]

(ii) Higher liquidation value α increases the safe rate, the volume of safe debt, and the level of aggregate investment. The level of equity decreases with α, whereas the impact on excess return on equity is ambiguous.[14]

(iii) A mean-preserving compression (i.e., a safer investment achieved by a lower R or γ,

[12]To this end, we maintain throughout the regulatory condition on the distribution of self-insurance returns, $(r - \alpha) f(r) > F(r)$, which holds for the uniform distribution, for example.

[13]Where $\hat{\delta} = \frac{r^* - \alpha}{r^*}$, with r^* being the equilibrium safe rate given in Proposition 3.

[14]The comparative statics are qualitatively similar even if a higher liquidation value leads to a shift from a regime with scarcity to a regime with abundant safety capacity.

and a higher α, for unchanged PV) increases the safe rate, the volume of safe debt, and the level of aggregate investment. The level of equity decreases, whereas the impact on excess return on equity is ambiguous.[15]

(iv) *A downward shift in the distribution of personal returns $F(r)$ (FOSD) decreases the safe rate and the level of aggregate investment. The impact on debt and equity is ambiguous. Excess return on equity increases.*

(v) *A higher reference level S decreases the safe rate and the level of aggregate investment. The impact on debt and equity is ambiguous. Excess return on equity increases.*

Proof. See Appendix C.4. □

A main takeaway of Proposition 4 (summarized in Table 2) is that factors contributing to more scarce safety capacity decrease the safe rate and aggregate investment and increase excess return on equity. Parts (i) - (iv) of Proposition 4 are illustrated in Figure 4 whereas part (v) is illustrated in Figure 5. The intuition follows.

Better investment opportunities do not affect safety capacity of intermediaries nor demand for safety by investors, as they both depend on the liquidation value rather than the upside potential; thus, the safe rate r^* is unaffected.[16] Interestingly, better investment opportunities make it more tempting for high-return investors to invest directly instead of holding equity. Thus, excess return on equity increases to counteract this effect.

A higher liquidation value increases safety capacity of intermediaries, which allows them to offer a higher safe rate and support more debt. At the same time, a higher liquidation value weakens the need for loss-absorbing equity. Given that self-insurance is chosen only by equity holders, the level of self-insurance drops as well. Lower self-insurance implies that aggregate investment increases. The impact on excess return on equity is ambiguous; on the one hand, a higher liquidation value increases the incentive of high-return investors to invest directly instead of holding equity; thus, the return on equity should increase to counteract

[15] The comparative statics are qualitatively similar even if a safer investment leads to a shift from a regime with scarcity to a regime with abundant safety capacity.

[16] This result relies critically on the assumption that the liquidation value is exogenous. If, instead, the liquidation value increased as investment prospects improve, better investment prospects would increase the safe rate and the level of investment.

	safe rate	excess return	debt	equity	aggregate investment
Better investment prospects	(−)	(↑)	(−)	(−)	(−)
Higher liquidation value	(↑)	(?)	(↑)	(↓)	(↑)
Safer investment project	(↑)	(?)	(↑)	(↓)	(↑)
Worse personal returns	(↓)	(↑)	(?)	(?)	(↓)
Higher reference level S	(↓)	(↑)	(?)	(?)	(↓)

Table (2) Comparative statics when safety capacity is scarce: (↑) indicate increase, (↓) decrease, (−) no effect, (?) ambiguous effect.

this effect. On the other hand, a higher liquidation value increases the safe rate, which implies that a lower excess return on equity is required to convince investors to hold equity.

Part (i) and (ii) imply that a mean-preserving compression (i.e., a safer investment achieved by a lower R or γ, and a higher α, for unchanged PV) has the same impact as an increase in the liquidation value α, as neither the change in γ nor R have an effect on safety capacity of intermediaries.

The intuition in part (iv) is that, all else constant, worsening personal returns increases demand for safety and decreases intermediaries' loss-absorbing equity. Hence, the safe rate drops. Excess return on equity increases to convince investors to become equity holders, such that the higher demand for safety is met. The intuition of part (v) is similar as higher S corresponds to higher safety needs which increases demand for safety and, at the same time, decreases the level of loss-absorbing equity and safety capacity of intermediaries. Note that the level of loss-absorbing equity drops because high-return investors who are the potential equity holders need to allocate more resources to self-insurance to secure safety.

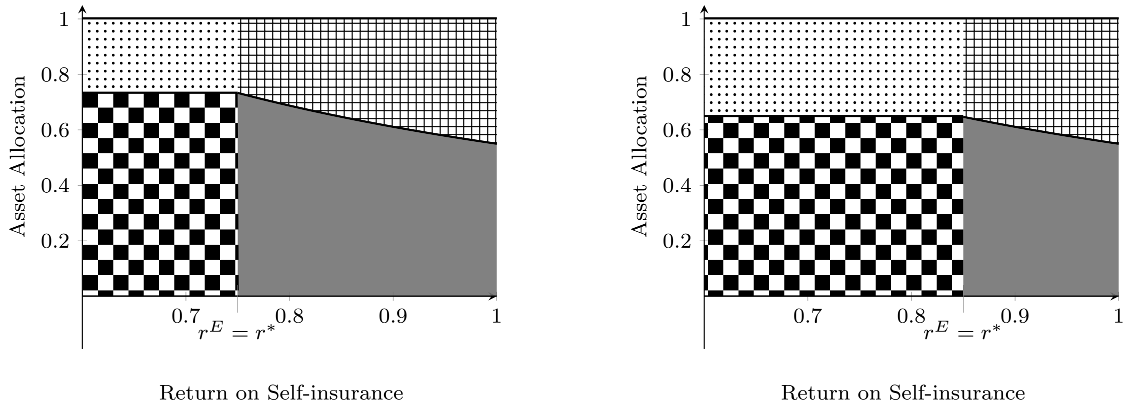

Figure (4) Asset allocation when safety capacity is scarce. The left panel depicts the case of low liquidation value / safer investment project / low personal returns. The right panel depicts the case of high liquidation value / riskier investment project / high personal returns. Darkly shaded area: self-insurance; gridded area: equity; checkerboard area: debt; dotted area: direct investment.

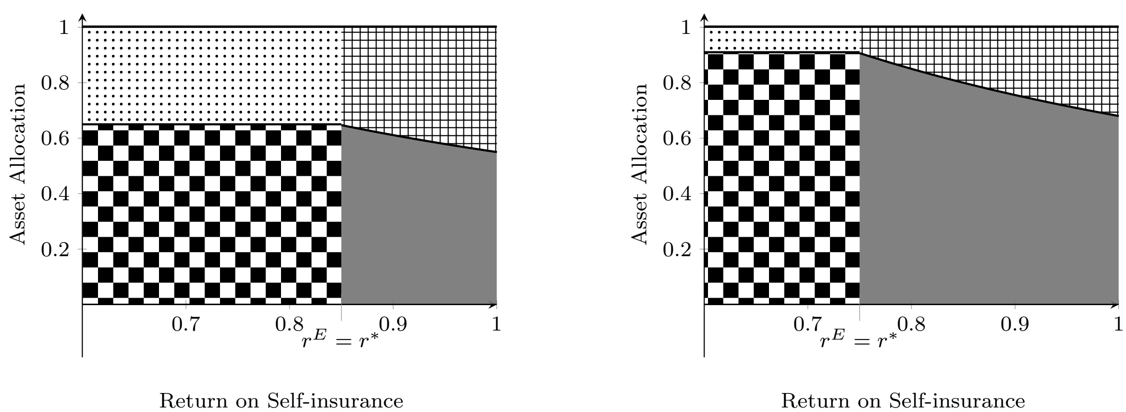

Figure (5) Asset allocation when safety capacity is scarce. The left (right) panel depicts the case of low (high) reference level S. Darkly shaded area: self-insurance; gridded area: equity; checkerboard area: debt; dotted area: direct investment.

Implications for household finance. Our work studies how agents (households) make financial decisions related to risk-management, self-insurance and investment. Thus, our findings have implications for household finance.

Corollary 2. *Investors with high self-insurance returns achieve a higher expected return on their risky part of their portfolio. The more scarce safety capacity is, the higher the discrepancy in the returns of the risky portfolio of investors.*

Corollary 2 is a key implication of Proposition 3. The intuition is the following. Investors with low self-insurance returns prefer investing directly than holding equity because they wish to maintain the option to liquidate in state RR, which is necessary for securing safety. Conditional on liquidation in state RR, investing directly promises an expected return $\delta\gamma R + (1 - \delta\gamma)\alpha < PV$. In contrast, investors with high self-insurance returns can take care of their own safety and thus afford to invest in risky equity and benefit from its higher return, which is strictly smaller than the expected return on equity, $PV + \epsilon$, where ϵ is higher the more scarce safety capacity is.

Although we do not that model that explicitly, to the extent that wealthier investors have better self-insurance returns (due to, for example, better access to education), Corollary 2 can also account for large differences in risky returns across the wealth distribution (Fagereng et al., 2016). This insight is also in line with Bach et al. (2020), who find that the higher return earned by wealthier investors on risky assets appears to reflect "compensations for high systematic risk that the rest of the population seem unwilling or unable to take".

4 Public provision of safety

In Section 3, we considered private provision of safety and showed that intermediaries issuing demandable debt, backed by sufficient loss-absorbing equity, implement the *constrained efficient* allocation. This outcome is the best a private arrangement can achieve, given that personal asset returns are non-contractible. In this section, we explore whether a government can do better than the private sector. Critically, we do not assume that a government can create safety. Instead, we emphasize that a government's statutory power allows taxation of personal asset returns, even though these proceeds cannot be transferred via contracts. We show that although governments cannot create safety, they can *redistribute* it.

In this section, we consider public provision of safety that takes the form of deposit insurance; in Appendix A, we study provision of public debt. Both forms are backed by a lump-sum tax T on all investors such that the government breaks even.[17] The latter allows us to internalize the negative ex-ante effects of public provision of safety.

Note that taxing liquidation proceeds at $t = 1$ simply redistributes scarce safe assets, while taxing proceeds from risky investment at $t = 2$ does not produce any revenue if the state turns out to be low. Hence, for deposit insurance to be credible (and public debt to be safe), taxation of personal asset returns at $t = 2$ is required. We show that in contrast to public debt, public provision of safety via deposit insurance can crowd in private provision of safety and boost aggregate investment.

4.1 Deposit Insurance

We consider a deposit insurance fund that insures a fraction $\phi \in (0, 1)$ of the debt issued by intermediaries. We use subscript DI to denote quantities with deposit insurance. Proposition 5 presents the impact of introducing deposit insurance.

Proposition 5. ***Impact of deposit insurance.*** *For deposit insurance ϕ, the safe rate solves* (18)*. When safety capacity is sufficiently scarce, the introduction of deposit insurance increases the safe rate, $\left.\frac{dr_{DI}}{d\phi}\right|_{\phi\to 0} > 0$, private provision of safety, $\left.\frac{dd_{DI}}{d\phi}\right|_{\phi\to 0} > 0$, and aggregate investment, $\left.\frac{dI_{DI}}{d\phi}\right|_{\phi\to 0} > 0$.*

Proof. See Appendix C.5. □

Recall that safety capacity constraint is given by (11). By incorporating a deposit insurance policy ϕ into (11), safety capacity constraint becomes

[17] Allowing for lump-sum taxation shuts down its redistribution effect and enables us to better highlight the impact of public provision of safety.

$$(1-\phi)r_{DI}\overbrace{F(r_{DI})\left(\frac{S+T-\alpha}{r_{DI}-\alpha}\right)}^{\text{debt, } d_{DI}} \leq \alpha\left[\overbrace{F(r_{DI})\left(\frac{S+T-\alpha}{r_{DI}-\alpha}\right)}^{\text{debt, } d_{DI}}+\overbrace{\int_{r_{DI}}^{r^H}\left(1-\frac{S+T}{r}\right)dF(r)}^{\text{equity, } e_{DI}}\right]. \quad (18)$$

The balanced-budget condition requires that $T = \phi r_{DI} F(r_{DI})\left(\frac{S+T-\alpha}{r_{DI}-\alpha}\right)$, which implies that $T = \frac{\phi r_{DI} F(r_{DI})(S-\alpha)}{(1-\phi F(r_{DI}))r_{DI}-\alpha}$. Internalizing the impact of taxes on safety-seeking incentives gives rise to two conflicting forces. On the one hand, even allowing for the additional demand for safety coming from investors' incentives to cover prospective taxes, the demand for safety that intermediaries face (captured by the LHS of (18)) is lower. The latter is true because intermediaries are responsible for fraction $1-\phi$ of the total demand for safety. On the other hand, safety capacity of intermediaries decreases: equity holders need to self-insure more to cover prospective taxes, which decreases the amount of loss-absorbing equity (captured by the second term in the square bracket in the RHS of (18)). We show in the proof of Proposition 5 that if safety capacity is sufficiently scarce, for $\phi \to 0$, the first effect dominates and deposit insurance relaxes intermediaries' safety capacity constraint and leads to a higher safe rate.

The introduction of deposit insurance can lead to higher or lower aggregate investment, as it has both investment gains and losses. Investment gains come from the fact that deposit insurance and the accompanying taxes, effectively, shift self-insurance from less efficient to more efficient investors, improving allocation of resources. Investment losses come from the need of productive investors, who rely on self-insurance, to increase self-insure to cover prospective taxes. Again, if safety capacity is sufficiently scarce, for $\phi \to 0$, the share of resources allocated to self-insurance (instead of investment) to cover prospective taxes is also small, and investment gains exceed investment losses.

Deposit insurance when safety capacity is abundant. For completeness, note that deposit insurance increases the safe rate and can lead to higher aggregate investment even if safety capacity is abundant (see Online Appendix). However, the underlying intuition differs: by ensuring a safe payment at $t = 2$, deposit insurance weakens debt holders' incentives to withdraw. As a result, deposit insurance alleviates the problem of value-decreasing liquidation in state RR, which increases expected asset value and investment.

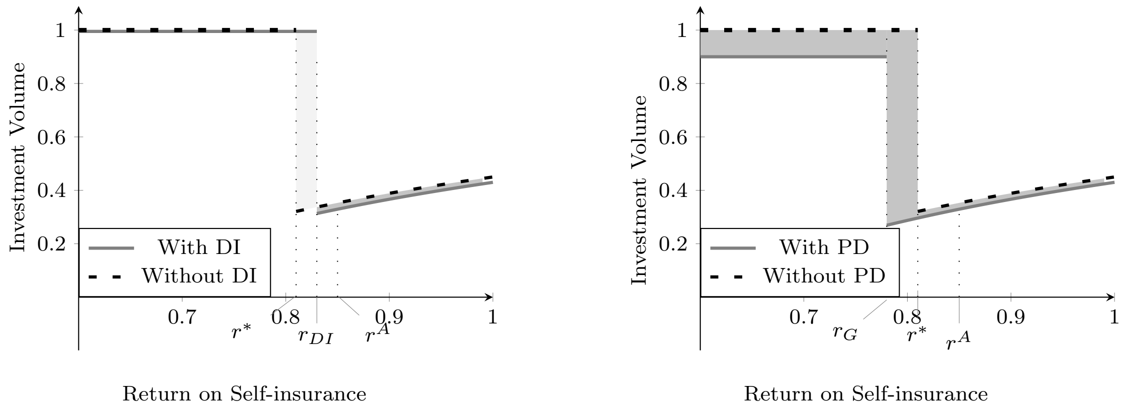

Figure (6) Impact of deposit insurance and public debt when safety capacity is scarce. The left (right) panel depicts the case of deposit insurance (public debt). The solid (dashed) line illustrates investment with (without) public provision of safety. The light-shaded (dark-shaded) area captures investment gains (losses) from public provision of safety. Parameters: $S = 0.55$, $r_L = 0.6$, $r_H = 1$, $\gamma = \alpha = \delta$, $R = 4$, $r \sim G[0.6, 1]$ with $G(r) = 12.5r - 6.25r^2 - 5.25$. For these values, $r^A = 0.85$, $r^E = r^* = 0.81$. For deposit insurance covering $\phi = 0.1$ of safety demand, $r_{DI} = 0.83$. For public debt covering $\psi = 0.1$ of safety demand, $r_G = 0.78$.

Deposit Insurance versus Public Debt. This section shows that with deposit insurance, private and public provision of safety can be complements. Interestingly, in Appendix A, we show that this complementarity breaks down when public provision of safety takes the form of public debt. The latter verifies that findings of Gorton et al. (2012), Sunderam (2015), Lenel et al. (2017), Jackson and Pennacchi (2021), and Gorton and Ordonez (2022) in a setting where public debt is backed by taxation guaranteeing its safety. The implication of crowding out of private safety provision and investment by public debt replicates the result of Krishnamurthy and Vissing-Jorgensen (2015) in a context with safety preferences and is consistent with evidence documented in their paper. Corollary 3 compares the impact of deposit insurance and public debt. The left (right) panel of Figure 6 illustrates the impact of deposit insurance (public debt) on the safe rate and aggregate investment.

Corollary 3. *Deposit insurance leads to weakly better allocations of resources than public debt. Deposit insurance can increase the safe rate, in contrast to public debt which decreases it. Public debt crowds out investment, in contrast to deposit insurance which can crowd in or out investment.*

5 Conclusion

We offer a theory of financial intermediation and demandable debt based on investor preferences for safety. Our approach is consistent with recent evidence on the scale of safe assets as a share of wealth and on their segmented pricing. Our parsimonious setup analyzes how households achieve their reference level of consumption before the introduction of financial assets, thanks to direct control over personal assets and real investment.

Financial intermediaries funded by demandable debt can improve upon autarky by providing more efficient insurance, boosting investment and expected output. Specifically, by issuing demandable senior claims backed by adequate loss-absorbing capacity (equity capital), banks can commit to a safe payoff. Although intermediation produces a conflict between debt and equity holders in residual risk states, the option to withdraw upon demand completes the contract and implements a safe payoff via partial liquidation of investment under residual risk. This novel view of demandable debt is driven by direct safety provision (rather than liquidity cross-insurance, as in Diamond and Dybvig (1983))

In contrast to the well-known result of crowding out induced by public debt, we find that public provision of safety—via deposit insurance—can crowd in its private provision via bank debt and increase the safe rate and aggregate investment. Interestingly, the latter is true even when we allow for the negative ex-ante implications of the prospective taxation backing deposit insurance. The underlying mechanism is that deposit insurance reduces inefficient liquidation (when safety capacity is abundant) and can relax intermediaries' safety capacity constraint (when safety capacity is scarce).

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A Public debt issuance

In this section, we study the case where public safety provision takes the form of public debt. We assume that at $t = 0$, a government issues an amount of public debt $G > 0$ to be repaid at $t = 2$. We use subscript G to denote quantities with public debt. We denote by r_{PD} the endogenous return on public debt and by r_G the safe rate. For public debt to be safe, it has to be backed by taxation equal to $T = Gr_{PD}$. We focus on the interesting case where the supply of government debt cannot satisfy the entire demand for safety, i.e., $G \in (0, \hat{\hat{G}})$.[18] Proposition 6 summarizes the impact of public debt.

Proposition 6. ***Impact of public debt.*** *For $G \in (0, \hat{\hat{G}})$, public debt and private debt are perfect substitutes, thus $r_{PD} = r_G^*$, where r_G^* solves* (20)*. Public debt decreases the safe rate r_G^*, crowds out private provision of safety, and decreases aggregate investment.*

Proof. See Appendix C.6. □

For $G \in (0, \hat{\hat{G}})$, by incorporating public debt into (11) we obtain the following safety capacity constraint of intermediaries:

$$\overbrace{\left[F(r_G)\left(\frac{S+T-\alpha}{r_G-\alpha}\right)-G\right]}^{\text{debt } d_G} r_G \leq \alpha \left[\overbrace{F(r_G)\left(\frac{S+T-\alpha}{r_G-\alpha}\right)-G}^{\text{debt } d_G}+\overbrace{\int_{r_G}^{r^H}\left(1-\frac{S+T}{r}\right)dF(r)}^{\text{equity } e_G}\right]. \quad (19)$$

Suppose that public debt covers fraction ψ of total demand for safety, which implies that $Gr_G = \psi r_G F(r_G)\left(\frac{S+T-\alpha}{r_G-\alpha}\right)$. Then, (19) reduces to:

$$(1-\psi)F(r_G)\left(\tfrac{S+T-\alpha}{r_G-\alpha}\right) r_G \leq \alpha \left[(1-\psi)F(r_G)\left(\tfrac{S+T-\alpha}{r_G-\alpha}\right)+\int_{r_G}^{r^H}\tfrac{r-S-T}{r}dF(r)\right], \quad (20)$$

where the balanced-budget constraint requires $T = Gr_G = \psi r_G F(r_G)\left(\frac{S+T-\alpha}{r_G-\alpha}\right)$ which implies $T = \frac{\psi r_G F(r_G)(S-\alpha)}{(1-\psi F(r_G))r_G-\alpha}$. Note that issuing public debt decreases the amount of loss-absorbing

[18] For $G > \hat{\hat{G}}$, the accompanied tax implies that safety capacity of intermediaries falls below safety demand. Formally, $\hat{G}$ solves $(S+T)F(r_G) - Gr_G = \alpha(1 - \int_{r_G}^{r^H} \frac{S+T}{r} dF(r))$, where $T = Gr_G$.

equity as some resources are allocated to taxes. We show in the proof that independently of the distribution $F(.)$ and of whether safety demand decreases or increases, the decline in loss-absorbing equity decreases the safe rate. Therefore, public debt increases the level of self-insurance and crowds out aggregate investment. For completeness, we show in the Online Appendix that the findings of Proposition 6 extend to the case where safety capacity is abundant.

B Auxiliary Lemmas

B.1 Investors' portfolio choice

Investors' portfolio choice depends on whether the face value r^* of demandable debt is higher or lower than r^A (defined in (3)). As both cases can arise in equilibrium depending on safety capacity, we explore them separately. We denote the return on equity as $PV + \epsilon$, where ϵ can be thought of as the *excess return* on equity, i.e., the return achieved above and beyond the return of investing directly.

Lemma 2. ***Portfolio choice when*** $\mathbf{r}^* \geq \mathbf{r}^{\mathbf{A}}$***.*** *Suppose that the intermediary offers demandable debt with face value* $r^* \geq r^A$ *backed by enough loss-absorbing equity* $\underline{e}$ *and promises a return on equity* $PV + \epsilon$*, with* $\epsilon \in [0, \hat{\epsilon}]$*. Investors with* $r \leq r^*$ *prefer to achieve safety by holding debt* $d(r^*) = \frac{S}{r^*}$*, whereas investors with* $r > r^*$ *prefer to achieve safety by self-insuring* $x = \frac{S}{r}$*. The remaining resources are invested in equity. The total amount of debt and equity is given by* (23) *and* (24)*.*

Lemma 3. ***Portfolio choice when*** $\mathbf{r}^* < \mathbf{r}^{\mathbf{A}}$***.*** *Suppose that the intermediary offers demandable debt with face value* $r^* < r^A$ *backed by enough loss-absorbing equity* $\underline{e}$ *and promises a return on equity* $PV + \epsilon$*, with* $\epsilon \in [0, \hat{\epsilon}]$*. Investors with* $r < r^*$ *prefer to hold demandable debt* $d(r^*) = \frac{S-\alpha}{r^*-\alpha}$ *and invest their remaining resources directly, such that they can liquidate if the state is RR or L. Investors with* $r \in [r^*, r^\epsilon]$ *prefer to self-insure* $x = \frac{S-\alpha}{r-\alpha}$ *and invest their remaining resources directly, such that they can liquidate if the state is RR or L. Fi-*

nally, investors with $r > r^\epsilon$ prefer to self-insure fully, i.e., $x = \frac{S}{r}$, and invest their remaining resources in equity. The total amount of debt and equity is given by (28) *and* (29).[19]

The intuition of Lemma 2 and Lemma 3 is the following. Investors who have a maximum guaranteed return of at least r^A (independently if this is via personal assets or demandable debt), find it optimal to have *full* insurance at $t = 0$, and allocate their remaining resources to risky, but more profitable, equity. In contrast, if the maximum guaranteed return is lower than r^A, investors find it optimal to have *partial* insurance at $t = 0$ (either by self-insuring or holding demandable debt, depending on which alternative implies a higher guaranteed return), and invest their remaining resources directly, such that they maintain the option of liquidating at $t = 1$ if the state is RR or L. The proofs of Lemma 2 and Lemma 3 follows.

Proof of Lemma 2. Suppose that the intermediary offers demandable debt with face value $r^* \geq r^A$ backed by enough loss-absorbing equity $\underline{e}$ and promises a return on equity $PV + \epsilon$, with $\epsilon \in [0, \hat{\epsilon}]$. Proposition 1 implies that for a guaranteed return $r' \geq r^A$, investors always prefer full to partial insurance. The latter allows them to allocate their remaining resources to risky equity. The investor can secure S either by demandable debt $d(r^*) = \frac{S}{r^*}$ or by self-insurance $x = \frac{S}{r}$. The expected return of each strategy is

$$\frac{S}{r^*}r^* + \left(1 - \frac{S}{r^*}\right)(PV + \epsilon), \tag{21}$$

$$\frac{S}{r}r + \left(1 - \frac{S}{r}\right)(PV + \epsilon). \tag{22}$$

For $r^* > r$ the first strategy dominates, and vice versa. The total amount of debt and equity is given by

$$\hat{d} = \int_{r_L}^{r^A} \frac{S}{r} dF(r), \tag{23}$$

$$\hat{e} = \int_{r_L}^{r^A} \left(1 - \left(\frac{S}{r^A}\right)\right) dF(r) + \int_{r^A}^{r_H} \left(1 - \left(\frac{S}{r}\right)\right) dF(r). \tag{24}$$

[19] Where $r^\epsilon = \frac{\alpha[\epsilon + \alpha\delta(1-\gamma) + \gamma R]}{\alpha\delta + \epsilon + (1-\delta)\gamma R}$. If $\epsilon = 0$, then $r^\epsilon = r^A$, whereas if $\epsilon = \hat{\epsilon}$, then $r^\epsilon = r^*$, where $\hat{\epsilon} = \frac{\alpha\delta[\alpha(1-\gamma) - r] + \gamma R[\alpha - (1-\delta)r]}{r - \alpha}$.

Proof of Lemma 3. Suppose that the intermediary offers demandable debt with face value $r^* \geq r^A$ backed by enough loss-absorbing equity $\underline{e}$ and promises a return on equity $PV + \epsilon$, with $\epsilon \in [0, \hat{\epsilon}]$. Proposition 1 implies that for a guaranteed return $r' > r^A$, investors always prefer full to partial insurance, and vice versa. In general, there are three possible strategies: (i) hold demandable debt $d = \frac{S-\alpha}{r^*-\alpha}$ and invest directly such that liquidation is possible in state RR; (ii) self-insure $x = \frac{S-\alpha}{r-\alpha}$ and invest directly such that liquidation is possible in state RR (iii) self-insure fully $x = \frac{S}{r}$ and invest equity. The expected return of each strategy is

$$\frac{S-\alpha}{r^*-\alpha}r^* + \left(1 - \frac{S-\alpha}{r^*-\alpha}\right)[\delta\gamma R + (1-\delta\gamma)\alpha], \tag{25}$$

$$\frac{S-\alpha}{r-\alpha}r + \left(1 - \frac{S-\alpha}{r-\alpha}\right)[\delta\gamma R + (1-\delta\gamma)\alpha], \tag{26}$$

$$\frac{S}{r}r + \left(1 - \frac{S}{r}\right)(PV + \epsilon). \tag{27}$$

The expected return is higher under strategy (i) when $r < r^*$; strategy (ii) when $r \in [r^*, r^\epsilon]$; strategy (iii) when $r > r^\epsilon$, where $r^\epsilon = \frac{\alpha[\epsilon + \alpha\delta(1-\gamma) + \gamma R]}{\alpha\delta + \epsilon + (1-\delta)\gamma R}$. Note that if $\epsilon = 0$, then $r^\epsilon = r^A$, whereas if $\epsilon = \hat{\epsilon}$, then $r^\epsilon = r^*$, where $\hat{\epsilon} = \frac{\alpha\delta[\alpha(1-\gamma)-r] + \gamma R[\alpha - (1-\delta)r]}{r-\alpha}$. The total amount of debt and equity is given by

$$\hat{d} = \int_{r_L}^{r^*} \frac{S-\alpha}{r^*-\alpha} dF(r), \tag{28}$$

$$\hat{e} = \int_{r^*}^{r_H} \left(1 - \left(\frac{S}{r}\right)\right) dF(r). \tag{29}$$

C Omitted Proofs

C.1 Proof of Proposition 1

Let x be the share allocated to self-insurance; the residual $1-x$ is invested. Applying backward indication, first, consider the liquidation decision on the interim date. Full liquidation is always optimal in state L (since $\alpha > 0$). In contrast, liquidation is always suboptimal in state H (since $R > \alpha$). In state RR, given that liquidation is value-destroying (since $\gamma R > \alpha$), investors will liquidate the minimum fraction necessary to secure safety, $\ell \equiv \frac{S-rx}{\alpha(1-x)} \in [0,1]$.[20]

Next, consider the choice of x. Following the restrictions in the parameter space (summarized in (2)), $x \in [\underline{x}, \overline{x}]$. It is never optimal to self-insure more than the amount necessary to secure safety, i.e., $\overline{x} \equiv \frac{S}{r}$, as investing has a higher expected return. Similar, it is never optimal to self-insure less than $\underline{x} \equiv \frac{S-\alpha}{r-\alpha} < \overline{x}$, as even if investment is liquidated fully, the recovered amount does not secure safety.

Anticipating their future liquidation actions, at $t = 0$ investors choose x to maximize expected output Y^A subject to achieving safety in all states:

$$x^A \equiv \arg \max_{x \in [\underline{x}, \overline{x}]} Y^A(x; r), \tag{30}$$

where

$$Y^A(x,r) = xr + (1-x)\left\{\delta\left[\gamma R + (1-\gamma)\alpha\right] + (1-\delta)\left[\ell\alpha + (1-\ell)\gamma R\right]\right\}. \tag{31}$$

Finally, substituting $\ell \equiv \frac{S-rx}{\alpha(1-x)}$ into (31), we can express Y^A as:

$$Y^A(x,r) = PV - x(PV - r) - (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)(S - rx). \tag{32}$$

Intuitively, Y^A equals the present value of investment reduced by the opportunity cost of self-insurance and the expected loss from liquidation in state RR.

Since all expected returns are linear in x, the optimal portfolio choice in Autarky is a corner solution, i.e., $(x^A = \underline{x}, \ell = 1)$ or $(x^A = \overline{x}, \ell = 0)$. The expected return of the strategy involving full self-insurance is $S + \left(1 - \frac{S}{r}\right) PV$, whereas the expected return of the strategy involving partial self-insurance is $S + \frac{\delta\gamma(\alpha - R)(S-r)}{r-\alpha}$. The latter strategy is more (less) attractive to investors whose personal assets promise a large (small) return, $r > r^A$ ($r < r^A$), where r^A is given by (3). Investors with $r = r^A$ are indifferent between the two strategies. We restrict attention to the case where $r^A \in (r_L, r_H)$, where the threshold r^A is interior if $\delta \in (\tilde{\delta}, \tilde{\tilde{\delta}})$, where $\tilde{\delta} \equiv \frac{\gamma R\left(\frac{r_L}{\alpha} - 1\right)}{(1-\gamma)\alpha + r_L\left(\frac{\gamma R}{\alpha} - 1\right)} \in (0,1)$ and $\tilde{\tilde{\delta}} \equiv \frac{\gamma R\left(\frac{r_H}{\alpha} - 1\right)}{(1-\gamma)\alpha + r_H\left(\frac{\gamma R}{\alpha} - 1\right)} \in (\tilde{\delta}, 1)$.

[20] Note that $\ell(\overline{x}) = 0$ and $\ell(\underline{x}) = 1$ for all r.

C.2 Proof of Proposition 2

We formally state and solve the planner's problem. An investor's type is its self-insurance return, $r \in [r_L, r_H]$, with probability density function $f(r)$ and cumulative distribution $F(r)$. Critically, the return on self-insurance is non-contractible, so the proceeds of self-insurance cannot be credibly promised to another investor or taxed and redistributed by the planner. Thus, redistribution across investors requires contractible investment and contingent liquidation at $t = 1$, as will become clear below.

The state of nature s_t is $s_1 \in \{H, RR, L\}$ and $s_2 \in \{H, L\}$. The contractible return on investment at $t = 2$ is $R(s_2) = R\,\mathbf{1}_{\{s_2=H\}}$, with non-contingent liquidation value α. The history of states σ_t is $\sigma_1 = s_1$ and $\sigma_2 = \{(H, H), (RR, H), (RR, L), (L, L)\}$. The probabilities of a history, $\pi(\sigma_t)$, are $\pi(H) = \gamma\delta = \pi((H, H))$, $\pi(L) = (1-\gamma)\delta = \pi((L, L))$, $\pi(RR) = 1-\delta$, $\pi((RR, H)) = \gamma(1-\delta)$, and $\pi((RR, L)) = (1-\gamma)(1-\delta)$.

The choice variables are (i) self-insurance by investors of type r for their own safety purposes, $x(r) \in [0, 1]$; (ii) the proportion of state-contingent interim liquidation of investment, $\ell(s_1) \in [0, 1]$; (iii) the allocation of proceeds from investment at $t = 2$ contingent on type and the history of states, $\chi(r, \sigma_2) \geq 0$; and (iv) consumption levels contingent on type and the history of states, $\{c_1(r, s_1), c_2(r, \sigma_2)\}$, with $c_t(\cdot) \geq 0$. The aggregate volumes are $X \equiv \int_{r_L}^{r_H} x(r)dF(r)$ for self-insurance and $I \equiv 1 - X$ for investment. The objective is to maximize the expected consumption of investors:

$$\max\ W \equiv \int_{r_L}^{r_H} \sum_{\sigma_2} \pi(\sigma_2)\big[c_1(r, \sigma_2[1]) + c_2(r, \sigma_2)\big]dF(r), \tag{33}$$

where $\sigma_2[1]$ is the first element of σ_2, for example RR in the history (RR, L). There are several constraints. A resource constraint at $t = 1$ states that aggregate consumption at $t = 1$ comes from liquidation of investment in each state, where we ignore weak inequalities because the objective function is strongly monotone in consumption, $\int_{r_L}^{r_H} c_1(r, s_1)dF(r) = \ell(s_1)\alpha I$ for all s_1. The proceeds from liquidation can be freely redistributed between investors, so only an aggregate constraint is relevant. A resource constraint at $t = 2$ states that consumption for each type and in each state comprises the proceeds from self-insurance and allocated proceeds from investment, where the allocation shares add up to one:

$$c_2(r,\sigma_2) = rx(r) + \chi(r,\sigma_2)\left[1-\ell(\sigma_2[1])\right] R(\sigma_2)\, I, \quad \forall (r,\sigma_2), \tag{34}$$

$$1 = \int_{r_L}^{r_H} \chi(r,\sigma_2) dF(r), \quad \forall \sigma_2. \tag{35}$$

A safety constraint states that each investor achieves the reference consumption level, $c_1(r,\sigma_2[1]) + c_2(r,\sigma_2) \geq S, \quad \forall (r,\sigma_2)$. Finally, a participation constraint of each investor states that each investor achieves at least its autarky level of expected consumption:

$$\sum_{\sigma_2} \pi(\sigma_2)\left[c_1(r,\sigma_2[1]) + c_2(r,\sigma_2)\right] \geq Y^A(r), \quad \forall r, \tag{36}$$

where $Y^A(r) \equiv Y^A(x^A(r), r)$ is the expected autarky output under the optimal autarky portfolio choice, $x^A(r)$. Thus, the aggregate expected output in autarky is

$$\begin{aligned} Y^A &\equiv \int_{r_L}^{r_H} Y^A(r) dF(r) \qquad (37) \\ &= S + PV\left[1 - \frac{\alpha}{r^A}\int_{r_L}^{r^A} \frac{r-S}{r-\alpha} dF(r) - \int_{r_L}^{r^A} \frac{S-\alpha}{r-\alpha} dF(r) - \int_{r^A}^{r_H} \frac{S}{r} dF(r)\right]. \end{aligned}$$

We turn to solving for the efficient allocation. Ignore the participation constraints; we show below how they are satisfied. Integrating the resource constraint at $t=2$ over investors, we obtain for aggregate consumption:

$$\int_{r_L}^{r_H} c_2(r,\sigma_2) dF(r) = \int_{r_L}^{r_H} rx(r) dF(r) + (1-\ell(\sigma_2[1]))R(\sigma_2)I, \quad \forall \sigma_2, \tag{38}$$

since the allocation shares add up to one. Critically, this approach requires us to restrict attention to $x \in [0, \frac{S}{r}]$, since self-insurance cannot be done on behalf of others and it is never optimal to self-insure more than what already achieves safety in all states, given that the opportunity cost of self-insurance is $PV > r$.

In history (H,H), it is optimal not to liquidate, $\ell^*(H) = 0 = c_1^*(r,H)$, because $R > \alpha$ and the reference consumption level can be achieved at either date. The aggregate resource constraint at $t=2$ implies that $\int_{r_L}^{r_H} c_2(r,(H,H)) dF(r) = \int_{r_L}^{r_H} rx(r) dF(r) + RI$. In history

(L, L), it is optimal to liquidate fully, $\ell^*(L) = 1$, since $\alpha > 0$. The aggregate resource constraints imply that $\int_{r_L}^{r_H} c_1(r, L)dF(r) = \alpha I$ and $\int_{r_L}^{r_H} c_2(r, (L, L))dF(r) = \int_{r_L}^{r_H} rx(r)dF(r)$. Consider state $s_1 = RR$. Let the liquidation proportion be $\ell \equiv \ell(RR) \in [0, 1]$, so the resource constraint at $t = 1$ implies $\int_{r_L}^{r_H} c_1(r, RR)dF(r) = \ell\alpha I$. The resource constraint at $t = 2$ depends on which return on investment is realized: $\int_{r_L}^{r_H} c_2(r, (RR, L))dF(r) = \int_{r_L}^{r_H} rx(r)dF(r)$ or $\int_{r_L}^{r_H} c_2(r, (RR, H))dF(r) = \int_{r_L}^{r_H} rx(r)dF(r) + RI$.

Since the proceeds from investment can be rearranged across investors at either date, it suffices to satisfy the safety constraint in the aggregate,

$$\int_{r_L}^{r_H} \big[c_1(r, \sigma_2[1]) + c_2(r, \sigma_2)\big] dF(r) \geq S, \quad \forall \sigma_2. \tag{39}$$

We study the safety constraint for each history. For (H, H), it reduces to $R + \int (r - R)x(r)dF(r) \geq S$, where we used the definition of I. This inequality holds strictly for any choice of $x(r)$, since $R > r \geq S$. For (L, L), the safety constraint reduces to $\int_{r_L}^{r_H}(r - \alpha)x(r)dF(r) \geq S - \alpha$, which can be rewritten as:

$$\alpha I \geq S - \int_{r_L}^{r_H} rx(r)dF(r), \tag{40}$$

so some self-insurance by investors is required. The history (RR, L) is more restrictive than (RR, H) since fewer resources are available at $t = 2$. It can be stated as $\ell \geq \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}$. Since $\ell \leq 1$, this constraint is more restrictive than that in history (L, L).

Using the above resource constraints for each history, we simplify the objective function to the following reduced optimization problem subject to safety constraints:

$$\max_{\ell \in [0,1], \{x(r) \in [0, \frac{S}{r}]\}} W \quad = \quad PVI + \int_{r_L}^{r^H} rx(r)dF(r) - (1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\ell\alpha I \tag{41}$$

$$\text{s.t.} \qquad I = 1 - \int_{r_L}^{r^H} x(r)dF(r), \quad \ell \geq \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}. \tag{42}$$

Since $\frac{dW}{d\ell} = -(1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)\alpha I < 0$, $\ell^* = \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}$ and $W(\ell^*) = PVI + \int_{r_L}^{r^H} rx(r)dF(r) - (1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\left(S - \int_{r_L}^{r_H} rx(r)dF(r)\right)$, the reduced problem is:

$$\max_{\{x(r)\in[0,\frac{S}{r}]\}} W = PV - \int_{r_L}^{r^H}(PV-r)x(r)dF(r) - (1-\delta)\left(\frac{\gamma R}{\alpha}-1\right)\left(S-\int_{r_L}^{r_H} rx(r)dF(r)\right)$$
$$\text{s.t.} \quad S-\int_{r_L}^{r_H} rx(r)dF(r) \leq \alpha\left(1-\int_{r_L}^{r^H} x(r)dF(r)\right). \tag{43}$$

Note that $\frac{dW}{dx(r)} = PVf(r)\left[\frac{r}{r^A}-1\right]$. Therefore, it is optimal to use a threshold strategy, whereby investors of high types fully self-insure, $x^E(r) = \frac{S}{r}\mathbf{1}_{\{r\geq r^E\}}$, for some r^E to be determined. Moreover, it follows from $\frac{dW}{dx(r)}$ that $r^E \leq r^A$, depending on whether the safety constraint (43) is slack, i.e., safety capacity is abundant or scarce. We explore each case separately.

Abundant safety capacity. By Definition 1, abundant safety capacity corresponds to the case where the safety constraint (43) is slack. Following the discussion above, the efficient threshold above which full self-insurance occurs is $r^E = r^A$. For the safety constraint to be indeed slack, we require $SF(r^A) \leq \alpha I(r^A)$, where $I(r^A) \equiv 1 - \int_{r^A}^{r_H} = \frac{S}{r}dF(r) \equiv 1 - X(r^A)$. We next characterize this inequality and when it holds. In particular, we state and then prove the following Lemma.

Lemma 4. *Safety capacity constraint binds at a unique threshold, $r_{sc} \in (r_L, r_H)$. Moreover, there exists a unique level, $\bar{S} > \alpha$, such that $r^A \leq r_{sc}$ if and only if $S \leq \bar{S}$.*

Consider the implicit function $H(r) \equiv SF(r) - \alpha(1 - X(r))$. We first show that H equals zero once in its domain $[r_L, r_H]$, thus defining a threshold value r_{sc} at which safety capacity binds: $H(r_{sc}) \equiv 0$. Uniqueness follows from strong monotonicity, since $\frac{dH(r)}{dr} = Sf(r)(1-\frac{\alpha}{r}) > 0$. Existence follows from different signs of its bounds, since $H(r_L) = -\alpha\left(1-\int_{r_L}^{r_H}\frac{S}{\rho}dF(\rho)\right) < 0$ and $H(r_H) = S - \alpha > 0$. Thus, a unique $r_{sc} \in (r_L, r_H)$ exists. The aggregate safety constraint can be expressed as $r^E \leq r_{sc}$.

We turn to the construction of the bound $\bar{S}$. By the implicit function theorem, $\frac{dr_{sc}}{dS} < 0$ because $\frac{dH}{dS} = F(r) + \alpha\int_{r^E}^{r_H}\frac{dF(\rho)}{\rho} > 0$. This strong monotonicity ensures the uniqueness of $\bar{S}$. To ensure $\bar{S} > \alpha$ (so $r^A < r_{sc}$ at $S \to \alpha$), it suffices to show $H(r^A) < 0$ when $S \to \alpha$. This condition always holds because $\alpha F(r^A) - \alpha\left(1-\int_{r^A}^{r_H}\frac{\alpha}{\rho}dF(\rho)\right) < 0 \Leftrightarrow \int_{r^A}^{r_H}\left(\frac{\alpha}{\rho}-1\right)dF(\rho) <$

0. This boundary condition ensures the existence of $\bar{S}$. Taking these results together, we have $r^E = r^A$ when the constraint is slack.

Scarce safety capacity. If $S > \bar{S}$, then the safety constraint (43) is violated at r^A, so the efficient threshold return has to be lower. As a result, $r^E = r_{sc} < r^A$, since $\frac{dW}{dx}\big|_{r=r_{sc}} < 0$ and the planner does not wish to increase self-insurance beyond what is required to satisfy the aggregate safety constraint.

Participation constraints. Finally, we need to show that the participation constraints bind. Since the proceeds from investment can be freely rearranged across investors (the liquidation proceeds at $t = 1$ and the return at $t = 2$), it suffices to show that the expected output under the efficient allocation, Y^E, is no smaller than the expected autarky output, Y^A. The latter always holds, independently of whether $S \leq \bar{S}$ or $S > \bar{S}$. This is intuitive, as the planner can always implement the autarky allocation, thus, the aggregate output in the efficient allocation can never be lower than the aggregate output in autarky.

C.3 Proof of Proposition 3

First, note that when safety capacity is scarce, intermediaries cannot offer a safe rate r^A: even if they liquidate their entire investment ($\ell = 1$), the resulting value will not be sufficient to repay all depositors, thus, debt would no longer be safe. As a result, $r^* < r^A$, and $\hat{d}$ and $\hat{e}$ are determined by (28) and (29) in Lemma 3.

Similar to Bertrand competition on prices, in equilibrium, competing intermediaries offer the highest feasible safe rate, defined by feasibility constraint (11), which should bind in equilibrium. This can be shown with the method of contradiction. Suppose that constraint (11) holds with strict inequality. If this was the case, that would imply that the intermediary has excessive equity, i.e., more than the minimum amount necessary to guarantee safety of debt. Recall that by Lemma 3, the supply of equity is weakly increasing in excess return on equity ϵ. Hence, the intermediary can decrease e without violating feasibility constraint (11) by decreasing excess return on equity ϵ. At the same time, by Lemma 3, decreasing ϵ would increase the safe rate r which leaves investors indifferent between debt and equity, as equity would become less attractive compared to debt. However, a higher safe rate would contradict with initial hypothesis that in equilibrium, intermediaries offer the highest feasible safe rate. Therefore, constraint (11) cannot hold with strict inequality in equilibrium.

Next, solving (11) with respect to e and substituting it into (10) implies that condition (10) holds with equality. Hence, (10) is redundant. Also, solving (11) with respect to e and substituting it into (9), we obtain

$$V(r, \epsilon, d, e) = d\frac{\delta\gamma r, (R - \alpha)}{\alpha} \tag{44}$$

which increases in the level of debt d, therefore, constraint (12) binds. Combining that $V(r, \epsilon, d, e)$ increases in debt with the observation that debt capacity increases in the level of loss-absorbing equity, which implies that, for given ϵ, the intermediary fully absorbs the supply of equity, i.e., (13) binds.

So far we have shown that optimal safe rate r^* solves (11) with equality conditional on $d = \hat{d}$ and $e = \hat{e}$, where $\hat{d}$ and $\hat{e}$ are determined by (28) and (29) in Lemma 3. In what follows, we characterize the optimal excess return on equity ϵ^*. As shown in the proof of Lemma 3, investors with $r = r^*$ are indifferent between holding debt and equity for $\epsilon = \epsilon^*$, where ϵ^* solves

$$\frac{S - \alpha}{r^* - \alpha} r^* + \left(1 - \frac{S - \alpha}{r^* - \alpha}\right)[\delta\gamma R + (1 - \delta\gamma)\alpha] = \frac{S}{r^*} r^* + \left(1 - \frac{S}{r^*}\right)(PV + \epsilon), \tag{45}$$

which implies that

$$\epsilon^* = \frac{\alpha\delta[\alpha(1 - \gamma) - r^*] + \gamma R[\alpha - (1 - \delta)r^*]}{r^* - \alpha}. \tag{46}$$

Finally, for r^* and ϵ^*, by Lemma 3, $\hat{d}$ and $\hat{e}$ is given by (15) and (16) in the main text. Gathering everything together implies that the equilibrium safe rate equates demand and supply of safety, i.e., the equilibrium safe rate solves equation (17).

C.4 Proof of Proposition 4

Part (i). Consider an improvement in investment prospects. As we show in Proposition 3, the safe rate is determined by condition (17), which is not a function of γ, δ or R. Thus, better investment prospects do not affect the equilibrium safe rate r^*. Following that, the level of debt and equity given by (15) and (16) respectively remain unchanged. The same holds for aggregate self-insurance $X(r^*) = \int_{r^*}^{r^H} \left(\frac{S}{r}\right) dF(r)$ and aggregate investment $I(r^*) = 1 - X(r^*)$.

The only variable affected is excess return on equity (given by (14)) that implements r^*. Note that $\frac{\partial \epsilon^*}{\partial R} = \frac{\gamma[\alpha-(1-\delta)r^*]}{r^*-\alpha}$, which is positive for $\delta > \hat{\delta}$, where $\hat{\delta} = \frac{r^*-\alpha}{r^*}$. Also, $\frac{\partial \epsilon^*}{\partial \delta} = \frac{(1-\gamma)\alpha^2-\alpha r^*+\gamma R r^*}{r^*-\alpha} > 0$ and $\frac{\partial \epsilon^*}{\partial \gamma} = \frac{\alpha(R-\alpha\delta)+(1-\delta)Rr^*}{r^*-\alpha} > 0$ for any admissible value of parameters.

Part (ii). Consider an increase in liquidation value, captured by higher α. All else constant, an increase in α increases the RHS of (17) which captures the safety capacity of intermediaries. For the equilibrium condition to be restored, r^* has to increase. Let r^* ($r^{*'}$) denote the safe rate before (after) the increase in liquidation value. $r^* < r^{*'}$ implies that aggregate self-insurance decreases: $X(r^*) = \int_{r^*}^{r^H} \left(\frac{S}{r}\right) dF(r) > X(r^{*'}) = \int_{r^{*'}}^{r^H} \left(\frac{S}{r}\right) dF(r)$. Equivalently, the aggregate investment increases, i.e., $I(r^*) = 1 - X(r^*) < I(r^{*'}) = 1 - X(r^{*'})$. Also, $r^* < r^{*'}$ implies that equity decreases: $\int_{r^*_{sc}}^{r_H} \left(1 - \left(\frac{S}{r}\right)\right) dF(r) > \int_{r^{*'}}^{r_H} \left(1 - \left(\frac{S}{r}\right)\right) dF(r)$.

In addition, for the distributional assumption $(r - \alpha)\, f(r) > F(r)$ (which holds for the uniform distribution, for example), the level of debt d^* increases with r. Finally,

$$\frac{\partial \epsilon^*}{\partial \alpha} = \frac{-\alpha^3(1-\gamma) - \alpha[\alpha(-2+\gamma)+\gamma R]r^* - (\alpha-\gamma R)(r^*)^2 + \alpha\delta\gamma(\alpha - R)\frac{\partial r^*}{\partial \alpha}}{(r^*-\alpha)^2}$$

can be positive or negative, thus the impact of α on ϵ^* is ambiguous.

Part (iii). The proof of this part is identical to the proof of part (i) and part (ii). The only difference is that we consider changes while holding the value of PV constant.

Part (iv) and (v). We present the proof of part (iv); the proof of part (v) follows the same logic. Consider an increase in S. All else constant, an increase in S decreases the

RHS of (17), which captures safety capacity of intermediaries, and increases the LHS of (17), which captures the safety demand. For the equilibrium condition to be restored, r^* has to decrease. Let r^* ($r^{*'}$) denote the safe rate before (after) the increase of the reference level from S to S'. $r^* > r^{*'}$ implies that aggregate self-insurance increases as $X(r^*) = \int_{r^*}^{r^H} \left(\frac{S}{r}\right) dF(r) < X(r^{*'}) = \int_{r^{*'}}^{r^H} \left(\frac{S'}{r}\right) dF(r)$. Equivalently, aggregate investment increases: $I(r^*) = 1 - X(r^*) < I(r^{*'}) = 1 - X(r^{*'})$. Finally, note that excess return on equity (given by (14)) is not a function of S, thus, it is only affected via the safe rate, where $\frac{\partial \epsilon^*}{\partial r^*_{sc}} < 0$.

C.5 Proof of Proposition 5

As we show in Proposition 3, the equilibrium safe rate is determined by the binding safety capacity constraint (11). By incorporating a deposit insurance policy ϕ into (11), we obtain (18). First, we show that for a sufficiently small safe rate (which implies that safety capacity is very scarce), the introduction of deposit insurance increases the safe rate.

Let r_{DI} denote the equilibrium safe rate with deposit insurance. Note that the equilibrium condition (18) can be expressed as

$$F(r_{DI}) = \alpha \int_{r_{DI}}^{r^H} \frac{r - (S+T)}{r} \frac{\alpha - r_{DI}(1 - \phi F(r_{DI}))}{\alpha - r_{DI}(1-\phi)} dF(r). \tag{47}$$

Recall that the balanced-budget condition requires that $T = \phi r_{DI} F(r_{DI}) \left(\frac{S+T-\alpha}{r_{DI}-\alpha}\right)$, which implies that $T = \frac{\phi r_{DI} F(r_{DI})(S-\alpha)}{(1-\phi F(r_{DI}))r_{DI}-\alpha}$. Substituting T into (47), we obtain

$$F(r_{DI}) = \alpha \int_{r_{DI}}^{r^H} \underbrace{\frac{((\alpha - r_{DI})(r-S) + r_{DI}(-\alpha F(r_{DI}) + r - (1 - F(r_{DI}))S)\psi)(\alpha - r_{DI}(1 - F(r_{DI})))}{r(\alpha - r_{DI}(1-\psi))^2}}_{M} dF(r). \tag{48}$$

Note that if $\left.\frac{\partial M}{\partial \phi}\right|_{\phi=0} > 0$, then the RHS of the equilibrium condition (48) increases when deposit insurance is introduced; thus, for the equilibrium to be restored, the LHS of (48) has to increase, which implies that the safe rate has to increase as $F(r_{DI})$ is an increasing function of the safe rate r_{DI}. It holds that:

$$\left.\frac{\partial M}{\partial \phi}\right|_{\phi=0} = \frac{(r-S) + (\alpha - r)F(r_{DI})}{r(r_{DI}) - \alpha}. \tag{49}$$

Note that since $r_{DI} > S > \alpha$, the denominator is always positive, whereas the numerator can be positive or negative. Hence, identifying the sign requires additional assumptions regarding the distribution $F(.)$. However, note that in the case where safety capacity is very scarce, it holds that $r_{DI} \to r_L$, which implies that $F(r_{DI}) \to 0$, and $\frac{\partial M}{\partial \phi}\Big|_{\phi=0} > 0$. As a result, when safety capacity is very scarce, the introduction of deposit insurance increases the safe rate, i.e., $\frac{r^*_{DI}}{\partial \phi}\Big|_{\phi=0} > 0$.

Next, we explore how the introduction of deposit insurance affects aggregate investment $I(r_{DI})$, given by

$$I(r_{DI}) = 1 - \int_{r_{DI}}^{r^H} \frac{S+T}{r} dF(r), \tag{50}$$

which can be expressed as

$$I(r_{DI}) = F(r_{DI}) + \int_{r_{DI}}^{r^H} \left(1 - \frac{S+T}{r}\right) dF(r). \tag{51}$$

Note that the equilibrium condition (18) can be expressed as

$$\alpha \int_{r_{DI}}^{r^H} \left(1 - \frac{S+T}{r}\right) dF(r) = \frac{(S-\alpha)(\alpha - r_{DI}(1-\phi))}{\alpha(\alpha - r_{DI}(1-\phi F(r_{DI})))} F(r_{DI}). \tag{52}$$

Substituting (53) into (51), we obtain

$$I(r_{DI}) = F(r_{DI}) \left(1 + \frac{(S-\alpha)(\alpha - r_{DI}(1-\phi))}{\alpha(\alpha - r_{DI}(1-\phi F(r_{DI})))}\right). \tag{53}$$

Finally, differentiating with respect to ϕ and evaluating at $\phi = 0$, we obtain

$$\frac{\partial I(r_{DI})}{\partial \phi}\Big|_{\phi=0} = \frac{(\alpha - S) r_{DI} F(r_{DI})(1 - F(r_{DI})) + S(r_{DI} - \alpha)\frac{dF(r_{DI})}{dr_{DI}}\frac{dr_{DI}}{d\phi}|_{\phi=0}}{r(r_{DI} - \alpha)}, \tag{54}$$

where the first term in the numerator is negative and the second is positive. Identifying the sign requires additional assumptions regarding the distribution $F(.)$. However, note that in the case where safety capacity is very scarce, it holds that $r_{DI} \to r_L$, which implies that $F(r_{DI}) \to 0$, and $\frac{\partial I(r_{DI})}{\partial \phi}\Big|_{\phi=0}$. As a result, when safety capacity is very scarce, the introduction of deposit insurance increases aggregate investment.

C.6 Proof of Proposition 6

Since both public and private debt are substitutes for achieving safety, in equilibrium, their returns are equalized, i.e., $r_G^* = r_{PD}$. If $r_{PD} < r_G^*$, demand for public debt is zero and its market fails to clear (since $G > 0$), so its return must rise. If $r_{PD} > r_G^*$, there is no demand for private debt. Hence, in what follows we use that $r_{PD} = r_G^*$.

As we show in Proposition 3, the equilibrium safe rate is determined by the binding safety capacity constraint. Recall that when safety capacity is scarce, safety capacity constraint is given by (11). By incorporating that public debt covers a fraction ψ of demand for safety into (11), we obtain (20). First, we show that the introduction of public debt decreases the safe rate. Let r_G denote the equilibrium safe rate with public debt. Note that the equilibrium condition (20) can be expressed as

$$F(r_G) = \alpha \int_{r_G}^{r^H} \frac{r-(S+T)}{r} \frac{1}{S+T-\alpha} dF(r), \tag{55}$$

Recall that the balanced-budget condition requires that $T = Gr_G = \psi r_G F(r_G)\left(\frac{S+T-\alpha}{r_G-\alpha}\right)$ which implies $T = \frac{\psi r_G F(r_G)(S-\alpha)}{(1-\psi F(r_G))r_G-\alpha}$. Substituting T into (55) we obtain

$$F(r_G) = \alpha \int_{r_G}^{r^H} \underbrace{\frac{\alpha(-r+S+F(r_G)r_G\psi) - r_G(s+r(F(r_G)\psi-1)}{r(1-r_G)(\alpha-S)}}_{M'} dF(r). \tag{56}$$

Note that if $\frac{\partial M'}{\partial \phi} < 0$, the RHS of equilibrium condition (56) decreases in ψ, thus, for the equilibrium to be restored, the LHS of (56) has to decrease, which implies that the safe rate has to decrease as $F(r_G)$ is an increasing function of the safe rate r_G. It holds that:

$$\frac{\partial M'}{\partial \phi} = \frac{F(r_G)(a-r)r_G}{r(\alpha-r_G)(\alpha-S)} < 0, \tag{57}$$

thus, the safe rate decreases with ψ. Finally, aggregate investment is given by

$$I(rG = 1 - \int_{r_G}^{r^H} \frac{(S+T)}{r} \tag{58}$$

which decreases in ψ as $\frac{dr_G}{d\psi} < 0$ and $\frac{dT}{d\psi} > 0$.

D Figure: Safe assets as a fraction of wealth and GDP

Figure 7 replicates Gorton et al. (2012), where our estimates follow their estimates reduced by long-term private debt.

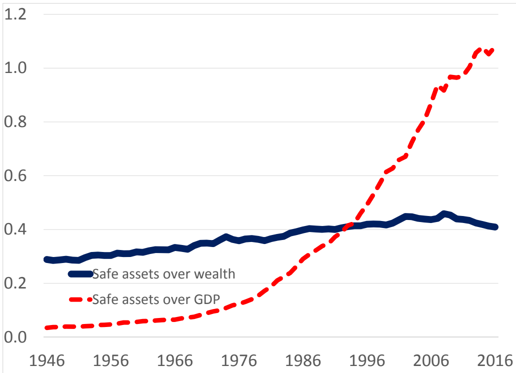

Figure (7) Safe assets as a fraction of wealth and GDP.