Information Choice and Amplification of Financial Crises

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We propose an amplification mechanism of financial crises based on the information choices of investors. Information acquisition makes investors more likely to act against their prior. Deteriorating public news under an initially strong (weak) prior increases (reduces) the value of private information and induces more (less) information acquisition. Deteriorating public news increases the probability of a crisis, since the initially strong (weak) prior induces no attacks (attacks). This effect is amplified with endogenous information choices. To enhance financial stability, a policy maker affects information acquisition via taxes and subsidies. We derive and discuss testable implications for the magnitude of amplification. (*JEL* G01, D82, F31, G21, G12, G28)

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Financial investors devote substantial economic activity to acquiring information. In particular, information acquisition played an important role in the most recent financial crisis. In Section 1, we review evidence about the importance of such information acquisition in the run-up of several recent crisis episodes. In this paper, we show how the acquisition of information by investors amplifies the probability of a financial crises.

Financial crises historically were explained by either weak fundamentals or panics.[1] The global games literature pioneered by Carlsson and van Damme

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[1] A self-fulfilling crisis can be caused by a panic among bank depositors, as in Bryant (1980) and Diamond and Dybvig (1983), or among currency speculators, as in Obstfeld (1996). By contrast, fundamental-based crises have been analyzed by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998) for bank runs, and by Krugman (1979) for currency crises. See also Goldstein (2012) for a recent review.

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(1993) reconciles both of these views, since weak fundamentals cause the self-fulfilling beliefs about a financial crisis. In particular, global coordination games of regime change are used to study bank runs, currency attacks, and debt crises.[2] A crisis occurs if sufficiently many depositors withdraw funds from a bank, currency speculators attack a peg, or creditors do not roll over debt. In these models, financial investors base their decisions on an exogenous endowment of public and private information about an unobserved fundamental that measures the profitability of a bank, the foreign reserves of a central bank, or the solvency of a debtor.

In Section 2, we offer a parsimonious model of information choice in a standard global coordination game of regime change. Investors choose ex ante whether to improve, at a cost, the quality of their private information about the fundamental, for example, by hiring analysts or purchasing data. A heterogeneous information cost ensures the existence of a unique equilibrium, in which investors with a sufficiently low cost acquire information.[3]

In Section 3, we explain how the information choices of investors amplify the probability of a financial crisis. To illustrate the mechanism, we consider a debt rollover game. Each investor wishes to roll over debt whenever the debtor is solvent; this more likely occurs when other investors also roll over. Suppose that public information about debtor solvency is strong, for example, the debtor's credit rating is high. What happens after a rating downgrade or an earnings warning, either of which weakens the public information about debtor solvency?

After a downgrade, debt holders have a higher incentive to acquire private information, which is more valuable for two reasons. First, since the public signal casts more doubt about the solvency of the debtor, acquiring private information helps a debt holder determine the solvency of the debtor. Second, acquiring private information helps a debt holder anticipate the rollover decisions of other debt holders. For these fundamental and strategic reasons, a debt holder with more precise private information is more likely to roll over debt when the debtor is solvent, and more likely to withdraw when a debt run occurs and the debtor is insolvent. As a result, more investors acquire private information after a downgrade.

A larger proportion of informed investors increases the probability of a debt crisis. Uninformed investors have less precise private information about debtor solvency and therefore rely on public information more. Since the initial credit rating is high, uninformed investors tend to roll over debt. By contrast, informed investors have more precise private information and therefore tend

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[2] See Rochet and Vives (2004) and Goldstein and Pauzner (2005) for bank runs; Morris and Shin (1998) and Corsetti et al. (2004) for currency attacks; Morris and Shin (2004) and Corsetti et al. (2006) for debt crises. See Bebchuk and Goldstein (2011) for credit freezes and also Vives (2005, 2014).

[3] Endogenous information in global coordination games can lead to multiple equilibria. In Angeletos and Werning (2006), a public market price aggregates dispersed private information, similar to Grossman and Stiglitz (1980). See also Hellwig et al. (2006) and Angeletos et al. (2006).

to disregard the initially favorable public information, the high credit rating. As a result, informed investors refuse to roll over debt for a larger range of private signals. Hence, a larger proportion of informed investors increases the solvency threshold below which a debt crisis occurs and the ex ante probability of a debt crisis.[4]

The case of strong, but deteriorating, public information (e.g., a downgrade of an initially high rating) is arguably empirically relevant for many recent financial crisis episodes. However, amplification also occurs for low and deteriorating public information, though the workings of the mechanism differ. After a downgrade of a debtor with an initially low credit rating, the value of private information is lower and fewer investors acquire private information. Intuitively, there is little doubt about the insolvency of the debtor and the rollover behaviour of other creditors. A smaller proportion of informed investors, however, increases the probability of a debt crisis for a low rating. The intuition is again that uninformed investors are less likely to act against public information, which is unfavorable in this case. The different workings of the amplification mechanism suggest a different policy response.

In Section 4, we study how a policy maker can enhance financial stability. We consider a deep-pocketed policy maker concerned about reducing the probability of a financial crisis. We start by studying taxes and subsidies on information acquisition, which alter the information choice of investors. However, the appropriate policy response to deteriorating public information depends on the solvency of the debtor. For a creditworthy debtor, taxation is desirable to discourage information acquisition. In contrast, for a less creditworthy debtor, a subsidy is desirable to encourage information acquisition. Similar results hold when the policy maker taxes and subsidizes investor payoffs instead.

We also study the consequences of an improvement in the quality of public information. Policies that make public information more precise include the regulation of credit rating agencies to make future ratings more informative or a commitment to publish the results of future bank stress tests. These interventions occur ex ante, that is, before investors observe public information about debtor solvency or asset quality.[5] The first effect is increased coordination on the more informative public signal. The second effect is that fewer investors acquire private information (crowding out). In sum, improving the quality of public information has an *ambiguous* effect on the probability of a crisis; this effect is lower whenever the public signal is strong. This policy may have

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[4] This result on the extensive margin (the proportion of informed investors) complements the result on the intensive margin (precision of private information) in Metz (2002).

[5] In contrast, the tax or subsidy on information acquisition or investor payoffs is ex post, that is, after investors observed the public signal about debtor solvency. Note that either policy intervention occurs *before* investors decide whether to acquire private information.

unintended consequences, for example, committing to releasing bank stress tests could amplify future banking crises.

Turning to testable implications in Section 5, we derive and characterize the magnitude of amplification. Our theory implies that the magnitude of amplification is (1) nonmonotone in the public information about debtor solvency, such as credit ratings; (2) higher for less dispersed distributions of the information cost; (3) higher when the precision improvement in private information is larger. Intuitively, the magnitude is larger the more investors alter their information choice and the more precise private information of informed investors is. To obtain these testable implications, we generalize our model to a generic distribution of information costs and limited precision improvement. These extensions also illustrate the robustness of the amplification mechanism. Throughout this section, we discuss several environments in which these implications about the magnitude of amplification can be tested.

In Section 6, we study extensions to probe the robustness of the mechanism further. An important feature of the amplification mechanism is how a higher proportion of informed investors affects the probability of a crisis. Analyzing a specification with generalized payoffs proposed by Iachan and Nenov (2015), who generates a more general link between the proportion of informed investors and the probability of a crisis, we state sufficient conditions for amplification to occur. We also consider an extension with a homogeneous information cost that yields multiple equilibria (Hellwig and Veldkamp 2009). We explain the cases in which amplification still obtains. A conclusion and appendices with derivations and proofs follow.

Other theories of amplification have been proposed. Fire sales occur when the natural buyer of an asset experiences financial stress (Shleifer and Vishny 1992, 1997; Kiyotaki and Moore 1997). Such sales can be induced by predatory trading (Brunnermeier and Pedersen 2005). Investors may disengage from markets due to complexity (Caballero and Simsek 2013) or Knightian uncertainty (Caballero and Krishnamurthy 2008). Under adverse selection in secondary debt markets (Gorton and Pennacchi 1990), information production may be destabilizing (Dang et al. 2012; Gorton and Ordonez 2014).

Our model is related to the literature of information choice in coordination games. Hellwig and Veldkamp (2009) first studied the optimal information choice in strategic models. In a beauty contest, they show that the information choices of investors inherit the underlying strategic motive (complementarity or substitutability).[6] We confirm their "inheritance result" in the context of a global coordination game with parsimonious information choice. Furthermore, Hellwig and Veldkamp (2009) showed that multiple equilibria arise from a binary information choice and homogeneous information cost. In our model, a unique equilibrium obtains since the information cost is *heterogeneous*

[6] Myatt and Wallace (2012) and Colombo et al. (2014) also studied information choice in beauty contests.

across investors. In a regime change game, Szkup and Trevino (2015) studied continuous information choice subject to a convex information cost that is *homogeneous* across investors. They investigated the efficiency of equilibrium, when information choices are complements or substitutes, and the trade-off between public and private information, by focusing on the *precision* of public information. In contrast, we focus on the *level* of public information, which is crucial for our amplification mechanism. Specifically, we study how changes in the public signal affect both the incentives of investors to acquire information and the probability of a crisis.

Our paper contributes to a literature on dynamic global games of regime change. Dasgupta (2007) studied the option to delay foreign direct investment in emerging markets, where the benefit of more precise information is traded off with a lower return on investment. Angeletos et al. (2007) studied an infinite-horizon version with the arrival of additional private information over time that generates rich equilibrium dynamics. Like Szkup and Trevino (2015), we focus on costly information acquisition in a first stage that affects the actions of investors in a second stage, such as their decision to roll over debt. In Yang (2015), the information cost is proportional to the implied reduction in entropy, which generates a coordination motive in information choices and multiple equilibria.[7]

He and Manela (2016) studied the acquisition of information about bank liquidity and the dynamic withdrawal decisions of investors. Building on the asynchronous awareness model of Abreu and Brunnermeier (2003), their model yields rich time-series implications about run behavior. In contrast, our static coordination game is a very different framework and we emphasize the acquisition of information about bank *solvency*.[8]

1. Information Acquisition and Financial Crises

In this section, we review existing evidence about the importance of information acquisition in several recent crisis episodes. We also state a noisy but direct measure of information acquisition before the crises at Bear Stearns, Lehman, and Greek debt. Private information acquisition seems to occur before each crisis event after bad public news arrived.

Kacperczyk and Schnabl (2010) argued that information acquisition was an important factor in the run on commercial paper in 2007–2008. Public information about the safety of commercial paper deteriorated due to both concerns about the quality of the underlying collateral and counterparty risk. They stated, "[B]efore the financial crisis, most investors believed that

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7 Kendall (2015) offers another alternative specification of information costs. Modeling a financial market, information acquisition has a time cost due to the expected adverse price movements induced by other traders.

8 Nikitin and Smith (2008) studied costly verification of solvency in a Diamond and Diamond and Dybvig (1983) setup.

commercial paper almost never defaults and therefore had little incentive to invest in information gathering about issuers of commercial paper. [...] However, during the crisis, investors decided to invest more resources in information gathering activities [...]" (p. 45). This is consistent with our model.

According to the Financial Crisis Inquiry Report (2011), the hedge fund investors of Bear Stearns acquired information after Bear Stearns reported its first quarterly loss, but before the eventual run in March 2008. Consistent with our model's prediction of more private information acquisition after public information about the financial health of Bear Stearns deteriorated, the Financial Crisis Inquiry Report (2011) stated, "The hedge funds that were clients of Bear's prime brokerage services were particularly concerned that Bear would be unable to return their cash and securities. Lou Lebedin, the head of Bear's prime brokerage, told the FCIC that hedge fund clients occasionally inquired about the bank's financial condition in the latter half of 2007, but that such inquiries picked up at the beginning of 2008" (p. 286).

He and Manela (2016) argued that information acquisition played a role in the run on the U.S. commercial bank Washington Mutual and the run by U.S. money market mutual funds on European banks, specifically those with exposure to Greek debt. According to Reuters 2011, information acquisition differed across money market funds, which yielded different conclusions by fund managers, whereby some managers rolled over bank debt, while others withdrew.

Finally, we report an imperfect but direct measure of investor attention and information acquisition for three crises episodes. Da et al. (2011) suggested Google search frequency as a real-time measure that can be accessed via Google Trends. Figure 1 reports this measure for the crises of Bear Stearns, Lehman Brothers, and Greek debt. It shows an increase in search volumes before each crisis event after bad public news arrived.

2. A Stylized Crisis Model with Information Choice

We propose a model of parsimonious information choice in a global coordination game of regime change. A unit continuum of risk-neutral investors $i \in [0,1]$ simultaneously decide whether ($a_i = 1$) or not ($a_i = 0$) to attack the regime. Regime change occurs if enough investors attack; that is, if the aggregate attack size $A \equiv \int_0^1 a_i \, di$ exceeds a fundamental $\theta \in \mathbb{R}$ that measures the strength of the regime. Building on Vives (2005), the payoff from attacking is a benefit $b \in (0,1)$ if regime change occurs, and a loss $\ell \in (0,1)$ otherwise:

$$u(a_i = 1, A, \theta) = b \, \mathbf{1}_{\{A \geq \theta\}} - \ell \, \mathbf{1}_{\{A < \theta\}}, \qquad (1)$$

where $\mathbf{1}$ is the indicator function. An investor's incentive to attack is given by the conservativeness ratio $\kappa \equiv \frac{\ell}{b+\ell} \in (0,1)$. The constant payoff from not attacking is normalized to zero, so the differential payoff from attacking increases in the attack size A and decreases in the fundamental θ. As a result, there is a

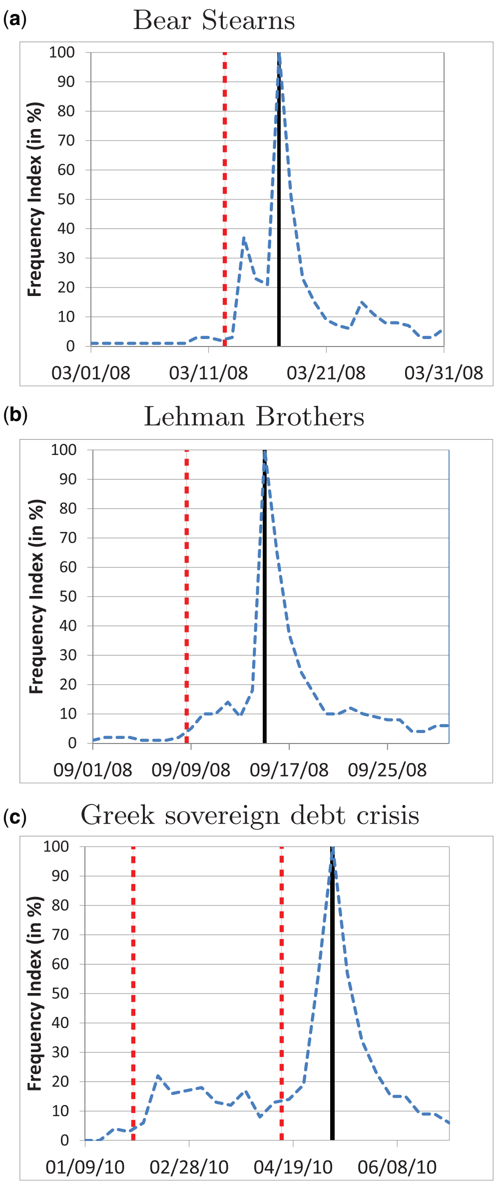

Figure 1
Worldwide web search interest on Google for three crisis episodes
Panel (a) plots the key word "Bear Stearns" during March 2008; panel (b) plots "Lehman Brothers" during September 2008; and panel (c) plots "Greek crisis" during the first half of 2010 as a single dashed blue line. All values are normalized to 100%. The solid black line represents the crisis event: March 16, 2008 (sale of Bear Stearns to JP Morgan), September 15, 2008 (failure of Lehman Brothers), and May 2, 2010 (political agreement of first Greek bailout package). The dashed red line represents the arrival of bad public news: March 14, 2008 (*New York Times*, frontpage: "Who could buy Bear Stearns?"), September 10, 2008 (*New York Times*, frontpage: "Wall Street fears on Lehman Bros. batters markets"), and February 3, 2010 (*The Economist*, frontpage: "A Greek bailout, and soon?"), and April 22, 2010 (*Wall Street Journal*, frontpage: "Investors desert Greek bond market"). *Source*: GoogleTrends.

Table 1
Time line

Information stage	Coordination stage
1. Public signal μ about fundamental θ	1. Private signal x_i about fundamental θ • signal more precise if informed
2. Simultaneous information choice • binary action $n_i \in \{I, U\}$ • heterogeneous information cost c_i	2. Simultaneous attack decision • binary action $a_i \in \{0, 1\}$
	3. Outcome of regime and payoffs

coordination motive among investors whose attack decisions exhibit global strategic complementarity.

Our preferred interpretation of a regime change is a financial crisis, such as a currency crisis, a bank run, or a sovereign debt crisis. The fundamental θ can be interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998; Corsetti et al. 2004), as the measure of investment profitability (Rochet and Vives 2004; Goldstein and Pauzner 2005; Corsetti et al. 2006) or a sovereign's taxation power or willingness to repay. Investors can be interpreted as currency speculators, as retail or wholesale bank creditors who withdraw funds, or as sovereign debt holders who refuse to roll over.

There is incomplete information about the fundamental, which is drawn from an improper uniform prior. Investors receive a public and a private signal (Morris and Shin 2003):

$$\mu \equiv \theta + \nu, \quad \nu \sim \mathcal{N}\left(0, \alpha^{-1}\right), \tag{2}$$

$$x_i \equiv \theta + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}\left(0, \gamma^{-1}\right), \tag{3}$$

where the aggregate noise ν is normally distributed with zero mean and precision $\alpha \in (0, \infty)$, and is independent of the fundamental. Idiosyncratic noise ϵ_i is identically and independently distributed across investors and independent of both the fundamental and the aggregate noise. The idiosyncratic noise is normally distributed with zero mean and an *endogenous* precision γ, described below. The information structure is common knowledge.

Table 1 summarizes the timeline of events. At the information stage, investors receive the public signal and simultaneously make a costly binary information choice, $n_i \in \{I, U\}$. At the subsequent coordination stage, informed investors ($n_i = I$) receive more precise private information:

$$\gamma_U < \gamma_I. \tag{4}$$

Unless stated otherwise, we focus on vanishing noise for informed investors, $\gamma_I \to \infty$, which provides a parsimonious benchmark and maintains uniqueness at the coordination stage.[9]

9 We analyze an extension with limited precision improvement, $\gamma_I \in (\gamma_U, \infty)$, in Section 5.4.

An information cost captures the resources required to acquire and process information. Skill differences in generating and processing information are captured by heterogeneity in the information cost that is uniformly distributed over a unit interval:

$$c_i \sim \mathcal{U}[0,1].^{10} \tag{5}$$

2.1 Equilibrium

We start with a definition of the equilibrium concept. We focus on symmetric equilibria.

Definition 1. A pure-strategy perfect Bayesian equilibrium comprises an information choice, $n_i^* = z \in \{I, U\}$ of each investor $i \in [0,1]$, an aggregate proportion of informed investors, $n^* \in [0,1]$, attack rules, $a_I^*(\cdot) \in \{0,1\}$ and $a_U^*(\cdot) \in \{0,1\}$ for informed and uninformed investors, respectively, and an aggregate attack size, $A^* \in [0,1]$, such that:

1. For a given proportion of informed investors n^* and a given information choice $n_i^* = z$, the attack rule specifies an optimal behavior for each investor i at the coordination stage:

$$\begin{aligned} a_z^*(\cdot) = \arg\max_{a_i} a_i \big[& b \Pr(A^* \geq \theta | n^*, n_i^* = z; \mu, x_i) \\ & - \ell \Pr(A^* < \theta | n^*, n_i^* = z; \mu, x_i) \big]. \end{aligned} \tag{6}$$

2. For a given proportion of informed investors n^*, the aggregate attack size is consistent with the individually optimal attack behavior:

$$A^* = n^* \int_0^1 a_I^*(\cdot) di + (1 - n^*) \int_0^1 a_U^*(\cdot) di. \tag{7}$$

3. The aggregate proportion of informed investors is consistent with the individually optimal information choices:

$$n^* = \int_0^1 \mathbf{1}\{n_i^* = I\} di. \tag{8}$$

4. For a given proportion of informed investors n^* and attack rules $a_I^*(\cdot)$ and $a_U^*(\cdot)$, the private information choice n_i^* is optimal for each investor i:

$$n_i^* = \arg \max_{n_i \in \{I,U\}} \mathbf{1}\{n_i = I\}[EU^I - c_i] + \mathbf{1}\{n_i = U\} EU^U, \tag{9}$$

   where $EU^z = EU^z(n^*, a_I^*(\cdot), a_U^*(\cdot); \mu)$ is the expected utility of an investor who chooses $n_i^* = z$ at the information stage.

[10] We analyze an extension with a generic distribution of the information cost, $f(c)$, in Section 5.3.

Proposition 1. Existence of a unique equilibrium.

If private information is sufficiently precise, $\gamma_U > \underline{\gamma} \equiv \left(\frac{\alpha}{\sqrt{2\pi}-2}\right)^2$, then there generically exists a unique pure-strategy monotone perfect Bayesian equilibrium. It is characterized by (1) a threshold information cost, $\overline{c}$, and (2) signal thresholds for informed investors, $\overline{x}_I(\overline{c})$, and uninformed investors, $\overline{x}_U(\overline{c})$, and a fundamental threshold, $\overline{\theta}(\overline{c})$. At the information stage, investors acquire information if and only if their individual information cost is below the threshold, $n_i^* = I \Leftrightarrow c_i < \overline{c}$. The proportion of informed investors is $n^* = \overline{c}$. At the coordination stage, each investor attacks the regime if and only if it receives a private signal below the signal threshold specific to its information choice, $x_i < \overline{x}_z(\overline{c})$ for $n_i^* = z \in \{I, U\}$, and a regime change occurs if and only if the fundamental is below the threshold, $\theta < \overline{\theta}(\overline{c})$.

Proof. See Appendix A. ∎

The expected utility of an investor with information choice $n_i = z$ comprises two terms. An investor receives the benefit b when attacking ($x_i < \overline{x}_z$) if regime change occurs ($\theta < \overline{\theta}$), and the loss ℓ when attacking ($x_i < \overline{x}_z$) if no regime change occurs ($\theta > \overline{\theta}$):

$$
\begin{aligned}
EU^z &\equiv b\int_{-\infty}^{\overline{\theta}}\int_{-\infty}^{\overline{x}_z} f^z(x|\theta)dx\,dG(\theta) - \ell\int_{\overline{\theta}}^{\infty}\int_{-\infty}^{\overline{x}_z} f^z(x|\theta)dx\,dG(\theta) \\
&= bG(\overline{\theta}) - \ell\int_{\overline{\theta}}^{\infty}\int_{-\infty}^{\overline{x}_z} f^z(x|\theta)dx\,dG(\theta) \qquad (10) \\
&\quad - b\int_{-\infty}^{\overline{\theta}}\int_{\overline{x}_z}^{\infty} f^z(x|\theta)dx\,dG(\theta),
\end{aligned}
$$

where $G(\theta) \equiv \Phi\left(\sqrt{\alpha}[\theta - \mu]\right)$ is the cumulative distribution function (cdf) of the fundamental, $f^z(x) \equiv \sqrt{\gamma_z}\phi\left(\sqrt{\gamma_z}[x - \theta]\right)$ is the probability density function (pdf) of the private signal conditional on the fundamental and the information choice, and $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and cdf of the standard Gaussian random variable, respectively.

The first term in Equation (10) is the gain from attacking the regime if the investor had perfectly precise information about the fundamental. The second and third terms measure the mistakes of an investor due to imprecise information. The second term is the type I error, whereby an investor attacks ($x_i < \overline{x}_z$) but no regime change occurs ($\theta > \overline{\theta}$). The third term is the type II error, whereby an investor does not attack although a regime change occurs.

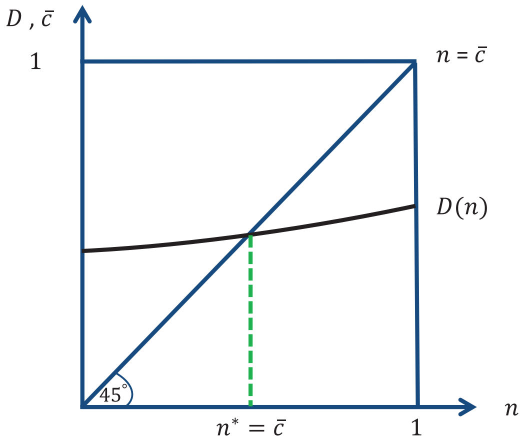

Figure 2
Value of more precise private information and the proportion of informed investors: A unique fixed point of the threshold information cost exists
Existence follows from the bounds on the value of information, $D \in (0,1)$ (Lemma 3). Uniqueness follows from the bounds on its slope, $\frac{dD}{dn^*} \in (0,1)$ (Lemma 2).

The value of more precise private information is thus $D \equiv EU^I - EU^U$:

$$D = \ell \int_{\overline{\theta}}^{\infty} \Gamma(\theta, \overline{x}_I, \overline{x}_U) dG(\theta) - b \int_{-\infty}^{\overline{\theta}} \Gamma(\theta, \overline{x}_I, \overline{x}_U) dG(\theta) \qquad (11)$$

$$\Gamma(\theta, \overline{x}_I, \overline{x}_U) \equiv \int_{-\infty}^{\overline{x}_U} f^U(x|\theta) dx - \int_{-\infty}^{\overline{x}_I} f^I(x|\theta) dx \equiv \Phi_U - \Phi_I, \qquad (12)$$

where $\Gamma(\theta, \overline{x}_I, \overline{x}_U)$ measures, for any given fundamental, the difference in the probability of attacking between an uninformed and informed investor. This difference is generically non-zero, since informed investors receive more precise information than uninformed investors and therefore use a different signal threshold. Informed investors receive a precise private signal and do not make errors, since they attack if and only if regime change occurs. The value of private information is therefore the sum of the errors made by uninformed investors, weighted by the appropriate payoff parameter: uninformed investors sometimes attack although no regime change occurs and sometimes do not attack although a regime change occurs.

An investor with the threshold information cost $\overline{c}$ is indifferent between acquiring and not acquiring information, $\overline{c} = D(\overline{c})$. Figure 2 illustrates the unique solution to this fixed-point problem, which determines the threshold information cost.

We establish bounds on the value of private information, $D \in (0,1)$, in Lemma 3 in Appendix A.2. These bounds ensure the existence of a threshold information cost. The value of private information is positive for both fundamental and strategic reasons. First, informed investors form a more precise and more accurate belief about the fundamental than uninformed investors:

$$\theta|n_j = z, x_j \sim \mathcal{N}\left(\frac{\alpha\mu+\gamma_z x_j}{\alpha+\gamma_z}, \frac{1}{\alpha+\gamma_z}\right), \tag{13}$$

where $z \in \{I, U\}$ and $\lambda_z^j \equiv \frac{\alpha\mu+\gamma_z x_j}{\alpha+\gamma_z}$ is the posterior mean. Second, as a result, informed investors form a more accurate and more precise belief about the private information received by other investors. For example, the posterior belief of an informed investor j about an informed investor i is

$$x_i^I|x_j^I \sim \mathcal{N}\left(\lambda_I^j, \frac{1}{\alpha+\gamma_I}+\frac{1}{\gamma_I}\right), \tag{14}$$

and the posterior belief of an uninformed investor j about an informed investor i is

$$x_i^I|x_j^U \sim \mathcal{N}\left(\lambda_U^j, \frac{1}{\alpha+\gamma_U}+\frac{1}{\gamma_I}\right). \tag{15}$$

We show in Lemma 2 in Appendix A.2 that the information choices of investors are strategic complements, $\frac{dD}{dn^*} > 0$. To obtain intuition, we consider the two effects of more informed investors on the value of private information. First, the precision of the belief about the fundamental is independent of the proportion of informed investors. Second, the precision of the belief about the aggregate attack size increases in the proportion of informed investors. Mechanically, the attack behavior of informed investors matters more for the aggregate attack size as more investors are informed. Moreover, an informed investor has a more precise belief about the private information of informed investors than about the private information of uninformed investors:

$$Var\left(x_i^I|x_j^I\right) = \frac{1}{\alpha+\gamma_I}+\frac{1}{\gamma_I} < \frac{1}{\alpha+\gamma_I}+\frac{1}{\gamma_U} = Var\left(x_i^U|x_j^I\right). \tag{16}$$

Uniqueness rests on heterogeneous information costs and precise private information. First, for a homogeneous information cost, multiple equilibria arise for a binary information choice (Hellwig and Veldkamp 2009). Despite the strategic complementarity in information choices, the amount of ex ante heterogeneity suffices for uniqueness. The support of the information cost includes all possible values of private information, resulting in dominance regions at the information stage. Specifically, there exists a lower dominance region $[0, D(0))$ in which acquiring information is a dominant action, and an upper dominance region $(D(1), 1]$ in which not acquiring information is a dominant action. Second, sufficiently precise private information of uninformed investors ensures that $\frac{dD}{dn^*} < 1$, whereby a larger threshold information cost raises the proportion of informed investors and, via strategic complementarity in information choice, the value of private information at a sufficiently low rate (Lemma 2). The required lower bound on the precision of private information,

$\underline{\gamma}$, is more restrictive than in a stand-alone global coordination game, such as Morris and Shin (2003).[11]

Definition 2. Let $\hat{\mu} \equiv 1-\kappa-\frac{\sqrt{\alpha+\gamma_U}-\sqrt{\gamma_U}}{\alpha}\Phi^{-1}(\kappa)$ denote a threshold level of the public signal. The public signal about the fundamental is **strong** if $\mu > \hat{\mu}$, and it is **weak** if $\mu < \hat{\mu}$.

A unique equilibrium exists generically. The inequality $\mu \neq \hat{\mu}$ ensures that the information and contagion stages are linked, whereby changes in the aggregate proportion of informed investors affect the fundamental threshold, $\frac{d\overline{\theta}}{dn^*} \neq 0$, as shown in Lemma 1 in Appendix A.1. All of the subsequent results also hold generically.

3. Amplification

A unique equilibrium is a solid foundation for comparative statics. Changes in the public signal, $d\mu$, give rise to amplification based on the information choice of investors. We study the ex ante probability of a financial crisis determined by the fundamental threshold:

$$\Pr\{\theta \leq \overline{\theta}\} = \Phi\left(\sqrt{\alpha}\left[\overline{\theta}(\overline{c}) - \mu\right]\right). \tag{17}$$

To illustrate the amplification mechanism, we compare endogenous and exogenous information. Let $\tilde{\theta}$ be the fundamental threshold if information is exogenous. A well-known result in the literature is that a weaker public signal raises the fundamental threshold:

$$\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n},\overline{\theta}} = \frac{A_\mu}{1-A_{\overline{\theta}}} < 0, \tag{18}$$

where $A_{\overline{\theta}}$ and A_μ are the partial derivatives of the aggregate attack size with respect to the fundamental threshold and the public signal, respectively, derived in Appendices A.1 and B.1. All partial derivatives are evaluated at the fundamental threshold $\overline{\theta}$ and an (exogenous) proportion of informed investors $\overline{n}$. To ensure comparability, this proportion is set to the equilibrium proportion of informed investors in the case of endogenous information, $\overline{n} = n^*$.

A novel effect arises if information is endogenous. Changes in the public signal now also affect the incentives to acquire information (captured by D_μ). In turn, changes in the proportion of informed investors affect the aggregate attack size (captured by A_{n^*}). In Appendix B.1, we derive the total effect of a change

[11] We further analyze the lower bound on the precision of private information required for uniqueness in the case of a generic distribution of the information cost in Section 5.3.

in the public signal on the fundamental threshold by totally differentiating the equilibrium conditions of both stages:

$$\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} = \frac{A_\mu + A_{n^*} D_\mu}{1 - A_{\overline{\theta}} - A_{n^*} D_{\overline{\theta}}} < 0, \tag{19}$$

where A_{n^*} is the partial derivative of the aggregate attack size with respect to the proportion of informed investors, while D_μ and $D_{\overline{\theta}}$ are the partial derivatives of the value of private information with respect to the public signal and the fundamental threshold. We derive $D_{\overline{\theta}}$ in the proof of Lemma 2 in Appendix A.2, and D_μ in Appendix B.1.

Proposition 2. Amplification. If private information is sufficiently precise, $\gamma_U > \underline{\gamma}$, then the information choices of investors amplify the impact of changes in public information on both the fundamental threshold and on the probability of a financial crisis:

$$-\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} > -\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n}=n^*,\overline{\theta}}. \tag{20}$$

Proof. See Appendix B. ■

The condition sufficient for uniqueness also suffices for amplification. The lower bound on the precision of private information ensures a positive denominator of Equation (19).

To provide intuition for the amplification result, we take a closer look at the constituting forces of the mechanism in three steps. First, we describe how a change in the public signal affects the incentives of investors to acquire private information. In equilibrium, there is a nonmonotonic relationship between the public signal about the fundamental and the value of private information, $D_\mu(\mu - \hat{\mu}) < 0$, shown in Figure 3 and proven in Appendix B.

Using the signal threshold of uninformed investors, $\overline{x}_U = \overline{\theta} + \frac{\alpha}{\gamma_U}[\overline{\theta} - \mu] - \frac{\sqrt{\alpha+\gamma_U}}{\gamma_U}\Phi^{-1}(\kappa)$, the value of private information can be expressed as follows for $\gamma_I \to \infty$:

$$D \to \ell \int_{\overline{\theta}}^{\infty} \Phi\Big(\sqrt{\gamma_U}\,[\overline{\theta} - \theta] + \frac{\alpha}{\sqrt{\gamma_U}}[\overline{\theta} - \mu] - \sqrt{1 + \frac{\alpha}{\gamma_U}}\,\Phi^{-1}(\kappa)\Big)$$

$$\phi\Big(\sqrt{\alpha}\,[\theta - \mu]\Big) d\theta + b \int_{-\infty}^{\overline{\theta}} \Big[1 - \Phi\Big(\sqrt{\gamma_U}\,[\overline{\theta} - \theta] + \frac{\alpha}{\sqrt{\gamma_U}}[\overline{\theta} - \mu]$$

$$- \sqrt{1 + \frac{\alpha}{\gamma_U}}\,\Phi^{-1}(\kappa)\Big)\Big]\,\phi\Big(\sqrt{\alpha}\,[\theta - \mu]\Big) d\theta. \tag{21}$$

To obtain intuition for the nonmonotonicity of the value of private information in the public signal, we build on Szkup and Trevino (2015). If

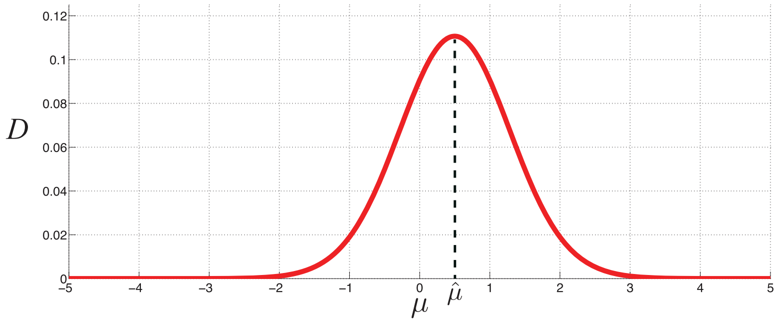

Figure 3
The relationship between the public signal μ and the value of private information D is non-monotonic
For low values of the public signal, $\mu < \hat{\mu}$, the value of private information increases in the public signal. The maximum value of private information is reached at $\mu = \hat{\mu}$. For high values of the public signal, $\mu > \hat{\mu}$, the value of private information decreases in the public signal. Parameter values are $b = \ell = 0.75$, $\alpha = 1$, and $\gamma_U = 4$, so $\underline{\gamma} = 3.89$ and $\hat{\mu} = 0.5$.

the public signal is weak, the fundamental threshold is high and a crisis is likely from an ex-ante perspective. Hence, there is a low probability of a type I error ex post, which is attacking the regime when no regime change occurs. As shown by the first term in Equation (21), the high realized levels of the fundamental required for no regime change to occur are unlikely (the term $\phi(\cdot)$ is small for all $\theta > \overline{\theta}$). There is also a low probability of a type II error, which is not attacking the regime although regime change occurs, as the second term in Equation (21) shows. The reason is different. Uninformed investors are likely to attack for a weak public signal (the term $1 - \Phi_U(\cdot)$ is small). Taken together, the value of private information is low for a weak public signal.

As the public signal strengthens, the fundamental threshold falls. The value of private information increases because both those fundamentals consistent with a type I error are now more likely and the probability of an uninformed investor attacking the regime falls, which increases the expected utility loss due to type II errors. Since precise private information allows an investor to avoid errors of either type, the value of private information increases. Similarly, the value of private information is low but increasing for a strong but weakening public signal. The maximum value of private information is reached at $\mu = \hat{\mu}$, which implies a fundamental threshold of $\hat{\overline{\theta}} = 1 - \kappa$. At this point, the fundamental threshold is insensitive to changes in the proportion of informed investors, $\frac{d\overline{\theta}}{dn^*}\Big|_{\mu=\hat{\mu}} = 0$, shown in Lemma 1.

In a second step, a change in the value of private information directly affects the equilibrium proportion of informed investors, as shown in Figure 4. A unique solution to the fixed-point problem $\overline{c} = D(n^* = \overline{c})$ exists. An increase in the value of information from D_1 to D_2, for example, raises the equilibrium proportion of informed investors from n_1^* to n_2^*. The magnitude of this change is affected by the degree of strategic complementarity in information choices.

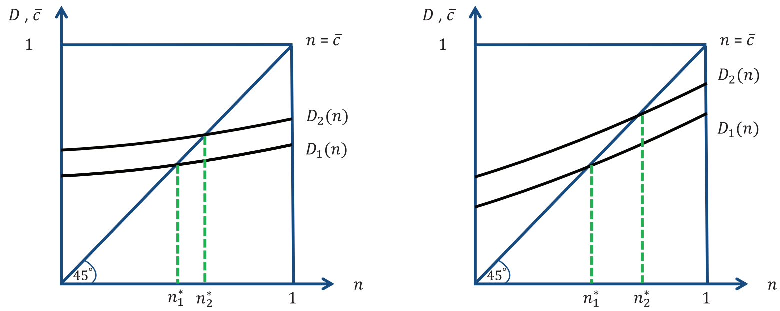

Figure 4
A higher value of private information increases the equilibrium proportion of informed investors
This effect is stronger the larger the degree of strategic complementarity in information choices. These strategic complementarities are stronger in the right panel.

If these strategic complementarities are stronger, such that the slope of the value of private information is steeper (right), then the increase is larger.

In a third step, changes in the proportion of informed investors affect the fundamental threshold below which a crisis occurs and, as a result, the probability of a financial crisis. For a strong public signal, $\mu > \hat{\mu}$, a larger proportion of informed investors leads to a higher threshold, while the converse result holds for a weak public signal, $\mu < \hat{\mu}$. Metz (2002) examined the dependence of the threshold on the precision of private and public information. Analyzing the intensive margin of private information, Metz (2002) found that the effect of more precise private information on the fundamental threshold depends on the public signal. More specifically, the threshold increases (decreases) in the precision of private information if the public signal is strong (weak). In contrast, we focus on the proportion of informed investors. We show in Lemma 1 in Appendix A.1 that the result of Metz (2002) also holds for the extensive margin of private information, $\frac{d\overline{\theta}}{dn^*}(\mu - \hat{\mu}) > 0$, as shown in Figure 5.

The fundamental threshold increases in the proportion of informed investors whenever $\Gamma(\overline{\theta}, \overline{x}_I, \overline{x}_U) < 0$. This condition, evaluated at the threshold value of the fundamental, holds if an informed investor is more likely to attack than an uninformed investor, $\Phi_I(\overline{\theta}) > \Phi_U(\overline{\theta})$, which occurs for a strong public signal (Lemma 1). Since their private signal is more informative about the fundamental, informed investors put a larger weight on private information and therefore a smaller weight on public information than uninformed investors, as shown by the posterior about the fundamental in condition (13). Therefore, informed investors are more likely to disregard a strong public signal and attack the regime than are uninformed investors. In other words, informed investors attack the regime for a larger range of private signals than uninformed investors when the public signal is strong. As a result, the fundamental threshold

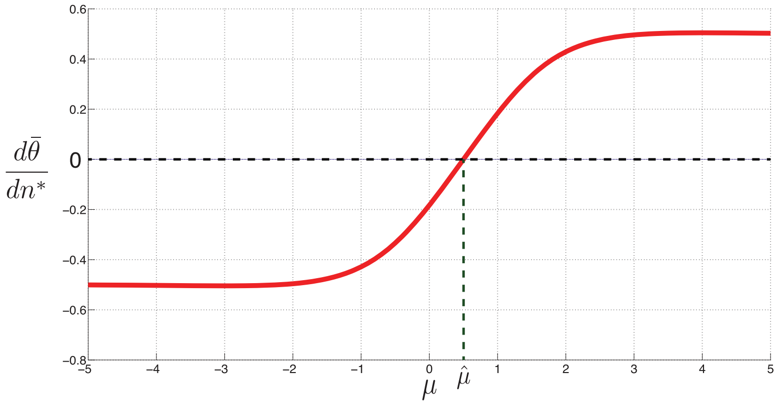

Figure 5
Impact of the proportion of informed investors on the fundamental threshold depends on the public signal
For $\mu > \hat{\mu}$, a larger proportion of informed investors leads to a higher fundamental threshold and to a higher probability of a financial crisis. The converse result holds for a weak public signal. Parameter values are $b = \ell = 0.75$, $\alpha = 1$, and $\gamma_U = 4$.

increases in the proportion of informed investors.[12] Similarly, for a weak public signal, informed investors are less likely to attack the regime than uninformed investors, so the threshold decreases in the proportion of informed investors.

3.1 Amplification for both weak and strong public signals

Amplification occurs independently of the strength of the public signal and the direction of its change, as can be seen by combining the previous three steps. However, the mechanism works differently for a strong versus a weak public signal. This different mechanism will have consequences for the design of policy interventions discussed in Section 4. To illustrate the amplification mechanism, we consider a deterioration in the public signal about the fundamental, which is purely expositional since the opposite result holds for an improvement.

3.1.1 Strong public signal. The deterioration of an initially strong public signal raises the value of private information, so more investors choose to acquire information. Since informed investors place a lower weight on public information, they are more likely to attack than uninformed investors for a strong public signal. Therefore, the fundamental threshold and the probability of a financial crisis rise further after the deterioration of the public signal. In short, amplification arises from the information choices of investors.

12 We revisit the link between the proportion of informed investors and the fundamental threshold below which a crisis occurs in Section 6.1.

3.1.2 Weak public signal. Likewise, as an initially weak public signal deteriorates, the value of private information falls and fewer investors acquire information. Since informed investors place a lower weight on public information, they are less likely to attack than uninformed investors for a weak public signal. As fewer investors are informed, the fundamental threshold and the probability of a financial crisis increase further, yielding amplification.

4. Policy

To explore potential policy interventions, we analyze a policy maker concerned about financial instability, defined as the ex ante probability of a financial crisis given by Equation (17). Financial instability is determined by the fundamental threshold below which a crisis occurs.

We consider two interventions. First, we study taxes and subsidies on information acquisition. (Similar results obtain for taxing payoffs.) These interventions are ex post; that is, once the public signal is observed. Second, we study an improvement of the quality of public information. This intervention is ex ante, that is before the public signal is observed. It captures the regulation of credit rating agencies to induce more informative ratings or the commitment of a regulator to publish the results of future bank stress tests.[13]

4.1 Taxes and subsidies

Suppose that the policy maker can tax or subsidize the information choice of investors, where we assume that the policy maker is deep-pocketed. The information cost of investor i changes to $c_i' \equiv (1-\tau)c_i$ for some $\tau < 1$, where a tax corresponds to $\tau < 0$ and a subsidy to $\tau > 0$. This policy affects optimality at the information stage only, where investor i acquires more precise private information if and only if $(1-\tau)c_i \leq \overline{c}$. Therefore, the proportion of informed investors is $n^* = \Pr\{c_i' \leq \overline{c}\} = \frac{\overline{c}}{1-\tau}$, where a subsidy raises the proportion of informed investors, while a tax lowers it. The fixed-point problem becomes $n^*(1-\tau) = \overline{c} = D$. Proposition 3 summarizes how changes in the tax or subsidy affect the fundamental threshold.

Proposition 3. Tax or subsidy on information acquisition. A change in the tax or subsidy on information acquisition affects the fundamental threshold according to:

$$\frac{d\overline{\theta}}{d\tau} = -\frac{\overline{c}}{(1-\tau)^2(1-A_{\overline{\theta}}) - (1-\tau)(b+\ell)g(\overline{\theta})\Gamma(\overline{\theta})^2}\,\Gamma(\overline{\theta}), \tag{22}$$

which is positive (negative) if and only if the public signal is strong (weak), $\frac{d\overline{\theta}}{d\tau}(\mu - \hat{\mu}) > 0$.[14]

[13] Since the policy maker is uninformed when choosing the quality of public information, there is no signaling to investors. Angeletos et al. (2006) studied signaling in a game of regime change without information choice.

[14] An analogous result holds for limited precision improvement with $\hat{\mu}$ replaced by $\tilde{\mu}$. See section 5.4.

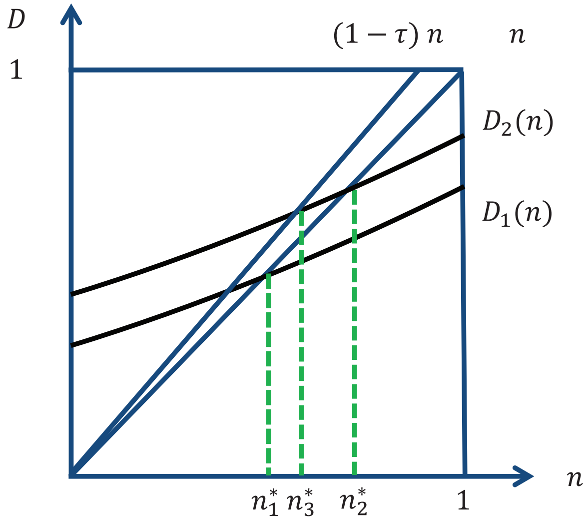

Figure 6
The impact of taxing the information acquisition of investors ($\tau < 0$) on the proportion of informed investors for a strong public signal, $\mu > \hat{\mu}$
A reduction in the public signal increases the value of private information from D_1 to D_2, increasing the proportion of informed investors from n_1^* to n_2^*. Taxing information acquisition raises the slope to $1-\tau > 1$ and limits the increase in the proportion of informed investors to $n_2^* < n_3^*$.

Proof. See Appendix C.1. ■

As a corollary of Proposition 3, the policy maker can use these taxes or subsidies to enhance financial stability after a reduction in the public signal, such as after a downgrade. However, the appropriate response (tax or subsidy) depends on the strength of the public signal, since the amplification mechanism works differently for a weak or strong public signal.

Corollary 1. Tax or subsidy policy. To reduce the probability of a financial crisis, a policy maker's adequate response to a reduced public signal is contingent on its strength. Taxation ($\tau < 0$) is desirable for a strong signal but a subsidy ($\tau > 0$) is desirable for a weak signal.

Figure 6 shows the case of a strong public signal, $\mu > \hat{\mu}$. A deterioration in the public signal, $d\mu < 0$, increases the value of private information, $D_\mu < 0$. Taxing information acquisition limits the increase in the proportion of informed investors and, as a result, the increase in the probability of a financial crisis. In contrast, for a weak public signal (not depicted), $\mu < \hat{\mu}$, a deterioration in the public signal decreases the value of private information, $D_\mu > 0$. Subsidizing information acquisition limits the decrease in the proportion of informed investors and the resultant increase in the probability of a financial crisis.

Similar results obtain if the policy maker can tax and subsidize the payoffs of investors. To illustrate this point, consider a tax on the benefit of attacking the regime changes, $b' \equiv (1-t)b$ for some $0 < t < 1$, combined with tax credit on the losses of attacking in the case of no regime change, $\ell' \equiv (1-t)\ell$. Since this policy leaves the conservatism ratio unchanged, $\kappa' = \kappa$, optimality at the coordination stage for any given proportion of informed investors is unaffected. Both the benefits and losses are reduced proportionally, so the value of private information shrinks accordingly, $D' \equiv (1-t)D$. Intuitively, acquiring more precise private information is less attractive because making errors of either type is now less costly. As a result, fewer investors acquire private information. The results of Corollary 1 carry over in that higher (lower) taxes are desirable for a strong (weak) public signal.

4.2 Improving the quality of public information

Since the information choice of investors may be difficult to verify (and thus difficult to tax), we study an improvement of the quality of public information as an alternative intervention. Suppose the policy maker can affect the precision of the public signal α. To simplify the exposition, we assume $\kappa = \frac{1}{2}$ and a precise private signal of informed investors, $\gamma_I > \underline{\gamma}_I$.

There are two consequences of improving the quality of public information. First, there is a *coordination effect*, whereby investors rely more on the public signal and less on the private signal at the coordination stage (see Equation (13)). As a result, fewer investors attack the regime, $A_\alpha < 0$, if the public signal is sufficiently strong. For $\kappa = \frac{1}{2}$, this condition simplifies to $\mu > \hat{\mu} = \frac{1}{2}$. Second, there is an *information choice effect*, whereby more precise public information reduces the value of private information and thus crowds out the acquisition of private information, $D_\alpha < 0$. Taken together, Proposition 4 states the total effect of improving the quality of public information.

Proposition 4. Improving the quality of public information. Suppose $\kappa = \frac{1}{2}$ and private information of informed investors is precise, $\gamma_I > \underline{\gamma}_I$. Improving the quality of public information affects the fundamental threshold according to:

$$\frac{d\overline{\theta}}{d\alpha} = \frac{A_\alpha + A_{n^*} D_\alpha}{1 - A_{\overline{\theta}} - A_{n^*} D_{\overline{\theta}}}, \tag{23}$$

which is negative (positive) if and only if the public signal is strong (weak), $\frac{d\overline{\theta}}{d\alpha}(\mu - \hat{\mu}) < 0$.

Proof. See Appendix C.2. ■

As a corollary of Proposition 4, the effect of improving the quality of public information ex ante on the probability of a financial crisis ex post is ambiguous. The consequences of this policy depend on the strength of the public signal. Specifically, more informative public information reduce the probability of a crisis if the public signal is strong. First, there is increased coordination on a more informative public signal. Second, the more informative public signal crowds out private information acquisition, which may have led to receiving bad news based on which an investor would have attacked. Since the reverse holds for a weak public signal, committing to releasing future stress tests could amplify future banking crises.

5. Magnitude of Amplification and Testable Implications

Having studied the qualitative aspects of the amplification mechanism, we now define and characterize its magnitude. Second, we study generalizations of our model to show the robustness of the mechanism and, more importantly, derive testable implications about the magnitude of amplification with respect to (1) the public signal, (2) the distribution of information costs, and (3) the precision of private information. Throughout this section, we suggest some environments in which these implications can be tested.

5.1 Magnitude of amplification

We define the magnitude of the amplification effect as $MoA \equiv \frac{\frac{d\overline{\theta}}{d\mu}\big|_{n^*,\overline{\theta}}}{\frac{d\tilde{\theta}}{d\mu}\big|_{\overline{n},\overline{\theta}}} - 1$ and derive a lower bound on this magnitude in Appendix D.2.

Proposition 5. Magnitude of amplification. For any precision improvement, the magnitude of amplification is

$$MoA = \frac{\frac{dD}{dn^*}}{(1-\frac{dD}{dn^*})A_{\overline{\theta}}} > 0. \tag{24}$$

It increases in the degree of strategic complementarity in information choices, $\frac{dD}{dn^*}$, and decreases in the sensitivity of the aggregate attack size to the fundamental threshold, $A_{\overline{\theta}}$.

Proof. See Appendix D.1. ■

In the following subsections, we derive three specific implications about the magnitude of amplification and discuss how these can be tested. The first result is that the magnitude of amplification is non-monotonic in the public signal (Section 5.2). This result is immediately testable in public debt markets where unexpected changes in ratings or earnings have implications for the magnitude of amplification. The second result highlights the dispersion in information costs for the magnitude of amplification (Section 5.3). We offer an empirical strategy based on exploiting differences in the design of over-the-counter versus centralized markets. The third result states that the magnitude of amplification is larger if informed investors have a more precise signal (Section 5.4). This result can be tested by using differences in the sophistication of institutional investors or differences in firm and bank characteristics.

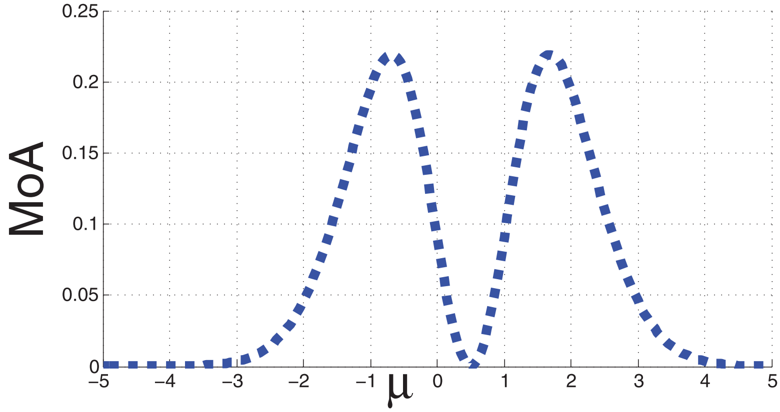

Figure 7
Non-monotonic relationship between the magnitude of amplification (MoA) and the public signal
Parameter values are $b = \ell = 0.75$, $\alpha = 1$, and $\gamma_U = 4$.

5.2 Public signal

Next, we study how the magnitude of amplification depends on the public signal.

Proposition 6. For any precision improvement, the magnitude of amplification is nonmonotonic in the public signal.

Proof. See Appendix D.1. ■

Figure 7 shows how the magnitude of amplification is non-monotonic in the public signal, such as a credit rating. For a weak or strong public signal, the changes in the value of private information after changes in the public signal are small (Figure 3), so the magnitude of amplification is small. Moreover, for values close to $\hat{\mu}$, the sensitivity of the aggregate attack size to the fundamental threshold, $A_{\overline{\theta}}$, is high and the fundamental threshold changes little with the proportion of informed investors. As a result, the magnitude of amplification is also small. At $\mu = \hat{\mu}$, the magnitude is zero, while it is strictly positive elsewhere. For other values of the public signal, however, the value of private information is quite sensitive to changes in the public signal and the fundamental threshold is sensitive to changes in the proportion of informed investors, resulting in a large magnitude of amplification.

The nonmonotonicity result can be tested in a number of markets, including the market for corporate debt. Consider a firm with debt to be rolled over by investors. A reduction in the public signal would correspond to an unexpected rating downgrade or earnings warning. Using the criteria stated in the next paragraph, an empiricist can separate firms according to whether information acquisition is likely or hard to occur. Our theory predicts that (1) amplification occurs for any given initial credit rating, and that (2) the magnitude of this effect depends on the initial rating. For example, within the class of creditworthy firms (e.g., those with investment-grade debt), our theory predicts the magnitude of amplification to be hump shaped in the initial rating.

An empiricist may use several potential proxies for the extent of information investors are able to acquire. First, some firms are publicly listed and their *equity* is traded, so these firms are subject to disclosure requirements imposed by the Securities and Exchange Commission. In contrast, many other firms are privately owned and therefore disclosure requirements do not apply. As a result, information acquisition by debt holders is more relevant in the former case. Second, firm size may proxy for information, since small firms are followed by fewer analysts than large firms. Third, the industry or the age of the firm form other proxies. Young and innovative companies tend to have a higher share of intangible assets, which are more difficult to evaluate (e.g. technology firms versus consumer goods). In addition, their business models and growth perspectives may be less tested and more difficult to evaluate than those of established firms, for which ample data exists.

Kisgen (2006) studies the impact of credit ratings on capital structure decisions. Since rating categories are broad (e.g., AA+, AA, and AA−), a downgrade by one notch has little effect if still in the same category, but it has a large impact if the firm will be in a lower category (i.e., A+). Firms close to an upgrade or downgrade are shown to issue less net debt relative to net equity. Similar to Kisgen (2006), net debt issuance can be constructed for each corporate firm. Beyond corporate debt, similar tests could be conducted for bank commercial paper. It is short-term (typically, with a maturity below 270 days) and is therefore rolled over frequently. Apart from a downgrade or an earnings warning, a reduction in the public signal could come from a downward revision of a bank's asset quality by its supervisor.

Again, an empiricist would need to separate circumstances under which information about the fundamentals of the bank is easy to acquire from those where it may be difficult. The distinction between privately owned and publicly listed companies also applies to banks. Second, the opacity of the bank's assets is a proxy for investor capacity to acquire information. For example, a bank invested in traditional and marketable assets is more transparent than a bank invested in complicated and perhaps illiquid structured products (e.g., exposure to products based on asset-backed securities). Also, the complexity of a bank in terms of its legal and organizational structure is another proxy (Cetorelli and Goldberg 2014; Goldberg 2016). It affects the ease with which investors can determine the profitability of the bank or, conversely, whether exposures or outright losses may be hidden.

5.3 Generic distribution of information cost

Consider a generic distribution of the information cost given by the probability density function $f(c)$ with support $[c_{min}, c_{max}]$, where $0 \leq c_{min} < c_{max}$. We offer sufficient conditions for a unique equilibrium to exist and show that the amplification result continues to hold. We also describe how the information cost distribution affects the magnitude of amplification.

Proposition 7. Generic distribution of information cost. Let $f(c)$ be the density function of the information cost with support $[c_{min}, c_{max}]$. If $c_{min} < D(0) < D(1) < c_{max}$ and $f(c) < \sqrt{\frac{\pi}{2}}$, then there exists a precision level $\tilde{\gamma} < \infty$ such that a unique equilibrium exists for any $\gamma_U \in (\tilde{\gamma}, \infty)$. Amplification via the information choice occurs in this equilibrium.

Proof. See Appendix D.3. ■

The first sufficient condition ensures that the information cost is sufficiently heterogeneous relative to the bounds on the value of private information. As a result, the required dominance regions at the information stage are ensured and the existence of a threshold information cost follows. The second condition limits, after a small increase in the threshold information cost $\bar{c}$, the increase in the proportion of investors who acquire information. Because of strategic complementarity, it also limits the value of private information, ensuring that a unique threshold information cost exists.

Proposition 8. Information cost distributions and the magnitude of amplification. Suppose the conditions of Proposition 7 hold. Then the following results for the magnitude of amplification hold uniformly, that is, for any given public signal μ:

1. If $f(c) > 1$, then the magnitude of amplification is larger than in the case of $c_i \sim U[0,1]$.
2. Consider two distributions of the information costs, f^1 and f^2, that satisfy the heterogeneity and slope requirements of Proposition 7. Let f^2 be a transformation of f^1 in the sense that $c^1_{min} \leq c^2_{min} < D(0) < D(1) < c^2_{max} \leq c^1_{max}$, where the probability mass between $[c^1_{min}, c^2_{min}]$ and $[c^2_{max}, c^1_{max}]$ is evenly distributed over $[c^2_{min}, c^2_{max}]$. Then, the magnitude of amplification for f^2 is higher than for f^1.
3. Consider two uniform distributions, $f^3 \sim U[c_{min}, c_{max}]$ and $f^4 \sim U[c_{min}+\varphi, c_{max}-\varphi]$, that satisfy the heterogeneity and slope requirements. Then the magnitude of amplification for f^4 is higher than for f^3. The difference in the magnitude increases in φ.

Proof. See Appendix D.3. ■

In case 1, if $1 < f(c) < \sqrt{\frac{\pi}{2}}$ (and sufficiently precise private information), changes in the public signal that affect the value of information and the threshold information cost lead to a larger change in the proportion of informed

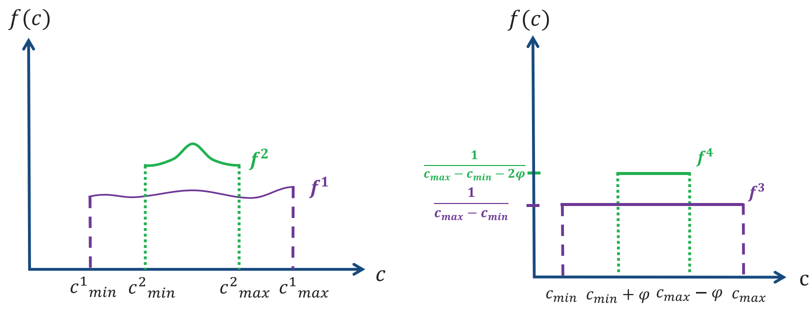

Figure 8
Distribution of information costs and the magnitude of amplification
The left panel shows case 2, and the right panel shows case 3.

investors, resulting in a larger magnitude of the amplification effect. In cases 2 and 3, a larger mass of the distribution in the relevant range of $[D(0), D(1)]$ implies a larger change in the proportion of informed investors as the threshold information cost changes. Therefore, the magnitude of the amplification effect after a change in the public signal is larger, as shown in Figure 8.

These results suggest that the magnitude of amplification tends to be lower for more dispersed skills in information acquisition, since a more dispersed distribution of information costs tends to induce smaller changes in the proportion of informed investors.

To test these implications for the magnitude of amplification, one can compare shocks to markets or asset classes that have different distributions of information costs. For example, the dispersion of information costs is higher in over-the-counter markets than in centralized markets. Ang et al. (2013) showed that markets in which investors trade over-the-counter are more opaque than centralized markets. Other proxies for the dispersion in information cost are the size of investors (larger investors have lower costs than smaller investors) or the sophistication of investors (retail investors have higher costs than professional investors). Garriott and Walton (2016) provided evidence about the lower informativeness of retail trades. Another proxy could be the (voluntary) disclosure policy of corporate firms and banks, which reduces the level and dispersion of information costs. Lang and Lundholm (1996) showed that such disclosures increase the accuracy of earning forecasts of analysts and reduces their dispersion.

5.4 Limited precision improvement

Consider next the case of limited precision improvement, $\gamma_I \in (\gamma_U, \infty)$. Investors who acquire information still receive a more precise signal than investors who acquire no information. However, the signal of informed investors is noisy, sometimes resulting in mistakes when (not) attacking the

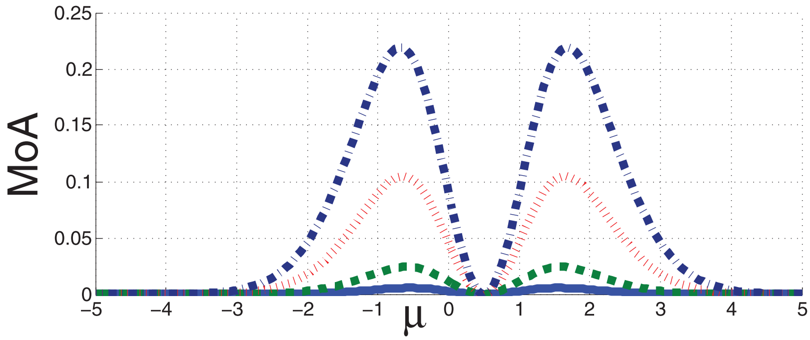

Figure 9
Precision of private information and the magnitude of amplification
More precise private information monotonically increases the magnitude of amplification. Parameter values are $b = \ell = 0.75$, $\alpha = 1$, $\gamma_U = 4$, and $\gamma_I \in \{6, 10, 50, \infty\}$.

regime. We generalize our result on the existence of a unique equilibrium and show that the amplification result continues to hold.

Proposition 9. Limited precision improvement. Let $\gamma_I \in (\gamma_U, \infty)$. If private information is sufficiently precise, $\gamma_U > \underline{\gamma}$, then there generically exists a unique equilibrium. Amplification via the information choices of investors occurs in this equilibrium.

Proof. See Appendix D.4. ■

Next, we study numerically how the degree of precision improvement affects the magnitude of amplification. Figure 9 shows the magnitude of amplification for various precision levels of private information of informed investors. The relationship between the private precision and the magnitude of amplification is monotonic, whereby greater precision raises the magnitude.

To test the implications of different degrees of precision improvement for the magnitude of amplification, the composition of institutional investors can be exploited. Hedge funds may be better at extracting information than mutual or pension funds, so the share of hedge funds among institutional investors is a proxy for the precision of private information. Specifically, following a negative shock, our model predicts a larger magnitude of amplification for a larger proportion of hedge fund investors. For the debt of a corporate firm, another proxy could be the industry and the age of the firm (as described above). The improvement in precision is small when much information about the firm is classified or protected (e.g., defense) or when little data or experience exists to evaluate a new business model (e.g., technology). In terms of bank debt, more complex banks and those invested in more opaque assets or with more opaque counterparties allow for smaller precision improvement.

6. Extensions

6.1 Payoff sensitivity

We have so far considered exogenous payoff parameters. However, Iachan and Nenov (2015) allowed the payoffs of investors to depend directly on the fundamental, which allows for a more general link between the proportion of informed investors and the fundamental threshold. To capture this idea in the context of our model, which has both a public signal and endogenous information choice, suppose that $b(\theta)\in(0,1)$ and $\ell(\theta)\in(0,1)$, where $b'(\theta)\in(-\infty,0)$ and $\ell'(\theta)\in(0,\infty)$. The payoff from attacking becomes

$$u(a_i=1,A,\theta)=b(\theta)\,\mathbf{1}_{\{A\geq\theta\}}-\ell(\theta)\,\mathbf{1}_{\{A<\theta\}}. \tag{25}$$

We maintain a uniformly distributed information cost, $c_i\sim\mathcal{U}[0,1]$, and consider limited precision improvement in this section.[15] Some tractability is lost when payoffs are sensitive to fundamentals but we can offer a condition sufficient for amplification to occur.

Proposition 10. Payoff sensitivity and amplification. Let the payoffs be sensitive to the fundamental, whereby $b(\theta)>0$ and $\ell(\theta)>0$ with $b'(\theta)<0$ and $\ell'(\theta)>0$. If

$$-g(\overline{\theta})\Big(b(\overline{\theta})+\ell(\overline{\theta})\Big)\Gamma(\overline{\theta})^2-\Gamma(\overline{\theta})\Big[\int_{\overline{\theta}}^{\infty}\ell'(\theta)\Gamma(\theta)dG(\theta)$$
$$-\int_{-\infty}^{\overline{\theta}}b'(\theta)\Gamma(\theta)dG(\theta)\Big]\leq 0, \tag{26}$$

amplification via the information choice of investors occurs after changes in the public signal.

Proof. See Appendix E. ■

In order to obtain more specific results, we study the linear case of $b(\theta)=b_0-b_1\theta$ and $\ell(\theta)=\ell_0+\ell_1\theta$, where all coefficients are strictly positive. Since $\overline{\theta}\in(0,1)$, we impose $b_0>b_1$ to ensure that $b(\theta)>0$ over the relevant range. Likewise, $\ell_0>0$ ensures that $\ell(\theta)>0$ over the relevant range. We also assume that the slope is identical, $b_1=\ell_1\equiv\lambda$.

Proposition 11. A linear case: Existence, uniqueness, and amplification. Consider the linear specification with identical slope coefficient, $b(\theta)=b_0-\lambda\theta$ and $\ell(\theta)=\ell_0+\lambda\theta$. If private information is sufficiently precise, $\gamma_U>\underline{\gamma}$, and payoff sensitivity is sufficiently low, $\lambda<\overline{\lambda}>0$, then there exists a unique

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[15] Iachan and Nenov (2015) also studied a threshold function $k(\theta)$, where $A\geq k(\theta)$ is required for regime change. Since the impact of this feature is minor, we set $k(\theta)=\theta$ to focus on the role of payoff sensitivity.

equilibrium. Amplification via the information choice of investors occurs in this equilibrium if

$$b_0+\ell_0 \geq \frac{\lambda}{g(\overline{\theta})\left|\Gamma(\overline{\theta})\right|}. \tag{27}$$

Proof. See Appendix E. ■

The sensitivity of the payoffs to the fundamental plays a crucial role for both the existence of a unique equilibrium and amplification. We show that a unique equilibrium in the overall game exists as long as the payoffs are not too sensitive to the fundamental. We also offer a simpler condition sufficient for amplification to occur in this equilibrium. Again, an upper bound on the sensitivity of payoffs suffices. These conditions are always met in the baseline case because payoffs are not sensitive to the fundamental, $\lambda=0$.

In the spirit of Iachan and Nenov (2015), we next consider the special cases in which only one payoff variable is sensitive to the fundamental. These cases may be interpreted as a stylized bank run ($b'(\theta)=0$) or a currency attack ($\ell'(\theta)=0$). For simplicity, we continue to consider a constant slope for the sensitive payoff, $\lambda > 0$. Iachan and Nenov (2015) show that more precise private information has different implications for the probability of regime change in these two cases. Therefore, we are interested in analyzing the magnitude of amplification in the cases where only one payoff variable is sensitive to the fundamental.

Proposition 12. One-sided payoff sensitivity. Consider the linear specification in which one payoff coefficient is insensitive to the fundamental, $b(\theta)= b_0-\lambda\theta$ and $\ell(\theta)=\ell_0$ or $b(\theta)=b_0$ and $\ell(\theta)=\ell_0+\lambda\theta$. If the private information of an informed investor is sufficiently precise, $\gamma_I > \underline{\gamma}_I$, the magnitude of amplification increases in $\Gamma(\overline{\theta})B(\overline{\theta})$, where

$$B(\overline{\theta})=\begin{cases}\lambda\int_{\overline{\theta}}^{\infty}\Gamma(\theta)dG(\theta)>0 & \text{if} \quad b'(\theta)=0\\ \lambda\int_{-\infty}^{\overline{\theta}}\Gamma(\theta)dG(\theta)<0 & \phantom{\text{if}} \quad \ell'(\theta)=0.\end{cases} \tag{28}$$

Proof. See Appendix E. ■

Proposition 12 states that the magnitude of amplification is different across the different cases of one-sided payoff sensitivity. It suggests that if informed investors are more likely to attack than uninformed investors, $\Gamma(\overline{\theta})<0$, the magnitude of amplification is higher in currency attacks than in bank runs. In contrast, if informed investors are less likely to attack, $\Gamma(\overline{\theta})>0$, the magnitude of amplification is higher in bank runs.

6.2 Homogeneous information cost

We study an extension with a homogeneous information cost for robustness. Such an information cost structure in a discrete information choice setup

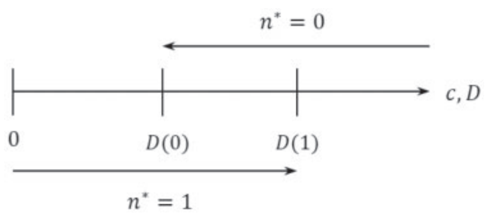

Figure 10
A homogeneous information cost: multiple equilibria exist for $D(0) \leq c \leq D(1)$**.**

yields multiple equilibria (Hellwig and Veldkamp 2009). We show that our amplification mechanism can obtain in this setup. First, Proposition 13 states the equilibrium in this environment without proof.

Proposition 13. Homogeneous information costs and multiple equilibria. Consider a homogeneous cost, $c_i \equiv c$. If private information is sufficiently precise, $\gamma_U > \underline{\gamma}$, the information cost determines the number of equilibria in the overall game.

If the information cost is low, $c < D(0)$, there exists a unique stable equilibrium in which all investors acquire information, $n^* = 1$. If $c > D(1)$, there exists a unique stable equilibrium without information acquisition, $n^* = 0$. In contrast, for an intermediate information cost, $D(0) \leq c \leq D(1)$, there exist three equilibria. Apart from the previous two equilibria, there is also an asymmetric and unstable equilibrium, in which investors are indifferent in their information choice and the aggregate proportion of informed investors is determined to ensure this indifference of the marginal investor: $n^* = D^{-1}(c)$.

The optimal behavior of investors at the coordination stage is again uniquely pinned down for a given proportion of informed investors, n^*. It is characterized by the thresholds $\overline{x}_I(n^*)$, $\overline{x}_U(n^*)$, $\overline{\theta}(n^*)$.

Figure 10 shows the link between the information cost and the number of equilibria. Given the strict monotonicity of the value of private information in the proportion of informed investors, the latter is uniquely determined in the asymmetric equilibrium. While the amplification effect does not occur in this equilibrium, we exclude the asymmetric equilibrium based on its instability. That is, we focus on the two stable equilibria with symmetric information choices of investors.

We assume a sunspot variable $s \in \{0, 1\}$ with $\Pr\{s = 1\} = q \in (0, 1)$. Whenever both symmetric equilibria exist, investors coordinate on one equilibrium according to the sunspot. That is, if $D(0) \leq c \leq D(1)$, the equilibrium with information acquisition, $n^* = 1$, occurs with probability q and the equilibrium without information acquisition, $n^* = 0$, occurs with probability $1 - q$. In what follows, we focus on a deteriorating public signal, but the analysis for an improving signal is analogous.

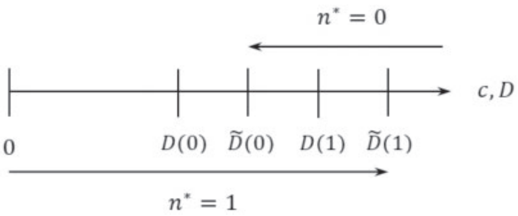

Figure 11
A homogeneous information cost and equilibrium selection via sunspots. Amplification occurs with positive probability for $D(0) < c < \tilde{D}(0)$ and $D(1) < c < \tilde{D}(1)$.

Figure 11 shows the case of a strong public signal, where the value of private information increases as the public signal deteriorates (denoted by $\tilde{D}(n)$). Three areas of inaction have no change in the equilibrium proportion of informed investors and therefore no amplification: $[0, D(0)]$, $[\tilde{D}(0), D(1)]$, and $[\tilde{D}(1), \infty)$. Consider next the range $(D(0), \tilde{D}(0))$, where before the deterioration in the public signal, information acquisition was the equilibrium with probability q, and it is now the unique equilibrium. Therefore, there is no change in the equilibrium proportion of informed investors with probability q but a change, $\Delta n^* = +1$, with probability $1-q$. Hence, with probability $1-q$, amplification occurs, since the probability of a financial crisis increases as more investors are informed for a strong public signal. Likewise for the range $(D(1), \tilde{D}(1))$, where $n^* = 0$ was the unique equilibrium before but $n^* = 1$ is now an equilibrium with probability q. Hence, $\Delta n^* = +1$ and therefore amplification occurs with probability q. The analogous case of a weak public signal yields $\Delta n^* = -1$ and is skipped for brevity.

7. Conclusion

We have proposed an amplification mechanism of financial crises based on the information choice of investors. In a debt rollover game, for instance, an investor wishes to roll over debt whenever the debtor is solvent; this more likely occurs when other investors also roll over. Adverse news about an initially creditworthy debtor, for example, a rating downgrade or an earnings warning, raises the value of private information and more investors acquire information about debtor solvency. In turn, informed investors are more likely to refuse to roll over debt than are uninformed investors. This amplifies the probability of a debt crisis.

We have shown how a policy maker can enhance financial stability. Taxes and subsidies, either on payoffs to investors or on information acquisition, alter the information choice of investors and therefore reduce the probability of a crisis. However, the optimal policy response to deteriorating public information depends on the solvency of the debtor. For an initially creditworthy debtor, taxes reduce the value of private information relative to the effective information cost. By discouraging information acquisition by investors, the probability of a debt crisis is reduced. For an initially less creditworthy debtor, subsidies to encourage information acquisition reduce the probability of a debt crisis. We have also studied the effects of an improvement in the quality of public information on financial stability.

We have derived testable implications about the magnitude of amplification. First, the magnitude of amplification is nonmonotone in the public information about debtor solvency, such as credit ratings. Second, we have characterized the effect of the distribution of information costs across investors. Amplification is larger the more investors change their information choice as a result of deteriorating public information, which tends to occur for less dispersed distributions. Third, amplification is larger when informed investors have more precise private information. We have discussed several environments in which to test these implications.

Appendix

A Derivations and Proof of Proposition 1

We construct the equilibrium by working backwards. In Section A.1, we study the optimal attack behavior of investors at the coordination stage for any given information choices $\{n_i^*\}$, and the optimal information choice of investors in Section A.2. We state all conditions for a general precision of informed investors, γ_I, to use these conditions also for the case of limited precision improvement. We also state specific results for vanishing private noise, $\gamma_I \to \infty$.

A.1 Coordination Stage

Given the information choices $\{n_i^*\}$, the global coordination game is standard. An investor's expected utility from attacking conditional on the public signal μ, the private signal x_i, the information choice n_i^*, and the aggregate proportion of informed investors n^* is:

$$E[u(a_i=1)|n_i^*,n^*;\mu,x_i] = -\ell + (b+\ell)\Pr[A>\theta|n_i^*,n^*;\mu,x_i]. \tag{A1}$$

Optimality for investor i at the coordination stage requires that his strategy maximizes the conditional expected utility, taking all other investors' strategies as given. Since each investor is atomistic, the aggregate attack size is unaffected by the individual attack decision.

Without loss of generality, we focus on symmetric monotone equilibria at the coordination stage throughout (Morris and Shin 2003; Frankel et al. 2003). For a given proportion of informed investors n^*, the equilibrium is fully characterized by an **signal threshold** for informed and uninformed investors, $\overline{x}_I(n^*;\mu)$ and $\overline{x}_U(n^*;\mu)$, and an **fundamental threshold**, $\overline{\theta}(n^*;\mu)$. Investor i attacks the regime if and only if the private signal is below an signal threshold specific to her information choice $n_i^* = z \in \{I,U\}$:

$$a_i^* = 1 \Leftrightarrow x_i \le \overline{x}(n_i^* = z, n^*;\mu) \equiv \overline{x}_z(n^*;\mu). \tag{A2}$$

Regime change occurs whenever the realized fundamental is below the fundamental threshold:

$$\theta < \overline{\theta}(n^*;\mu). \tag{A3}$$

These thresholds are determined by a critical mass condition at the aggregate level and indifference conditions at the individual level. An investor i uses both signals to form a posterior about the unobserved fundamental, where normality is preserved, the posterior mean is a weighted average of the public signal and the private signal, and the posterior precision is the sum of the precisions of the public and private signals (deGroot 1970):

$$h_z(\theta,x_i):\theta|n_i^*=z;\mu,x_i \sim \mathcal{N}\left(\frac{\alpha\mu+\gamma_z x_i}{\alpha+\gamma_z},\frac{1}{\alpha+\gamma_z}\right), \tag{A4}$$

where we use the distribution $h_z \equiv \sqrt{\alpha+\gamma_z}\,\phi\left(\sqrt{\alpha+\gamma_z}\left[\theta-\frac{\alpha\mu+\gamma_z x_i}{\alpha+\gamma_z}\right]\right)$ extensively in what follows. Each investor assigns the following probability to regime change, $\Pr\{\theta \le \overline{\theta}|n_i^*=z,n^*;\mu,x_i\}=\Phi\left(\sqrt{\alpha+\gamma_z}\left[\overline{\theta}-\frac{\alpha\mu+\gamma_z x_i}{\alpha+\gamma_z}\right]\right)$. An investor with information choice $n_i^*=z$ who receives the threshold signal $x_i=\overline{x}_z(n^*;\mu)$ is indifferent between attacking and not attacking.

This **indifference condition** states that the probability of regime change evaluated at the fundamental threshold equals the conservativeness ratio for both informed and uninformed investors, $\Pr\{\theta \le \overline{\theta}|n^*,n_i^*=z;\mu,x_i=\overline{x}_z\}\equiv\kappa$, and it yields the signal thresholds:

$$\overline{x}_z(n^*;\mu)=\overline{\theta}(n^*;\mu)+\frac{\alpha}{\gamma_z}\left[\overline{\theta}(n^*;\mu)-\mu\right]-\frac{\sqrt{\alpha+\gamma_z}}{\gamma_z}\Phi^{-1}(\kappa). \tag{A5}$$

Since all investors play the threshold strategy with thresholds $\overline{x}_I(n^*;\mu)$ if informed, and $\overline{x}_U(n^*;\mu)$ if uninformed, the aggregate attack size for any fundamental θ is:

$$\begin{aligned} A(n^*;\theta,\overline{x}_I,\overline{x}_U)&=\int_0^1 \mathbf{1}\{x_i \le \overline{x}_z|n_i^*=z,\theta\}di \equiv n^*\Phi_I(\theta)+(1-n^*)\Phi_U(\theta)\\ &=n^*\Phi\left(\sqrt{\gamma_I}\,[\overline{x}_I-\theta]\right)+(1-n^*)\Phi\left(\sqrt{\gamma_U}\,[\overline{x}_U-\theta]\right). \end{aligned}$$

The **critical mass condition** states that the aggregate attack size is just sufficient for regime change when the fundamental equals the fundamental threshold:

$$\overline{\theta}(n^*;\mu)=A\left(n^*;\overline{\theta}(n^*;\mu),\overline{x}_I(n^*;\mu),\overline{x}_U(n^*;\mu)\right). \tag{A6}$$

Inserting the indifference conditions in the critical mass condition yields the fundamental threshold for any given proportion of informed investors, $\overline{\theta}=\overline{\theta}(n^*;\mu)$, implicitly defined by:

$$\begin{aligned} \overline{\theta}&=n^*\Phi_I(\overline{\theta})+(1-n^*)\Phi_U(\overline{\theta}), \qquad \text{(A7)}\\ &=n^*\Phi\left(\frac{\alpha[\overline{\theta}-\mu]}{\sqrt{\gamma_I}}-\sqrt{1+\frac{\alpha}{\gamma_I}}\Phi^{-1}(\kappa)\right)+(1-n^*)\Phi\left(\frac{\alpha[\overline{\theta}-\mu]}{\sqrt{\gamma_U}}-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right). \end{aligned}$$

A unique solution to Equation (A7) for any given proportion of informed investors n^* is ensured by a sufficiently precise private signal of the uninformed investor, $\gamma_U > \underline{\gamma}' \equiv \frac{\alpha^2}{2\pi}$ (Morris and Shin 2003). Under this condition, the slope of the left-hand side of Equation (A7) exceeds the slope of the right-hand side, $1 > A_{\overline{\theta}} \equiv \frac{\partial A}{\partial \overline{\theta}} > 0$, which is evaluated at the equilibrium values $(\overline{\theta},n^*)$. To see this, we observe that

$$A_{\overline{\theta}}=n^*\frac{\alpha}{\sqrt{\gamma_I}}\phi\left(\frac{\alpha[\overline{\theta}-\mu]}{\sqrt{\gamma_I}}-\sqrt{1+\frac{\alpha}{\gamma_I}}\Phi^{-1}(\kappa)\right)+(1-n^*)\frac{\alpha}{\sqrt{\gamma_U}}\phi\left(\frac{\alpha[\overline{\theta}-\mu]}{\sqrt{\gamma_U}}-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right).$$

Since $\phi(\cdot) \le \frac{1}{\sqrt{2\pi}}$ and $\gamma_U < \gamma_I$, we have $A_{\overline{\theta}} \le \frac{\alpha}{\sqrt{2\pi\gamma_U}}$, resulting in the stated lower bound $\underline{\gamma}'$. Thus, there exists at most one solution. Since $A \in [0,1]$ and $A_{\overline{\theta}} > 0, \forall n^*$, there exists a unique fixed point $\overline{\theta}(n^*;\mu)$ in the interval $[0,1]$. Once the unique fundamental threshold is obtained, the signal thresholds $\overline{x}_z(n^*;\mu)$ are backed out from the indifference conditions.

In the limiting case of vanishing private noise of informed investors, $\gamma_I \to \infty$, informed investors base their posterior about the unobserved fundamental completely on their private signal, whereby $f^I(x|\theta)$ is the Dirac delta function at $x=\theta$, and the signal threshold of informed investors reduces to the fundamental threshold, $\overline{x}_I(n^*)\to\overline{\theta}(n^*)$. As a result, an informed investor attacks if and only if regime change occurs, whereby $\Phi_I(\theta)=\mathbf{1}_{\theta\leq\overline{\theta}}$ is a step function. Hence, the fundamental threshold converges to:

$$\overline{\theta}\to n^*(1-\kappa)+(1-n^*)\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right). \tag{A8}$$

Lemma 1 summarizes the responsiveness of the fundamental threshold to changes in the proportion of informed investors. It states a condition sufficient for a link between the coordination and information stages. This condition is generically satisfied.

Lemma 1. Suppose $\gamma_U>\underline{\gamma}'$. If $\mu\neq\hat{\mu}$, then the fundamental threshold at the coordination stage responds to changes in the proportion of informed investors:

$$\frac{d\overline{\theta}}{dn^*}\neq 0. \tag{A9}$$

Furthermore, the fundamental threshold increases (decreases) in the proportion of informed investors if the public signal is strong (weak):

$$\frac{d\overline{\theta}}{dn^*}\left(\mu-\hat{\mu}\right)>0. \tag{A10}$$

Proof. If the private information of uninformed investors is sufficiently precise, then $\overline{\theta}(n^*;\mu)$ is unique for any $n^*\in[0,1]$. The proof is in three steps. First, differentiating the fundamental threshold with respect to the proportion of informed investors yields

$$\frac{d\overline{\theta}}{dn^*}=\frac{\Phi\left(\frac{\alpha}{\sqrt{\gamma_I}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_I}}\Phi^{-1}(\kappa)\right)-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)}{1-A_{\overline{\theta}}} \tag{A11}$$

$$\to\frac{1-\kappa-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)}{1-A_{\overline{\theta}}},$$

for $\gamma_I\to\infty$. Second, recall that we have $0<A_{\overline{\theta}}<1$, where the second inequality follows from the sufficient condition for uniqueness at the coordination stage, $\gamma_U>\underline{\gamma}'$. Third, $\mu\neq\hat{\mu}$ ensures a nonzero numerator of the derivative $\frac{d\overline{\theta}}{dn^*}$. This can be proven by contradiction: suppose that the numerator is zero. Since $0=\Phi^{-1}(\kappa)+\Phi^{-1}(1-\kappa)$ for all $\kappa\in(0,1)$, we have $\hat{\theta}\equiv\mu+\frac{\sqrt{\alpha+\gamma_U}-\sqrt{\gamma_U}}{\alpha}\Phi^{-1}(\kappa)$. Inserting $\hat{\theta}$ in the defining equation of the fundamental threshold, Equation (A7), yields $\mu=\hat{\mu}$. Using the same argument, the numerator is positive (negative) if and only if the public signal is strong (weak). ■

A.2 Information Stage

Next, we evaluate the incentive of an investor to acquire information. This is achieved by comparing the expected utility of an informed investor (EU^I) with that of an uninformed investor (EU^U), as defined in Equation (10). The value of private information stated in Equation (11) is expressed using the difference in the probability of attacking between an uninformed and informed investor, $\Gamma(\theta,\overline{x}_I,\overline{x}_U)$. This difference is generically nonzero since informed investors receive more precise information than uninformed investors and therefore use a different signal threshold. For $b=\ell$, for example, $\overline{x}_I\neq\overline{x}_U$ whenever $\mu\neq\frac{1}{2}$. If the information advantage of informed investors vanishes, then both types of investors invest with the same probability conditional on the fundamental, that is, $\Gamma(\theta)\to 0$ as $\gamma_U \to \gamma_I$.

For the limiting case of vanishing private noise, $\gamma_I \to \infty$, we have:

$$D \to \ell \int_{\overline{\theta}}^{\infty} \Phi_U(\theta) dG(\theta) + b \int_{-\infty}^{\overline{\theta}} \left[1-\Phi_U(\theta)\right] dG(\theta). \tag{A12}$$

Lemma 2. If $\gamma_U > \underline{\gamma}'$, there is strategic complementarity in information choices:

$$\frac{dD}{dn^*} = (b+\ell)\left[1-A_{\overline{\theta}}\right] g(\overline{\theta}) \left(\frac{d\overline{\theta}}{dn^*}\right)^2 \geq 0, \tag{A13}$$

with strict inequality if $\mu \neq \hat{\mu}$. Furthermore, the more-restrictive lower bound on the precision of private information, $\gamma_U > \underline{\gamma}$, ensures that:

$$\frac{dD}{dn^*} < 1. \tag{A14}$$

Proof. First, $\gamma_U > \underline{\gamma}'$, so $\overline{\theta}(n^*;\mu)$ is unique for any n^*. Total differentiation yields $\frac{dD}{dn^*} = \frac{\partial D}{\partial \overline{\theta}} \frac{d\overline{\theta}}{dn^*} + \frac{\partial D}{\partial \overline{x}_U} \frac{d\overline{x}_U}{d\overline{\theta}} \frac{d\overline{\theta}}{dn^*}$. We prove below that $\frac{\partial D}{\partial \overline{x}_z} \equiv D_{\overline{x}_z} = 0$. By Leibniz rule, we have that $\frac{\partial D}{\partial \overline{\theta}} = g(\overline{\theta})$ $[b-(b+l)\Phi_U(\overline{\theta})]$. Inserting $\overline{x}_U$ from Equation (A5) and rewriting Lemma 1 to obtain an expression for $\Phi_U(\overline{\theta})$, we have:

$$\frac{\partial D}{\partial \overline{\theta}} \equiv D_{\overline{\theta}} = (b+\ell)(1-A_{\overline{\theta}}) \frac{d\overline{\theta}}{dn^*} g(\overline{\theta}). \tag{A15}$$

Therefore, by taking together, we can state $\frac{dD}{dn^*} = (b+\ell)\left[1-A_{\overline{\theta}}\right] g(\overline{\theta}) \left(\frac{d\overline{\theta}}{dn^*}\right)^2 \geq 0$. This derivative has four terms. The first three terms are strictly positive because $b>0$, $\ell>0$, $A_{\overline{\theta}}>0$, $A_{\overline{\theta}}<1$ if $\gamma_U > \underline{\gamma}'$, and $g>0$ (pdf of a standardized Gaussian). The fourth term is a square and thus nonnegative. It is strictly positive if $\mu \neq \hat{\mu}$ by Lemma 1. Thus, $\mu \neq \hat{\mu}$ suffices for the derivative to be strictly positive.

Next, $\frac{\partial D}{\partial \overline{x}_U} = 0$ by an envelope theorem argument. The threshold $\overline{x}_U$ is chosen by a first-order condition that balances the marginal cost of attacking absent regime change with the marginal benefit of attacking under regime change (see Appendix 7 for more details):

$$\frac{\partial D}{\partial \overline{x}_U} = -b \int_{-\infty}^{\overline{\theta}} f^U(\overline{x}_U|\theta) g(\theta) d\theta + \ell \int_{\overline{\theta}}^{\infty} f^U(\overline{x}_U|\theta) g(\theta) d\theta = 0. \tag{A16}$$

Using Lemma 1, the derivative of the value of private information with respect to the proportion of informed investors is:

$$\frac{dD}{dn^*} = \frac{(b+\ell) g(\overline{\theta}) \left[1-\kappa-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)\right]^2}{1-A_{\overline{\theta}}}, \tag{A17}$$

where $g \leq \frac{1}{\sqrt{2\pi}}$, $b+\ell<2$, and $[1-\kappa-\Phi(\cdot)]^2 \leq \max\{\kappa^2,(1-\kappa)^2\}<1$. Next, $A_{\overline{\theta}} = (1-n^*)\phi \left(\frac{\alpha}{\sqrt{\gamma_U}}\left[\overline{\theta}-\mu\right]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) \frac{\alpha}{\sqrt{\gamma_U}} \leq \frac{\alpha}{\sqrt{2\pi}\sqrt{\gamma_U}}$ since $n^* \geq 0$. Hence, $\frac{dD}{dn^*}<1$ is ensured by $\gamma_U > \underline{\gamma} > \underline{\gamma}'$. ■

Since $\mu \neq \hat{\mu}$ excludes a parameter space of zero measure, the value of private information increases strictly in the proportion of informed investors generically.

Lemma 3. If $\gamma_U > \underline{\gamma}'$, then the value of private information satisfies $D \in (0,1)$.

Proof. First, we show $D(n^*) > 0$ for all $n^* \in [0,1]$. Formally, $F^U(\theta) \in (0,1)$ and $g(\theta) > 0$ for all $\theta \in (-\infty, \infty)$ and $b > 0$ and $\ell > 0$, so $D > 0$. Intuitively, since $\gamma_U < \infty$, there is positive probability mass on the type I and type II errors of an uninformed investor. Second, we show $D(n^*) < 1$ for all n^*. Since $F^U(\theta) \le 1$ for all $\theta \in [\overline{\theta}, \infty)$ and $F^U(\theta) \ge 0$ for all $\theta \in (-\infty, \overline{\theta}]$, we have $D \le \ell + (b-\ell)G(\overline{\theta})$. Since $G(\overline{\theta}) \in [0,1]$, we have that $D \le \max\{b, \ell\} < 1$. ■

We are now ready to complete the construction of equilibrium. Consider an investor's optimal information choice. Given the binary action, an investor acquires information whenever the individual information cost is no larger than the value of private information:

$$n_i^* = I \Leftrightarrow c_i < D(n^*). \tag{A18}$$

Since each investor is atomistic and has no effect on the aggregate proportion of informed investors, the value of private information depends on the proportion of informed investors only. The perfect Bayesian equilibrium is constructed by combining these individual optimality conditions, Lemmas 2 and 3, and the consistency between individually optimal information acquisition choices and the aggregate proportion of informed investors.

The optimal information choice is characterized by a threshold strategy with some $\overline{c}$. An investor acquires information if and only if the information cost is below the threshold, $c_i < \overline{c}$, so the proportion of informed investors is $n^* = \overline{c}$. The marginal investor is indifferent between the information choices, $c_i = \overline{c}$, so this threshold is any solution of

$$\overline{c} = D(\overline{c}). \tag{A19}$$

Uniqueness requires that $D(\overline{c})$ has exactly one fixed point. First, the left-hand side of this equation is continuous, within $[0,1]$, and has a unit slope. Second, the right-hand side is continuous and strictly positive by Lemma 3, and its slope lies strictly within $(0,1)$ by Lemma 2. Therefore, if a solution exists, it is unique. Third, $D < 1$ by Lemma 3 ensures existence. This completes the proof of the existence of a unique interior solution $\overline{c}$.

B Amplification and Proof of Proposition 2

We prove the amplification result in three steps. The first two steps do not resort to the limiting case of vanishing private noise of informed investors. As a result, these two steps directly apply to the case of limited precision improvement studied in Section 5.4.

B.1 Derivation of Equation (19) and Intermediate Steps

We derive the total effect of changes in the public signal on the fundamental threshold, $\frac{d\overline{\theta}}{d\mu}$. The equilibrium is given by the following equations:

$$\overline{\theta} = A(n^*, \overline{\theta}, \mu),$$

$$n^* = \overline{c} = D(\overline{\theta}, \mu, \overline{x}_z), \tag{A20}$$

where the value of private information depends on the proportion of informed investors only indirectly. As for notation, m_x denotes the partial derivative $\frac{\partial m}{\partial x}$. Total differentiation yields:

$$\frac{d\overline{\theta}}{d\mu} = A_{n^*} \frac{dn^*}{d\mu} + A_{\overline{\theta}} \frac{d\overline{\theta}}{d\mu} + A_\mu,$$

$$\frac{dn^*}{d\mu} = D_{\overline{\theta}} \frac{d\overline{\theta}}{d\mu} + D_\mu, \tag{A21}$$

since $D_{\bar{x}_z} = 0$ for $z \in \{I, U\}$ (see Equation (A16)). Rewriting yields Equation (19). To evaluate the total derivative in Equation (19), we first obtain the partial derivatives of the aggregate attack size, which are all evaluated at the equilibrium quantities $(n^*, \overline{\theta})$:

$$A_\mu = -\frac{\alpha}{\sqrt{\gamma_I}} n^* \phi_I(\overline{\theta}) - (1-n^*)\frac{\alpha}{\sqrt{\gamma_U}}\phi_U(\overline{\theta}) < 0, \tag{A22}$$

$$A_{n^*} = \Phi_I(\overline{\theta}) - \Phi_U(\overline{\theta}) = [1 - A_{\overline{\theta}}]\frac{d\overline{\theta}}{dn^*}, \tag{A23}$$

where $\phi_z(\theta)$ is the probability density function associated with $\Phi_z(\theta)$.

Next, we determine the partial derivatives associated with the value of private information D. A change in the public signal μ affects it via (1) the fundamental threshold $\overline{\theta}$, (2) the signal thresholds $\overline{x}_z$, and (3) the distribution of fundamentals $g(\theta)$. First, for the fundamental threshold, we have already derived $D_{\overline{\theta}}$ in the proof of Lemma 2. Note that Equation (A15) generalizes to the case of limited precision improvement. Second, for the signal thresholds, we have already shown that $\frac{\partial D}{\partial \overline{x}_z} = 0$. Third, we consider the distribution of fundamentals. We use the following result (e.g., Bromiley 2013).

Lemma 4. The product of two normal probability density functions is a scaled normal probability density function. That is, if $f \sim \mathcal{N}(\mu_f, \sigma_f^2)$ and $g \sim \mathcal{N}(\mu_g, \sigma_g^2)$, then $f \circ g \sim S \times \mathcal{N}(\mu_{fg}, \sigma_{fg}^2)$, where

$$\sigma_{fg} = \sqrt{\frac{\sigma_f^2 \sigma_g^2}{\sigma_f^2 + \sigma_g^2}}, \quad \mu_{fg} = \frac{\mu_f \sigma_g^2 + \mu_g \sigma_f^2}{\sigma_f^2 + \sigma_g^2} \quad \text{and} \quad S = \frac{1}{\sqrt{2\pi(\sigma_f^2 + \sigma_g^2)}} \exp\left(-\frac{(\mu_f - \mu_g)^2}{2(\sigma_f^2 + \sigma_g^2)}\right).$$

To obtain the partial effect via a change in the distribution, D_μ, note that $\frac{\partial g}{\partial \mu} = \alpha(\theta - \mu)g(\theta) = -\frac{\partial g}{\partial \theta}$. The partial differentiation of the value of private information is:

$$D_\mu = \ell \int_{\overline{\theta}}^{\infty} \Big[\Phi_U(\theta) - \Phi_I(\theta)\Big]\alpha(\theta-\mu)g(\theta)d\theta - b\int_{-\infty}^{\overline{\theta}} \Big[\Phi_U(\theta) - \Phi_I(\theta)\Big]\alpha(\theta-\mu)g(\theta)d\theta. \tag{A24}$$

We proceed by integrating by parts. We set $u(\theta) \equiv \Phi_U(\theta) - \Phi_I(\theta)$ and $v'(\theta) \equiv \alpha(\theta-\mu)g(\theta)$, so $v(\theta) = -g(\theta)$ and $u'(\theta) = f^U(\overline{x}_U) - f^I(\overline{x}_I)$. From Result 4, $f^z(\overline{x}_z)(\theta)g(\theta) = S_z h_z(\theta)$, where $S_z \equiv \sqrt{\frac{\alpha\gamma_z}{\alpha+\gamma_z}}\phi\left(\sqrt{\frac{\alpha\gamma_z}{\alpha+\gamma_z}}[\mu - \overline{x}_z]\right) > 0$ is a constant and h_z is the probability density function of a normal random variable $\theta \sim \mathcal{N}\left(\frac{\alpha\mu+\gamma_z\overline{x}_z}{\alpha+\gamma_z}, \frac{1}{\alpha+\gamma_z}\right)$, with associated cumulative distribution function H_z. Using the first-order condition for the signal threshold $\overline{x}_z$, which can be written as $\kappa = H_z(\overline{\theta})$, the partial derivative simplifies to:

$$D_\mu = (b+\ell)g(\overline{\theta})\Big[\Phi_U(\overline{\theta}) - \Phi_I(\overline{\theta})\Big]. \tag{A25}$$

B.2 Proving the Inequality

Equipped with these preliminaries, we can now prove the inequality that constitutes the amplification result. Specifically, we need to show that

$$-\frac{A_\mu + A_{n^*} D_\mu}{1 - A_{\overline{\theta}} - A_{n^*} D_{\overline{\theta}}} > -\frac{A_\mu}{1 - A_{\overline{\theta}}}. \tag{A26}$$

To simplify this expression, we show in a first step that both denominators are positive. Consider $\gamma_I \to \infty$. As shown in the proof of Lemma 1, the denominator on the right-hand side of inequality

(A26) is positive if $\gamma_U > \underline{\gamma}'$. Using the expression for A_{n^*}, we rewrite the denominator on the left-hand side of inequality (A26):

$$\begin{aligned} 1-A_{\overline{\theta}}-A_{n^*}D_{\overline{\theta}} &= 1-A_{\overline{\theta}}-(1-A_{\overline{\theta}})\frac{d\overline{\theta}}{dn^*}D_{\overline{\theta}}, \\ &= (1-A_{\overline{\theta}})\left(1-D_{\overline{\theta}}\frac{d\overline{\theta}}{dn^*}\right), \\ &= (1-A_{\overline{\theta}})\left(1-\frac{dD}{dn^*}\right) > 0, \end{aligned} \tag{A27}$$

where the last line follows from the proof of Lemma 2 and the last inequality follows from equation (A14), which requires $\gamma_U > \underline{\gamma}$ and $\mu \neq \hat{\mu}$. In sum, both denominators are positive.

In a second step, we rewrite the inequality, using $A_{\overline{\theta}} = -A_\mu$ and $D_{\overline{\theta}} = -D_\mu$. Hence, amplification obtains if and only if

$$A_{n^*}D_\mu = -(b+\ell)g(\overline{\theta})\left(1-A_{\overline{\theta}}\right)^2\left(\frac{d\overline{\theta}}{dn^*}\right)^2 < 0, \tag{A28}$$

which holds generically, that is if and only if $\mu \neq \hat{\mu}$. To see this, observe that $b+\ell > 0$, $g > 0$, $A_{\overline{\theta}} \in (0,1)$ if $\gamma_U > \underline{\gamma}'$, and $\left(\frac{d\overline{\theta}}{dn^*}\right)^2 \geq 0$, with strict inequality if $\mu \neq \hat{\mu}$ by Lemma 1. Collecting the sufficient conditions, we require $\gamma_U > \underline{\gamma}$ and $\mu \neq \hat{\mu}$. This completes the proof of the amplification result regarding the fundamental threshold. Due to Equation (17), it extends directly to the probability of a financial crisis.

C Policy

C.1 Tax or Subsidy on Information Acquisition

To prove Proposition 3, note that the equilibrium in the case of tax or subsidy on information choice is given by the following two equations:

$$\overline{\theta} = A(n^*, \overline{\theta}) = n^*\Phi_I(\overline{\theta}) + (1-n^*)\Phi_U(\overline{\theta}), \tag{A29}$$

$$n^*(1-\tau) = \overline{c} = D(\overline{\theta}). \tag{A30}$$

Note that the tax or subsidy only enters on the left-hand side of the second equation. As a result, total differentiation with respect to τ yields Equation (22). Finally, we note that $\Gamma(\overline{\theta}) < 0$ if and only if the public signal is strong, as shown in Lemma 1.

C.2 Improving the Quality of Public Information

A higher precision of public information, $d\alpha > 0$, affects the equilibrium conditions at both the coordination stage and the information stage. Total differentiation yields Equation (23). Since the denominator of $\frac{d\overline{\theta}}{d\alpha}$ is positive, we need to evaluate its numerator. First, for $\kappa = \frac{1}{2}$, we have

$$A_\alpha = (\overline{\theta}-\mu)\left(\frac{n^*}{\sqrt{\gamma_I}}\phi_I(\overline{\theta}) + \frac{1-n^*}{\sqrt{\gamma_U}}\phi_U(\overline{\theta})\right), \tag{A31}$$

whose sign only depends on $(\overline{\theta}-\mu)$, since the second term is positive. Thus, $A_\alpha < 0$ if and only if $\overline{\theta} < \mu$, which holds if and only if $\mu > \frac{1}{2} = \hat{\mu}(\kappa = \frac{1}{2})$. Second, $A_{n^*} = -\Gamma(\overline{\theta}) > 0$ if and only if $\mu > \frac{1}{2}$. Thus, if $D_\alpha < 0$, then the claim $\frac{d\overline{\theta}}{d\alpha}(\mu - \hat{\mu}) < 0$ follows (generically).

Thus, it remains to show that $D_\alpha < 0$. The value of information depends on the precision of public information only via $g(\theta)$. Note that $2\frac{dg}{d\alpha} = g(\theta)\left[\frac{1}{\alpha} - (\theta-\mu)^2\right] \equiv m(\theta)$. One can prove, for example, by differentiation, that $M(\theta) \equiv \int m(\theta)d\theta = g(\theta)\frac{\theta-\mu}{\alpha}$. Thus,

$$2D_\alpha = \ell \int_{\overline{\theta}}^{\infty} \Gamma(\theta)m(\theta)d\theta - b\int_{-\infty}^{\overline{\theta}} \Gamma(\theta)m(\theta)d\theta. \tag{A32}$$

Using partial integration and Result 4, we observe that

$$\int \Gamma(\theta)m(\theta)d\theta = \Gamma(\theta)M(\theta) - \frac{S_U}{\alpha}\int h_U(\theta)(\theta-\mu)d\theta + \frac{S_I}{\alpha}\int h_I(\theta)(\theta-\mu)d\theta \tag{A33}$$

$$= \Gamma(\theta)M(\theta) - \frac{S_U}{\alpha}\left[\frac{\gamma_U(\overline{x}_U-\mu)}{\alpha+\gamma_U}H_U(\theta) - \frac{h_U(\theta)}{\alpha+\gamma_U}\right] + \frac{S_I}{\alpha}\left[\frac{\gamma_I(\overline{x}_I-\mu)}{\alpha+\gamma_I}H_I(\theta) - \frac{h_I(\theta)}{\alpha+\gamma_I}\right],$$

where h_z is evaluated at $\overline{x}_z$, and we define $\breve{\mu}_z \equiv \frac{\alpha\mu+\gamma_z\overline{x}_z}{\alpha+\gamma_z}$ as the mean of h_z, which allows us to rewrite and simplify the integral as follows: $\int h_z(\theta)(\theta-\mu)d\theta = \int h_z(\theta)(\theta-\breve{\mu}_z)d\theta + (\breve{\mu}_z-\mu)H_z(\theta) = -\frac{h_z(\theta)}{\alpha+\gamma_z} + \frac{\gamma_z(\overline{x}_z-\mu)}{\alpha+\gamma_z}H_z(\theta)$. (Recall that $\int y\phi(y)dy = -\phi(y)$.) It follows that:

$$2D_\alpha = -(b+\ell)\left(\Gamma(\overline{\theta})m(\overline{\theta}) + \frac{S_U}{\alpha}\frac{h_U(\overline{\theta})}{\alpha+\gamma_U} - \frac{S_I}{\alpha}\frac{h_I(\overline{\theta})}{\alpha+\gamma_I}\right). \tag{A34}$$

By continuity, there exists a $\underline{\gamma}_I$ such that $D_\alpha < 0$ for all $\gamma_I > \underline{\gamma}_I$. To see this, note that the third term of Equation (A34) vanishes as $\gamma_I \to \infty$. Also, note that $\Gamma(\overline{\theta})m(\overline{\theta}) > 0$ (generically), since $\Gamma(\overline{\theta}) < 0$ and $m(\overline{\theta}) < 0$ for $\mu > \frac{1}{2}$ and $\Gamma(\overline{\theta}) > 0$ and $m(\overline{\theta}) > 0$ for $\mu < \frac{1}{2}$.

D Magnitude of Amplification

D.1 Proof of Propositions 5 and 6

Using Equations (18) and (19), the magnitude of amplification (MoA) is given by:

$$\text{MoA}+1 = \frac{\left.\frac{d\bar{\theta}}{d\mu}\right|_{n^*,\bar{\theta}}}{\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\bar{n},\bar{\theta}}} = \frac{(A_\mu + D_\mu A_{n^*})(1-A_{\bar{\theta}})}{(1-A_{\bar{\theta}} - A_{n^*}D_{\bar{\theta}})A_\mu} = \frac{(A_\mu + D_\mu A_{n^*})}{(1-\frac{dD}{dn^*})A_\mu}, \tag{A35}$$

$$= \frac{(A_{\bar{\theta}} - D_\mu A_n)}{(1-\frac{dD}{dn^*})A_{\bar{\theta}}} = \frac{A_{\bar{\theta}} + (1-A_{\bar{\theta}})\frac{dD}{dn^*}}{(1-\frac{dD}{dn^*})A_{\bar{\theta}}} = 1 + \frac{\frac{dD}{dn^*}}{(1-\frac{dD}{dn^*})A_{\bar{\theta}}}, \tag{A36}$$

since $A_\mu = -A_{\bar{\theta}}$ and $-D_\mu A_{n^*} = (b+\ell)g(\bar{\theta})(1-A_\theta)^2(\frac{d\bar{\theta}}{dn^*})^2 = (1-A_{\bar{\theta}})\frac{dD}{dn^*}$.

Public signal. Consider first $\gamma_I \to \infty$. It follows that MoA$= \frac{\frac{dD}{dn^*}}{(1-\frac{dD}{dn^*})}\frac{1}{A_{\bar{\theta}}}$, which increases in $\frac{dD}{dn^*}$ and decreases in $A_{\bar{\theta}}$. We have $MoA(\mu=\hat{\mu})=0$ because of Lemma 1 but $MoA>0$ for all $\mu \in (-\infty,\hat{\mu})$ or $\mu \in (\hat{\mu},\infty)$. Moreover, $\lim_{\mu\to+\infty} MoA(\mu) = 0 = \lim_{\mu\to-\infty} MoA(\mu)$. This establishes the nonmonotonicity in the public signal. Finally, all of these results also hold for limited precision improvement, where $\hat{\mu}$ is replaced by $\tilde{\mu}$ (see also Section 5.4).

D.2 Lower Bound on MoA

Next, we characterize a lower bound on the magnitude of amplification. We first consider the case of vanishing private noise and subsequently study the case of limited precision improvement. In the latter case, the lower bound increases in the precision of private information of informed investors.

Proposition 14. Lower bounds on the magnitude of amplification.

1. If $\kappa > \frac{1}{2}$ and $\mu > \bar{\theta}$ (or $\kappa < \frac{1}{2}$ and $\mu < \bar{\theta}$), then

$$0 < \frac{\zeta\sqrt{2\pi\gamma_U}}{\alpha(1-\zeta)} < MoA, \tag{A37}$$

where $\zeta \equiv (b+\ell)\left[\Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-\kappa\right]^2 \min\{g(0), g(1)\}$.

2. If $\kappa = \frac{1}{2}$ and $1 < |\bar{\theta}-\mu|$, then we have two cases:

   (a.) For vanishing private noise, $\gamma_I \to \infty$, we have $0 < \frac{\zeta_0\sqrt{2\pi\gamma_U}}{\alpha(1-\zeta_0)} < MoA$, where $\zeta_0 \equiv 2b\left[\frac{1}{2}-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\right)\right]^2 \min\{g(0), g(1)\}$. The lower bound increases in b.

   (b.) For $\gamma_I < \infty$, and $|\bar{\theta}-\mu| < \sqrt{\frac{\ln(\gamma_I/\gamma_U)\gamma_I\gamma_U}{(\gamma_I-\gamma_U)\alpha^2}}$, we have $0 < \frac{\zeta_1\sqrt{2\pi\gamma_U}}{\alpha(1-\zeta_1)} < MoA$, where $\zeta_1 \equiv 2b\left[\Phi(\frac{\alpha}{\sqrt{\gamma_U}})-\Phi(\frac{\alpha}{\sqrt{\gamma_I}})\right]^2 \min\{g(0), g(1)\}$, which increases in both b and γ_I.

Proof. To determine a nontrivial lower bound, we assume $\kappa > \frac{1}{2}$ and $\mu > \bar{\theta}$. Due to equations (A5) and (A17), we have $\frac{dD}{dn^*} = (b+\ell)\frac{g(\bar{\theta})}{1-A_{\bar{\theta}}}[1-\kappa-\Phi(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa))]^2$. Thus, we have

$$1-\kappa-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) > 1-\kappa-\Phi\left(-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)$$
$$= \Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-\kappa > 0.$$

As a result:

$$\frac{dD}{dn^*} > (b+\ell)\frac{g(\bar{\theta})}{1-A_{\bar{\theta}}}\left[\Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-\kappa\right]^2,$$
$$> (b+\ell)g(\bar{\theta})\left[\Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-\kappa\right]^2,$$
$$> (b+\ell)\left[\Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-\kappa\right]^2 \min\{g(0), g(1)\} \equiv \zeta, \tag{A38}$$

where the second inequality follows from $0 < A_{\bar{\theta}} < 1$, and the third inequality follows from $\bar{\theta} \in [0,1]$ and since $g(\cdot)$ is unimodal at μ, so $g(\bar{\theta}) > \min\{g(0), g(1)\}$. Finally, since MoA increases in $\frac{dD}{dn^*}$ and since $A_{\overline{\theta}} < \frac{\alpha}{\sqrt{2\pi\gamma_U}}$, we have $\frac{\zeta\sqrt{2\pi\gamma_U}}{(1-\zeta)\alpha} < MoA$. Note that $\overline{\theta} = \mu$ obtains for $\mu(\overline{c}) = \overline{c}(1-\kappa)+(1-\overline{c})\left(1-\Phi\left(\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)\right)$, where $\overline{c}$ is the fixed point of $\overline{c} = \ell\int_{\mu(\overline{c})}^{\infty}\left[\mu(\overline{c})-\theta-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right]dG(\theta)+b\int_{-\infty}^{\mu(\overline{c})}\left[1-\mu(\overline{c})+\theta+\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right]dG(\theta)$.

The case of $\mu < \bar{\theta}$ and $\kappa < \frac{1}{2}$ follows similar steps and also results in the lower bound ζ. We obtain $\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) > \Phi\left(-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)$, and $\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) > 1-\kappa$. Therefore, $\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right) - (1-\kappa) > \Phi\left(-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)-(1-\kappa) > 0$.

Finally, consider the case of $\kappa=\frac{1}{2}$, so $\Phi^{-1}(\kappa)=0$. If $|\bar{\theta}-\mu|>1$, we have

$$\left[1-\kappa-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}[\bar{\theta}-\mu]-\sqrt{1+\frac{\alpha}{\gamma_U}}\Phi^{-1}(\kappa)\right)\right]^2 \geq \left[\frac{1}{2}-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\right)\right]^2, \tag{A39}$$

which leads to $\frac{dD}{dn^*}>2b\left[\frac{1}{2}-\Phi\left(\frac{\alpha}{\sqrt{\gamma_U}}\right)\right]^2 \min\{g(0),g(1)\}=\zeta_0$.

A lower bound for limited precision improvement. Suppose that $\gamma_I<\infty$. We first note a useful Lemma.

Lemma 5. Let $a>b>0$. Then, $\Phi(a\delta)-\Phi(b\delta)$ increases in δ if $|\delta|<\sqrt{\frac{2\ln(a/b)}{a^2-b^2}}$.

Proof. Let $\Delta(\delta)=\Phi(a\delta)-\Phi(b\delta)$. Then, $\frac{\partial\Delta(\delta)}{\partial\delta}=a\phi(a\delta)-b\phi(b\delta)>0$ if and only if $\delta^2<\frac{2\ln(a/b)}{a^2-b^2}$, completing the proof. ■

Let $\kappa=\frac{1}{2}$ and $|\bar{\theta}-\mu|>1$. To apply the above lemma, let $a=\frac{\alpha}{\sqrt{\gamma_U}}$, $b=\frac{\alpha}{\sqrt{\gamma_I}}$, and $\delta=\bar{\theta}-\mu$. Then, for $1<|\bar{\theta}-\mu|<\sqrt{\frac{\ln(\gamma_I/\gamma_U)\gamma_I\gamma_U}{(\gamma_I-\gamma_U)\alpha^2}}$, we have $|\Phi_U(\bar{\theta})-\Phi_I(\bar{\theta})|=|\Phi(\frac{\alpha}{\sqrt{\gamma_U}}(\bar{\theta}-\mu))-\Phi(\frac{\alpha}{\sqrt{\gamma_I}}(\bar{\theta}-\mu))|>|\Phi(\frac{\alpha}{\sqrt{\gamma_U}})-\Phi(\frac{\alpha}{\sqrt{\gamma_I}})|$. Equipped with this lower bound, equations (A5) and (A49) imply:

$$\frac{dD}{dn^*}=2b\frac{g(\bar{\theta})}{1-A_{\bar{\theta}}}[\Phi_U(\bar{\theta})-\Phi_I(\bar{\theta})]^2 \geq 2b[\Phi(\frac{\alpha}{\sqrt{\gamma_U}})-\Phi(\frac{\alpha}{\sqrt{\gamma_I}})]^2\min\{g(0),g(1)\}\equiv\zeta_1. \tag{A40}$$

Thus, we obtain a lower bound that increases in both γ_I and b:

$$0<\frac{\zeta_1\sqrt{2\pi\gamma_U}}{\alpha(1-\zeta_1)}<MoA<\infty. \tag{A41}$$

■

D.3 Generic Information Cost and Proofs of Propositions 7 and 8

The density function $f(c)$ has support $[c_{min},c_{max}]$ and let $F(c)$ be the associated cumulative distribution function. Investors use the threshold strategy in equilibrium, whereby an investor acquires information, $n_i^*=I$, if and only if the individual information cost is below the threshold information cost $\bar{c}$. As a result, the equilibrium proportion of informed investors is $n^*=F(\bar{c})$, and the equilibrium threshold information cost solves the fixed-point equation

$$\bar{c}=D(F(\bar{c})). \tag{A42}$$

Two conditions ensure the existence of a unique threshold information cost $\bar{c}$. First, the information cost has to be sufficiently heterogeneous relative to the bounds on the value of private information to ensure the required dominance regions at the information stage:

$$c_{min}<D(0)<D(1)<c_{max}, \tag{A43}$$

Existence of a threshold information cost follows. Second, uniqueness of the threshold information cost requires the slope of the left-hand side of condition (A42) to exceed the slope of the right-hand side:

$$1>\frac{dD}{dn^*}f(\bar{c}), \tag{A44}$$

where a higher threshold information cost raises the equilibrium proportion of informed investors and, because of the strategic complementarity in information acquisition, the value of private information. Paralleling the previous proof, an upper bound on the probability density function suffices:

$$f(c) \leq \frac{\sqrt{2\pi}}{2} - \frac{\alpha}{2\sqrt{\gamma_U}}, \tag{A45}$$

for $\gamma_U \in (\frac{\alpha^2}{2\pi}, \infty)$, where the lower bound ensures uniqueness at the coordination stage. If the distribution of information costs satisfies $f(c) < \sqrt{\frac{\pi}{2}}$, then there exists a sufficiently large but finite precision of an uninformed investor's private signal to support uniqueness.

For the uniform distribution over the interval $[0,1]$ considered in the main text, for example, the slope condition is satisfied, $f(c) = 1$, and implies a range of precision that ensure uniqueness, $\gamma_U \in (\underline{\gamma}, \infty)$. Moreover, since $D \in (0,1)$ by Lemma 3, the heterogeneity condition is also satisfied.

We generalize the amplification result to a generic distribution of the information cost:

$$\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} = \frac{A_\mu + A_{n^*} D_\mu f(\overline{c})}{1 - A_{\overline{\theta}} - A_{n^*} D_{\overline{\theta}} f(\overline{c})} < 0. \tag{A46}$$

Amplification always occurs since $A_{n^*} D_\mu f(\overline{c}) < 0$. Moreover, the magnitude of amplification is larger than in the main text if and only if $f(\overline{c}) > 1$, for which $f(c) > 1$ suffices.

More generally, the magnitude of amplification increases in the proportion of investors who change their information acquisition choice after a change in the public signal. The three specification considered in Proposition 8 are designed to ensure that one density function is above the other for any relevant information cost within $[D(0), D(1)]$. See also Figure 8.

D.4 Limited Precision Improvement and Proof of Proposition 9

For the extension of limited precision improvement, $\gamma_I \in (\gamma_U, \infty)$, we prove the existence and uniqueness of equilibrium and generalize the amplification result. The signal thresholds are given by Equation (A5) and the fundamental threshold is implicitly given by Equation (A7). Lemma 6 extends Lemma 1 and its proof parallels that of Lemma 1 closely and is omitted for brevity. To generalize the definition of a strong public signal, we replace $\hat{\mu}$ with $\tilde{\mu} \equiv \Phi\left(-\frac{\sqrt{\alpha+\gamma_I}-\sqrt{\alpha+\gamma_U}}{\sqrt{\gamma_I}-\sqrt{\gamma_U}}\Phi^{-1}(\kappa)\right) - \frac{\sqrt{\gamma_I(\alpha+\gamma_U)}-\sqrt{\gamma_U(\alpha+\gamma_I)}}{\alpha(\sqrt{\gamma_I}-\sqrt{\gamma_U})}\Phi^{-1}(\kappa)$, where $\tilde{\mu} \to \hat{\mu}$ as $\gamma_I \to \infty$. Moreover, $\hat{\theta}$ generalizes to $\tilde{\theta} \equiv \mu + \frac{\sqrt{\gamma_I(\alpha+\gamma_U)}-\sqrt{\gamma_U(\alpha+\gamma_I)}}{\alpha(\sqrt{\gamma_I}-\sqrt{\gamma_U})}\Phi^{-1}(\kappa)$.

Lemma 6. Suppose $\gamma_U > \underline{\gamma}'$. If $\mu \neq \tilde{\mu}$, then the fundamental threshold at the coordination stage responds to changes in the proportion of informed investors:

$$\frac{d\overline{\theta}}{dn^*} \neq 0 \tag{A47}$$

Furthermore, the fundamental threshold increases in the proportion of informed investors if and only if the public signal is strong, $\frac{d\overline{\theta}}{dn^*}\left(\mu - \tilde{\mu}\right) > 0$.

The value of private information is given by Equations (11) and (12). Lemma 2 and Lemma 3 also generalize to the case of limited precision improvement, as we show below.

Lemma 7. If $\gamma_U > \underline{\gamma}'$, there is strategic complementarity in information choices:

$$\frac{dD}{dn^*} = (b+\ell)\left[1 - A_{\overline{\theta}}\right] g(\overline{\theta}) \left(\frac{d\overline{\theta}}{dn^*}\right)^2 \geq 0, \tag{A48}$$

with strict inequality if $\mu \neq \tilde{\mu}$. Furthermore, the more-restrictive lower bound on the precision of private information, $\gamma_U > \left(\frac{\alpha}{\sqrt{2\pi}-2}\right)^2 = \underline{\gamma}$, ensures $\frac{dD}{dn^*} < 1$.

Proof. The proof follows the same steps as the proof of Lemma 2. If $\gamma_U > \underline{\gamma}'$, then $\overline{\theta}(n^*;\mu)$ is unique for any $n^* \in [0,1]$. One can show $\frac{dD}{dn^*} = (b+\ell)\left[1-A_{\overline{\theta}}\right] g(\overline{\theta}) \left(\frac{d\overline{\theta}}{dn^*}\right)^2 \geq 0$, with strict inequality if and only if $\mu \neq \tilde{\mu}$, where $\frac{\partial D}{\partial \overline{x}_z} = 0$ by an envelope theorem argument. Using Lemma 6 and rewriting yields

$$\frac{dD}{dn^*} = \frac{(b+\ell)g(\overline{\theta})\left[\Phi_I(\overline{\theta}) - \Phi_U(\overline{\theta})\right]^2}{1-A_{\overline{\theta}}}. \tag{A49}$$

Note that $g \leq \frac{1}{\sqrt{2\pi}}$, $b+\ell < 2$, and $[\Phi_I(\overline{\theta}) - \Phi_U(\overline{\theta})]^2 \leq 1$, and $A_{\overline{\theta}} \leq \frac{\alpha}{\sqrt{2\pi\gamma_U}}$ because $n^* \geq 0$ and $\phi \leq \frac{1}{\sqrt{2\pi}}$. Therefore, $\frac{dD}{dn^*} \leq \frac{2\frac{1}{\sqrt{2\pi}}}{1-\frac{\alpha}{\sqrt{2\pi\gamma_U}}}$. Hence, $\frac{dD}{dn^*} < 1$ is ensured by $\gamma_U > \underline{\gamma} > \underline{\gamma}'$. ■

Lemma 8. If $\gamma_U > \underline{\gamma}'$, then the value of private information satisfies $D \in (0,1)$.

Proof. If $\gamma_U > \underline{\gamma}'$, then $\overline{\theta}(n^*;\mu)$ is unique for any $n^* \in [0,1]$. First, we show $D < 1$ for all n^*:

$$D \leq \ell \int_{\overline{\theta}}^{\infty} \Phi_U(\theta) dG(\theta) + b \int_{-\infty}^{\overline{\theta}} \Phi_I(\theta) dG(\theta) \leq \ell[1-G(\overline{\theta})] + bG(\overline{\theta}) \leq \max\{b,\ell\} < 1, \tag{A50}$$

where, in the first line, we dropped $-\Phi_I(\theta)$ from the first term and $-\Phi_U(\theta)$ from the second term and, in the second line, used that $\Phi_z(\theta) \leq 1$ and $G(\overline{\theta}) \in [0,1]$.

Second, we show $D > 0$ for all n^*. Because of the strategic complementarity in information acquisition (Lemma 7), it suffices to show $D > 0$ at $n^* = 0$. At the lower bound, as $\gamma_I \to \gamma_U$, $\Gamma \to 0$ and $D \to 0$. At the upper bound, we have shown that $D > 0$ as $\gamma \to \infty$ (Lemma 3). Thus, a sufficient condition for $D > 0$ in case of limited precision improvement is $\left.\frac{dD}{d\gamma_I}\right|_{n^*=0} > 0$. In what follows, we use the result of $D_{\overline{x}_I} = 0$ and the result of $\left.\frac{d\overline{\theta}}{d\gamma_I}\right|_{n^*=0} = 0$, since changes in the precision of informed investors do not affect the fundamental threshold if a zero mass of investors acquires information. Hence, by total differentiation:

$$\left.\frac{dD}{d\gamma_I}\right|_{n^*=0} = \frac{\partial D}{\partial \gamma_I} = \frac{1}{2\sqrt{\gamma_I}}\left[-\ell \int_{\overline{\theta}}^{\infty} [\overline{x}_I - \theta]\phi_I(\theta) dG(\theta) + b \int_{\infty}^{\overline{\theta}} [\overline{x}_I - \theta]\phi_I(\theta) dG(\theta)\right] \tag{A51}$$

$$> \frac{1}{2\sqrt{\gamma_I}}\left[-\ell \int_{\overline{\theta}}^{\infty} [\overline{x}_I - \overline{\theta}]\phi_I(\theta) dG(\theta) + b \int_{\infty}^{\overline{\theta}} [\overline{x}_I - \overline{\theta}]\phi_I(\theta) dG(\theta)\right]$$

$$= \frac{[\overline{x}_I - \overline{\theta}]}{2\sqrt{\gamma_I}}\left[-\ell \int_{\overline{\theta}}^{\infty} \phi_I(\theta) dG(\theta) + b \int_{\infty}^{\overline{\theta}} \phi_I(\theta) dG(\theta)\right] = 0, \tag{A52}$$

where the first line uses $\phi_z(\theta)$ as the probability density function associated with $\Phi_z(\theta)$, the inequality of the second line arises from different weights used. Specifically, the expression on this line has a higher weight $[\overline{x}_I - \overline{\theta}] > [\overline{x}_I - \theta]$ for $\theta \geq \overline{\theta}$ on the loss and a lower weight $[\overline{x}_I - \overline{\theta}] < [\overline{x}_I - \theta]$ for $\theta \leq \overline{\theta}$ on the benefit. The third line is zero, since this is the first-order condition for the optimal threshold $\overline{x}_I$. This completes the proof. ■

As in Appendix A, existence and uniqueness follows from Lemmas 6 – 8. The final step is to prove amplification in the case of limited precision improvement. The proof in Appendix B generalizes. The first two steps did not require vanishing private noise of informed investors. The third step generalizes as well, where the relevant inequality becomes to $\mu \neq \tilde{\mu}$.

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In case of payoffs sensitive to the fundamental, where we follow Iachan and Nenov (2015), Bayesian updating is unchanged but the **indifference conditions** for $z \in \{I, U\}$ becomes

$$M^z(\overline{\theta}, \overline{x}_z) \equiv \int_{-\infty}^{\overline{\theta}} b(\theta) h_z(\theta, \overline{x}_z) d\theta - \int_{\overline{\theta}}^{\infty} \ell(\theta) h_z(\theta, \overline{x}_z) d\theta \equiv 0. \tag{A53}$$

where the signal threshold $\overline{x}_z$ enters implicitly via $h_z = h_z(\theta, \overline{x}_z)$. Without payoff sensitivity, Equation (A53) would again yield $\kappa = H_z(\overline{\theta}, \overline{x}_z)$, which implies an explicit expression of $\overline{x}_z$.

For future reference, the partial derivatives of $M^z(\overline{\theta}, \overline{x}_z; \mu)$ are

$$M^z_{\overline{\theta}} = h_z(\overline{\theta}) \Big(b(\overline{\theta}) + \ell(\overline{\theta}) \Big) > 0, \tag{A54}$$

$$M^z_{\overline{x}_z} = -\frac{\gamma_z}{\alpha + \gamma_z} \left[h_z(\overline{\theta}) \Big(b(\overline{\theta}) + \ell(\overline{\theta}) \Big) - \int_{-\infty}^{\overline{\theta}} b'(\theta) dH_z(\theta) + \int_{\overline{\theta}}^{\infty} \ell'(\theta) dH_z(\theta) \right] < 0, \tag{A55}$$

$$M^z_{\mu} = \frac{\alpha}{\gamma_z} M^z_{\overline{x}_z} < 0, \tag{A56}$$

where we used $\frac{\partial h_z}{\partial \overline{x}_z} = h_z(\theta) \gamma_z \left[\theta - \frac{\alpha\mu + \gamma_z \overline{x}_z}{\alpha + \gamma_z} \right] = \frac{\gamma_z}{\alpha} \frac{\partial h_z}{\partial \mu}$ and partial integration for $M^z_{\overline{x}_z}$ and M^z_{μ}.

The **critical mass condition** can be stated as:

$$\overline{\theta} \equiv n^* \Phi\left(\sqrt{\gamma_I} \left[\overline{x}_I - \overline{\theta} \right] \right) + (1 - n^*) \Phi\left(\sqrt{\gamma_U} \left[\overline{x}_U - \overline{\theta} \right] \right) \equiv A(\overline{\theta}, n^*, \overline{x}_U, \overline{x}_I). \tag{A57}$$

For future reference, the partial derivatives of $A(\overline{\theta}, n^*, \overline{x}_U, \overline{x}_I)$ are:

$$A_{n^*} = \Phi_I(\overline{\theta}) - \Phi_U(\overline{\theta}) = -\Gamma(\overline{\theta}), \tag{A58}$$

$$A_{\overline{x}_I} = n^* \sqrt{\gamma_I} \phi_I(\overline{\theta}) > 0, \tag{A59}$$

$$A_{\overline{x}_U} = (1 - n^*) \sqrt{\gamma_U} \phi_U(\overline{\theta}) > 0, \tag{A60}$$

$$A_{\overline{\theta}} = -A_{\overline{x}_I} - A_{\overline{x}_U} < 0, \tag{A61}$$

$$A_{\mu} = 0. \tag{A62}$$

These partial derivatives do not correspond to the main text directly, since one cannot solve for the signal thresholds $\overline{x}_z$ explicitly in case of payoff sensitivity. For example, A_μ captures only the direct effect of the public signal on the aggregate attack size (which is zero), without taking into account the indirect effect via the indifference condition of the marginal investor.

The value of private information can be stated as

$$D \equiv \int_{\overline{\theta}}^{\infty} \ell(\theta) \Gamma(\theta) dG(\theta) - \int_{-\infty}^{\overline{\theta}} b(\theta) \Gamma(\theta) dG(\theta), \tag{A63}$$

where the payoffs now depend directly on the fundamental. The partial derivatives are

$$D_{\overline{x}_z} = 0, \tag{A64}$$

$$D_{\overline{\theta}} = -g(\overline{\theta}) \big(b(\overline{\theta}) + \ell(\overline{\theta}) \big) \Gamma(\overline{\theta}), \tag{A65}$$

$$D_{\mu} = g(\overline{\theta}) \big(b(\overline{\theta}) + \ell(\overline{\theta}) \big) \Gamma(\overline{\theta}) + \int_{\overline{\theta}}^{\infty} \ell'(\theta) \Gamma(\theta) dG(\theta) - \int_{-\infty}^{\overline{\theta}} b'(\theta) \Gamma(\theta) dG(\theta), \tag{A66}$$

where the first line again follows from the optimality of $\overline{x}_z$, and the derivation of D_μ parallels the derivation in Appendix B.1. and again uses partial integration as well as the indifference condition in Equation (A53). Observe that D_μ now has additional terms that depend on the slopes of the payoff parameters with respect to the fundamental. Thus, $D_\mu \neq -D_{\overline{\theta}}$, in general. Since $c_i \sim \mathcal{U}[0,1]$ as in the main text, the equilibrium proportion of informed investors and the **threshold information cost** is again implicitly given by the fixed point $n^* = \overline{c} = D(\overline{c})$.

Exogenous information. If the proportion of informed investors is exogenous, the equilibrium is given by the following set of equations:

$$M^I(\overline{\theta}, \overline{x}_I; \mu) = 0, \tag{A67}$$

$$M^U(\overline{\theta}, \overline{x}_U; \mu) = 0, \tag{A68}$$

$$\overline{\theta} = A(\overline{\theta}, \overline{x}_U, \overline{x}_I, \overline{n}). \tag{A69}$$

Total differentiation of this system of equations with respect to μ and evaluating the resultant expression at the equilibrium values ($\overline{n} = n^*$, $\overline{\theta}$) yields

$$-\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n}=n^*,\overline{\theta}} = \frac{\frac{\alpha}{\gamma_I} A_{\overline{x}_I} + \frac{\alpha}{\gamma_U} A_{\overline{x}_U}}{1 - A_{\overline{\theta}} + A_{\overline{x}_I} \frac{M^I_{\overline{\theta}}}{M^I_{\overline{x}_I}} + A_{\overline{x}_U} \frac{M^U_{\overline{\theta}}}{M^U_{\overline{x}_U}}} \equiv \frac{\delta_0}{\delta_1} > 0, \tag{A70}$$

since $\delta_0 > 0$ and $\delta_1 > 0$ because of the uniqueness of equilibrium. We abstract from deriving sufficient conditions for the existence of a unique equilibrium in the general case but fully describe a special case with linear payoffs below.

Endogenous information choice. If the proportion of informed investors is endogenous, the equilibrium is given by the following set of equations:

$$M^I(\overline{\theta}, \overline{x}_I; \mu) = 0, \tag{A71}$$

$$M^U(\overline{\theta}, \overline{x}_U; \mu) = 0, \tag{A72}$$

$$\overline{\theta} = A(\overline{\theta}, \overline{x}_U, \overline{x}_I, n^*), \tag{A73}$$

$$n^* = \overline{c} = D(\overline{\theta}, \overline{x}_I, \overline{x}_U; \mu). \tag{A74}$$

Total differentiation of this system of equations with respect to μ and evaluating the resultant expression at the equilibrium values (n^*, $\overline{\theta}$) yields:

$$-\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} = \frac{\frac{\alpha}{\gamma_I} A_{\overline{x}_I} + \frac{\alpha}{\gamma_U} A_{\overline{x}_U} - A_{n^*} D_\mu}{1 - A_{\overline{\theta}} + A_{\overline{x}_I} \frac{M^I_{\overline{\theta}}}{M^I_{\overline{x}_I}} + A_{\overline{x}_U} \frac{M^U_{\overline{\theta}}}{M^U_{\overline{x}_U}} - A_{n^*} D_{\overline{\theta}}}. \tag{A75}$$

Amplification. Note that $-\left.\frac{d\overline{\theta}}{d\mu}\right|_{n^*,\overline{\theta}} > -\left.\frac{d\tilde{\theta}}{d\mu}\right|_{\overline{n}=n^*,\overline{\theta}}$, or amplification via the information choice of investors, occurs whenever:

$$-\delta_0 A_{n^*} D_{\overline{\theta}} + \delta_1 A_{n^*} D_\mu < 0, \tag{A76}$$

We have $A_{n^*} D_{\overline{\theta}} = g(\overline{\theta})\Big(b(\overline{\theta}) + \ell(\overline{\theta})\Big)\Gamma(\overline{\theta})^2 > 0$ generically. Thus, $A_{n^*} D_\mu \leq 0$ suffices for amplification, and this yields the stated condition, as can be seen from $A_{n^*} D_\mu \equiv \delta_3 + \delta_4$:

$$\delta_3 \equiv -g(\overline{\theta})\Big(b(\overline{\theta}) + \ell(\overline{\theta})\Big)\Gamma(\overline{\theta})^2 \leq 0, \tag{A77}$$

$$\delta_4 \equiv -\Gamma(\overline{\theta})\Big[\int_{\overline{\theta}}^{\infty} \ell'(\theta)\Gamma(\theta) dG(\theta) - \int_{-\infty}^{\overline{\theta}} b'(\theta)\Gamma(\theta) dG(\theta)\Big]. \tag{A78}$$

E.1 A Special Case with Linearity

Consider the linear case of $b(\theta)=b_0-b_1\theta$ and $\ell(\theta)=\ell_0+\ell_1\theta$, where all coefficients are strictly positive. Since $\overline{\theta}\in(0,1)$, we impose $b_0>b_1$ to ensure that $b(\theta)>0$ over the relevant range $(-\infty,1)$. Likewise, $\ell_0>0$ ensures that $\ell(\theta)>0$ over the relevant range $(0,\infty)$. We also assume identical slope coefficients $b_1=\ell_1\equiv\lambda$.

As a result, the partial derivatives of the indifference condition simplify to:

$$M^z_{\overline{\theta}}=h_z(\overline{\theta})\Big(b_0+\ell_0\Big)>0, \tag{A79}$$

$$M^z_{\overline{x}_z}=-\frac{\gamma_z}{\alpha+\gamma_z}\Big[M^z_{\overline{\theta}}+\lambda\Big]<0. \tag{A80}$$

These results allow us to simplify the expressions for δ_0 and δ_1. First, $\delta_0>0$ since $\phi(\cdot)\leq\frac{1}{2\pi}$, so $\delta_0=n^*\phi_I(\cdot)\frac{\alpha}{\sqrt{\gamma_I}}+(1-n^*)\phi_U(\cdot)\frac{\alpha}{\sqrt{\gamma_U}}\leq\frac{\alpha}{\sqrt{2\pi}}\Big[\frac{n^*}{\sqrt{\gamma_I}}+\frac{1-n^*}{\sqrt{\gamma_U}}\Big]\leq\frac{\alpha}{\sqrt{2\pi\gamma_U}}$. Thus, $\delta_0<1$ is again ensured by $\gamma_U>\underline{\gamma}'$. Second, one can show that $\delta_1>0$ under certain conditions. To see this, insert $A_{\overline{\theta}}$ into δ_1 to obtain:

$$\delta_1=1+A_{\overline{x}_I}\left(1+\frac{M^I_{\overline{\theta}}}{M^I_{\overline{x}_I}}\right)+A_{\overline{x}_U}\left(1+\frac{M^U_{\overline{\theta}}}{M^U_{\overline{x}_U}}\right), \tag{A81}$$

$$=1+n^*\sqrt{\gamma_I}\phi_I(\cdot)\frac{\lambda-\frac{\alpha}{\gamma_I}M^I_{\overline{\theta}}}{\lambda+M^I_{\overline{\theta}}}+(1-n^*)\sqrt{\gamma_U}\phi_U(\cdot)\frac{\lambda-\frac{\alpha}{\gamma_U}M^U_{\overline{\theta}}}{\lambda+M^U_{\overline{\theta}}}. \tag{A82}$$

If $\frac{d}{d\lambda}\frac{\lambda-\frac{\alpha}{\gamma_z}M^z_{\overline{\theta}}}{\lambda+M^z_{\overline{\theta}}}>0$, then $\delta_1>0$ is guaranteed by $\delta_1(\lambda=0)\equiv\delta_1^{min}>0$. As before, this is ensured by $\gamma_U>\underline{\gamma}'$. Lemma 9 states a sufficient condition for the positive total derivative.

Lemma 9. There exists a constant $\rho\in(0,\infty)$ such that $\frac{d}{d\lambda}\frac{\lambda-\frac{\alpha}{\gamma_z}M^z_{\overline{\theta}}}{\lambda+M^z_{\overline{\theta}}}>0$ if $\gamma_I\lambda<\rho$.

Proof. Taking a derivative with respect to λ implies

$$\frac{d}{d\lambda}\left(\frac{\lambda-\frac{\alpha}{\gamma_z}M^z_{\overline{\theta}}}{\lambda+M^z_{\overline{\theta}}}\right)=\frac{1+\frac{\alpha}{\gamma_z}}{(\lambda+M^z_{\overline{\theta}})^2}\left(M^z_{\overline{\theta}}-\lambda\frac{dM^z_{\overline{\theta}}}{d\lambda}\right)>0$$

if and only if $\frac{1}{\lambda}M^z_{\overline{\theta}}>\frac{dM^z_{\overline{\theta}}}{d\lambda}$. Thus, it suffices to show

$$\frac{1}{\lambda}M^z_{\overline{\theta}}>|\frac{dM^z_{\overline{\theta}}}{d\lambda}|. \tag{A83}$$

Thus, chain rule implies $|\frac{dM^z_{\overline{\theta}}}{d\lambda}|\leq|\frac{\partial M^z_{\overline{\theta}}}{\partial\overline{\theta}}||\frac{d\overline{\theta}}{d\lambda}|$. By following similar steps as in derivation of (A75), we have

$$-\frac{d\overline{\theta}}{d\lambda}\bigg|_{n^*,\overline{\theta}}=\frac{\frac{M^I_\lambda}{M^I_{\overline{x}_I}}A_{\overline{x}_I}+\frac{M^U_\lambda}{M^U_{\overline{x}_I}}A_{\overline{x}_U}-A_{n^*}D_\lambda}{1-A_{\overline{\theta}}+A_{\overline{x}_I}\frac{M^I_{\overline{\theta}}}{M^I_{\overline{x}_I}}+A_{\overline{x}_U}\frac{M^U_{\overline{\theta}}}{M^U_{\overline{x}_U}}-A_{n^*}D_{\overline{\theta}}}, \tag{A84}$$

where due to (A53) and (A63):

$$M^z_\lambda=-\frac{\alpha\mu+\gamma_z\overline{x}_z}{\alpha+\gamma_z}, \tag{A85}$$

$$D_\lambda=E[\theta\Gamma(\theta)]. \tag{A86}$$

Given the characterizations in (A58)-(A62), (A64)-(A66), (A85), and (A86), there exists a finite ρ such that $|\Big[\bar{\theta}-\frac{\alpha\mu+\gamma_z\overline{x}_z}{\alpha+\gamma_z}\Big]||\frac{d\bar{\theta}}{d\lambda}|<\rho^{-1}$. The existence of ρ follows by the finiteness of $\bar{\theta}$, $\frac{\alpha\mu+\gamma_z\overline{x}_z}{\alpha+\gamma_z}$

and $\frac{d\bar{\theta}}{d\lambda}$. Recall that $M^z_{\bar{\theta}} = h_z(\bar{\theta})(b_0+\ell_0)$, thus $-\frac{\partial M^z_{\bar{\theta}}}{\partial \bar{\theta}} = \left[\bar{\theta} - \frac{\alpha\mu+\gamma_z \bar{x}_z}{\alpha+\gamma_z}\right] h_z(\bar{\theta})\,(\alpha+\gamma_z)\,(b_0+\ell_0)$. Finally, plugging $\frac{\partial M^z_{\bar{\theta}}}{\partial \bar{\theta}}$ and (A84) into (A83) implies that the inequality holds when $\gamma_z \lambda < \rho$. Since $\gamma_I > \gamma_U$, thus it is sufficient to have $\gamma_I \lambda < \rho$. Let $\overline{\lambda} \equiv \frac{\rho}{\gamma_I}$ denote the upper bound on the sensitivity to the payoff, which completes the proof. ∎

We wish to establish the existence of a unique equilibrium. Paralleling the proof in Appendix A, we need to generalize Lemmas 2 and 3.

Lemma 10. If $\gamma_U > \underline{\gamma}$ and $\lambda < \overline{\lambda}$, then strategic complementarity generically obtains:

$$\frac{dD}{dn^*} = (b_0+\ell_0)g(\overline{\theta})\frac{\Gamma(\overline{\theta})^2}{\delta_1} > 0. \tag{A87}$$

Furthermore, $\frac{dD}{dn^*} < 1$.

Proof. The expression for $\frac{dD}{dn^*}$ is derived in the same way as in the main text. Using $g(\overline{\theta}) \le \frac{1}{\sqrt{2\pi}}$, $b_0+\ell_0 = b+\ell < 2$, $\Gamma^2 \le 1$, and $\delta_1 \ge \delta_1^{min}$ yields the lower bound on γ_U. ∎

Lemma 11. If $\gamma_U > \underline{\gamma}$ and $\lambda < \overline{\lambda}$, the value of private information satisfies $D \in (0,1)$.

Proof. First, $D > 0$ follows by a direct generalization of Lemma 8, where the generalized condition for the optimality of $\overline{x}_I$ is used. Second, since $b \in (0,1)$ and $\ell \in (0,1)$ and $\Gamma(\theta) \in [-1,1]$ for all θ, it follows that $D = \int_{\overline{\theta}}^{\infty} \ell(\theta)\Gamma(\theta)dG(\theta) - \int_{-\infty}^{\overline{\theta}} b(\theta)\Gamma(\theta)dG(\theta) < \int_{\overline{\theta}}^{\infty} dG(\theta) - \int_{-\infty}^{\overline{\theta}}(-1)dG(\theta) = 1$. ∎

Following the steps of the proof in the baseline model, a unique equilibrium exists. Simplifying the previous sufficient condition, amplification occurs whenever:

$$g(\overline{\theta})(b_0+\ell_0)\Gamma(\overline{\theta})^2 \ge -\lambda\Gamma(\overline{\theta})\int_{-\infty}^{\infty}\Gamma(\theta)dG(\theta). \tag{A88}$$

Using $\Gamma(\theta) \le 1$ for all θ, a simple condition sufficient for amplification is:

$$b_0+\ell_0 \ge \frac{\lambda}{g(\overline{\theta})\left|\Gamma(\overline{\theta})\right|}, \tag{A89}$$

which places another upper bound on the sensitivity of payoffs.

E.2 One Payoff Sensitive to the Fundamental

We derive the magnitude of amplification (MoA) in the case when payoffs are sensitive to the fundamental. Using the previous results, we can state

$$MoA = \frac{\delta_0 A_{n^*} D_{\overline{\theta}} - \delta_1 A_{n^*} D_{\mu}}{\delta_0[\delta_1 - A_{n^*} D_{\overline{\theta}}]}, \tag{A90}$$

$$= \frac{(\delta_0+\delta_1)A_{n^*} D_{\overline{\theta}} + \delta_1\Gamma(\overline{\theta})B(\overline{\theta})}{\delta_0[\delta_1 - A_{n^*} D_{\overline{\theta}}]}, \tag{A91}$$

where $B(\overline{\theta}) = \int_{\overline{\theta}}^{\infty} \ell'(\theta)\Gamma(\theta)dG(\theta) - \int_{-\infty}^{\overline{\theta}} b'(\theta)\Gamma(\theta)dG(\theta)$.

Thus, higher values of $\Gamma(\overline{\theta})B(\overline{\theta})$ lead to a higher magnitude of amplification. Under the assumption of one payoff being sensitive and the other depending linearly on the fundamental, we obtain the expression for $B(\overline{\theta})$ stated in the main text. Moreover, the signs follow from continuity and the observation that, for $\gamma_I \to \infty$, we have $\Gamma(\theta) \to F^U(\theta) > 0$ if $\theta > \overline{\theta}$ (perfectly informed investors never attack when no regime change occurs), while $\Gamma(\theta) \to F^U(\theta) - 1 < 0$ if $\theta < \overline{\theta}$ (perfectly informed investors always attack when regime change occurs). This completes the proof.

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