The digital economy, privacy, and CBDC

Toni Ahnert† Peter Hoffmann‡ Cyril Monnet§

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Abstract

We study a model of financial intermediation, payment choice, and privacy in the digital economy. While digital payments enable merchants to distribute their goods online, they also reveal information to banks. By contrast, cash only allows for inefficient offline sales, but guarantees anonymity. In equilibrium, merchants trade off the efficiency gains from online distribution (with digital payments) and the informational rents from staying anonymous (with cash). The introduction of central bank digital currency (CBDC) raises welfare because it reduces the privacy concerns associated with online distribution. Payment tokens issued by digital platforms crowd out CBDC unless the latter facilitates data-sharing.

Keywords: Central Bank Digital Currency, Privacy, Payments, Digital Platforms, Financial Intermediation.

JEL Codes: D82, E42, E58, G21.

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†European Central Bank and CEPR, toni.ahnert@ecb.int.
‡European Central Bank, peter.hoffmann@ecb.europa.eu.
§University of Bern and Study Center Gerzensee, cyril.monnet@unibe.ch
1 Introduction

The growing dominance of e-commerce has profound implications for the economics of payments. Since more and more transactions are conducted online, physical currency (“cash”) is becoming impractical as means of payment for a growing share of economic activity. At the same time, new electronic payment services (e.g. mobile wallets) provide increased speed and convenience to merchants and consumers. Accordingly, the use of cash is declining fast.¹ Seizing the opportunity, large technology firms (“BigTech”) are incorporating payment services into their digital ecosystems. While particularly salient in China, where WeChat and Alipay account for more than 90% of digital retail payments, the rest of the world is catching up fast.²

Unlike cash, digital payments generate troves of data, and private enterprises have incentives to use them for commercial purposes. This gives rise to privacy concerns because the increased availability of personal information can have important welfare implications.³ While a proliferation of data promises efficiency gains, policy makers have become increasingly uneasy about the dominance of data-centric business models and their potential to stifle competition, avoid creative destruction, and engage in price discrimination.⁴ At the same time, scandals such as the one surrounding Facebook and Cambridge Analytica have heightened public sensitivity about data privacy issues in the context of the digital economy.

Fuelled by this debate, policy makers have advanced the idea of creating a central bank digital currency (CBDC). One motivation is that public digital money has a comparative advantage at providing privacy because, unlike private sector alternatives, it is not bound by profit-maximization incentives.⁵ Although ultimately not realized, Facebook’s Libra proposal catapulted the entire debate

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¹See, for example, Table III.1 in Bank for International Settlements (2021).
²Most large technology firms have expanded into retail payments services, with popular products such as ApplePay or GooglePay growing at the expense of traditional instruments.
³See Acquisti et al. (2016) for a comprehensive overview of the economics of privacy.
⁴See, e.g., Bergemann et al. (2015), Jones and Tonetti (2020), and Ichihashi (2020).
⁵Consistent with this view, privacy has been named as number one concern in the Eurosystem’s public consultation on a digital euro (European Central Bank, 2021).
into the public limelight in 2019, and efforts towards the introduction of CBDCs have intensified since then.\(^6\) According to a 2020 survey by the Bank for International Settlements, more than 80% of all responding central banks were actively researching CBDCs (Boar and Wehrli, 2021).

This paper aims to speak to this debate. It develops a stylized model of financial intermediation to analyze the interconnections of payments and privacy in the context of the digital economy. In our model, sellers can distribute their goods offline (through a brick-and-mortar store) or online. Offline sales can be settled with both cash and a digital means of payment. Online distribution enables a more efficient matching with potential buyers, and thus generates a higher surplus. At the same time, online sales can only be settled with a digital means of payment.

Sellers are heterogeneous and require outside finance in two rounds of production. They privately learn their type (high (H) or low (L)) in the initial round of production. Only H-sellers can generate a continuation payoff that merits further financing for a second round of production. Since types are private information, financiers face an adverse selection problem and will only provide a continuation loan if they can learn the seller’s type. This refinancing decision also affects sales prices in the first round, which are negotiated in bilateral meetings between buyers and sellers through Nash bargaining.

We first study a setting in which a bank is the only financier. When bank deposits are the only digital means of payments, the bank directly observes sellers realized meetings through payment flows. By contrast, cash transactions provide no information to the bank. It therefore has to elicit information through contractual arrangements, which leaves informational rents to sellers.

We show that, in equilibrium, sellers opt for online distribution and settlement with bank deposits if the benefits of more efficient matching outweigh the loss of informational rents associated with privacy. This is the case if the resulting

\(^{6}\)See “Facebook gives up on crypto ambitions with Diem asset sale”, Financial Times, January 27, 2022.
efficiency gains that sellers can appropriate are large enough. Otherwise, goods are distributed offline, which is inefficient due to imperfect matching.

When sellers can use a CBDC—electronic cash—they can trade online without revealing any information to the bank. This enables sellers to capture the best of both worlds. They can reap some of the efficiency gains of online distribution, and at the same time earn informational rents from remaining anonymous. From a social welfare perspective, there are two efficiency gains from the introduction of CBDC. First, sellers are more likely to trade online when sales are settled with CBDC, which ensures efficient matching. Second, with CBDC, the bank always elicits information through a separating contract. This ensures that H-sellers are more likely to receive continuation financing from the bank, which further raises welfare.

We then extend the model to include a digital platform, which provides a settlement token and competes with the bank for continuation loans to sellers. The platform only observes sellers’ type whenever they use tokens as a means of payment. Perhaps surprisingly, we show that sellers always prefer settlement in tokens over CBDC or deposits. The reason is intuitive: since banks can elicit information through contracting for the initial loan, the use of tokens ensures that the platform and banks can compete for the continuation loan. By contrast, with either CBDC or deposits, only the bank is informed and acts as a monopolist.

We also highlight a “dark side” of token use. More specifically, we show that tokens enable the platform to fend off potential competitors by creating a so-called “walled garden”. While deposits or CBDC enable sellers to potentially benefit from switching to a more efficient entrant platform, the resulting lack of competition in the lending market ensures that all efficiency gains are appropriated by banks. Accordingly, sellers are better off with tokens.

Next, we enrich the CBDC with a data-sharing functionality. This enables sellers to reveal their type costlessly to both the bank and the platform. Importantly, they can do so after repaying their initial bank loan to avoid ceding any
surplus to banks. Sellers then enjoy perfect competition in the second round of lending. So they always opt for online sales through CBDC, which is the socially efficient outcome.

Finally, we show that a CBDC with a data-sharing feature also enhances competition among platforms by preventing the incumbent from creating a “walled garden”. Accordingly, sellers are able to reap the additional efficiency gains associated with entrant platforms.

**Literature.** Our paper is related to the literature on privacy in payments. In Kahn et al. (2005), cash payments preserve the anonymity of the purchaser. This provides protection against moral hazard, modelled as the risk of theft. This is different from the benefit of anonymity in our model, which is reduced rent extraction in the lending market. Moreover, we also study new trade-offs associated with the choice of trading venues and their interactions with different means of payments, including CBDCs and tokens issued by digital platforms.

The paper by Garratt and Van Oordt (2021) is also closely related. They study a setting in which merchants use information gleaned from current customer payments to engage in price discriminate against future customers. While customers can take costly actions to preserve their privacy in payments, they fail to appreciate the full social value of doing so and—similar to a public goods problem—insufficiently preserve their privacy. In contrast to their focus on an externality and the social value of privacy, our emphasis is on the private benefit of preserving privacy.

Our paper also builds on a literature studying the interaction of payments and lending. Empirical evidence suggests that payment flows are informative about borrower quality (see, e.g., Mester et al., 2007; Norden and Weber, 2010; Puri et al., 2017). Parlour et al. (2021) study a model where banks face competition for payment flows by FinTechs. While this may improve financial inclusion, it affects lending and payment pricing by threatening the information flow to banks.
He et al. (2021) study competition between banks and Fintech in lending markets with consumer data sharing. Data sharing enhances competition, but borrowers may still be worse off because their sign-up decisions reveal information about credit quality.

Finally, our paper is part of a fast-growing literature on CBDC. Brunnermeier and Payne (2022) develop a model of platform design under competition with a public marketplace and a potential entrant, and study how different forms of interoperability are affected by regulation (including CBDC). Their model is complementary to ours since it studies the nexus of CBDC and the digital economy, but abstracts from privacy issues altogether. In Garratt and Lee (2021), privacy features of CBDC are a way to maintain an efficient monopoly in data collection. Apart from privacy, the preservation of monetary sovereignty and an avoidance of digital dollarization can motivate the introduction of CBDC (Brunnermeier et al., 2019; Benigno et al., 2022). Several recent papers investigate how CBDC may affect credit supply (Keister and Sanches, 2022; Andolfatto, 2021; Chiu et al., 2021), bank runs (Fernández-Villaverde et al., 2020, 2021), the efficacy of government interventions (Keister and Monnet, 2020), and the monetary system (Niepelt, 2020).

**Structure.** The remainder of the paper is organized as follows. We introduce the basic model with cash and bank deposits in Section 2, and solve for the equilibrium in Section 3. We subsequently introduce a CBDC with anonymity in Section 4. We consider competition between the bank and a digital platform in Section 5. Finally, we examine data-sharing features of CBDC in Section 6. Section 7 concludes. All proofs are in the Appendices.

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7See Ahnert et al. (2022) for a comprehensive overview of recent work.
2 The basic model

The model has four dates \( t = 0, 1, 2, 3 \) and there is no discounting. There are three classes of risk-neutral agents: banks, buyers, and sellers of measure one each. There is a consumption good and an investment good. Both goods are indivisible.\(^8\)

Sellers have no resources at \( t = 0 \) and need to borrow from a bank to finance production. Sellers can produce one unit of the consumption good at \( t = 1 \) by using one unit of investment at \( t = 0 \). A mass \( q \in (0, 1) \) of sellers are of high type (H) and produce a good of high quality, while the remaining \( 1 - q \) sellers are of low type (L) and produce a good of low quality. Sellers are initially uncertain about their (persistent) type and privately learn it at beginning of \( t = 1 \). H-sellers can also produce \( \theta > 1 \) units of the consumption good at \( t = 3 \), using one unit of the investment good at \( t = 2 \). By contrast, L-sellers produce nothing at \( t = 3 \).

Buyers have deep pockets and are heterogeneous in their preferences. A measure \( q \) cares about quality and derives utility \( u_H \) from consuming one unit of the high-quality good, and \( u_L \) from consuming one unit of the low-quality good, with \( 1 < u_L < u_H \). We call them H-buyers. The remaining \( 1 - q \) L-buyers do not care about quality and obtain utility \( u_L \) independently of quality.\(^9\)

Banks are endowed with one unit of the investment good at \( t = 0 \) and \( t = 2 \), which they can lend to sellers. Their opportunity cost is 1 per unit of investment.\(^{10}\) Bankers can neither commit to long-term contracts, nor to not renegotiating loan terms. Hence, it is as if they could set the interest rates at \( t = 1 \) and \( t = 3 \). Banks make take-it-or-leave-it offers, but sellers can abscond with a fraction \( \lambda \) of their sales. If they use bank deposits as means of payment, absconding at \( t = 2 \) has a fixed effort cost of \( e \). This cost captures the notion that deposit flows enable

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\(^8\)Making goods indivisible greatly simplify the exposition and the analysis.

\(^9\)The assumption that the measure of H-sellers equals the measure of H-buyers is merely for analytical convenience. Assuming different measures would make the analysis more cumbersome, but not deliver additional insights.

\(^{10}\)This unit cost may reflect the bank’s cost of funding or an alternative safe investment opportunity.
the bank to monitor sellers’ activity more closely, which makes absconding more difficult and requires additional effort.

Sellers can distribute their goods through two types of venues, a brick-and-mortar store (“Offline” or OFF) or over the internet (“Online” or ON). Since their unit production is indivisible, sellers can only choose one trading venue. Offline, sellers and buyers are matched randomly. This gives rise to four types of meetings $m = (s, b)$, where $s$ and $b$ denote seller and buyer types, respectively. By contrast, matching is perfect when sellers distribute their goods online, so that there are only two types of meetings.\footnote{More specifically, we have the following offline meetings: a measure $q^2$ of $(H, H)$ meetings, a measure $q(1 - q)$ of $(H, L)$ meetings, a measure $(1 - q)q$ of $(L, H)$ meetings, and a measure $(1 - q)^2$ of $(L, L)$ meetings. Online, we have a measure $q$ of $(H, H)$ meetings and a measure $(1 - q)$ of $(L, L)$ meetings.} When meeting, buyers and sellers determine the price through bilateral Nash bargaining. We denote buyers’ market power by $\sigma \in [0, 1]$, which is constant across trading venues. If the negotiation fails, sellers consume their production to obtain utility $\lambda$.

We assume there are initially two means of payment (cash and bank deposits) and that buyers can costlessly exchange one for the other. Due to their physical nature, offline purchases can be settled both in cash (C) and in deposits (D), e.g. via debit or credit card. By contrast, the exchange of physical currency is too cumbersome for online sales, so they require a digital payment instrument such as deposits. We assume that the use of deposits enables banks to observe the sellers’ realized meeting $m$ because payment flows are informative about borrowers’ financial situation (Mester et al., 2007; Norden and Weber, 2010; Puri et al., 2017). This is not the case when cash is used. We refer to the combination of trading venue and payment means as a trading scheme, denoted by $\tau$. There are three possibilities: offline-cash (OFF-C), offline-deposits (OFF-D), and online-deposits (ON-D).

To simplify the exposition, we abstract from details about the exact way payments are made in our economy. However, Appendix C provides explicit foundations in the spirit of new monetarist models.
The timing shown in Figure 1 is as follows. At $t = 0$, sellers and banks are matched, sellers borrow one unit of the good and choose their trading scheme $\tau$. At $t = 1$, sellers learn their type and are then matched with a buyer for bargaining over the terms of trade $p_m$. At the end of $t = 1$, given the means of payment used, the bank sets the repayment $\{(r_m)\}$. At $t = 2$, the bank decides on a continuation loan $k \in \{0, 1\}$ at interest rate $i$. At $t = 3$, H-sellers who have received a loan produce $\theta$ and repay $i$ to the bank, or abscond with production to obtain a payoff $\lambda\theta$. L-sellers who have received a loan produce nothing and abscond with investment to obtain a payoff $\lambda$.

As a benchmark, consider the economy with full information. In this case, welfare is maximized whenever sellers choose to distribute their goods online and the bank gives a second loan to all H-sellers. Offline distribution is always inefficient. However, with private information, we will find conditions under which sellers prefer to distribute their goods offline.

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Figure 1: Timeline

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12Recall that the bank cannot commit at $t = 0$ to not renegotiating the loan rate at $t = 1$. 
3 Equilibrium

We now solve for the equilibrium. We proceed backwards, starting with banks’ choice on whether to extend a second loan at \( t = 2 \). We then solve for the sales prices and banks’ choice of loan contract at \( t = 1 \). We close by analyzing sellers’ choice of trading scheme at \( t = 0 \). Our equilibrium definition follows.

**Definition 1.** An equilibrium is a choice by banks of initial investment \( l \in \{0, 1\} \), repayment menu \( \{(r_m)\} \), continuation financing \( k \in \{0, 1\} \), and continuation repayment \( i \), a choice of trading scheme \( \tau \in \{OFF-D, OFF-C, ON-D\} \) by sellers, and a bilateral price \( p(m, k) \) for meeting \( m \) such that

1. banks maximize expected profits by choosing \((l, r, k, i)\), taking sellers’ choice \( \tau \) and bilateral prices \( p(m, k) \) as given,
2. sellers maximize expected profits by choosing \( \tau \), taking banks’ choices \((l, r, k, i)\) and bilateral prices \( p(m, k) \) as given, and
3. bilateral prices \( p(m, k) \) in meetings \( m \) are determined by Nash bargaining, taking \((r, k, i)\) as given.

3.1 Bank’s refinancing choice at \( t = 2 \)

Banks possibly face adverse selection, so their lending decision at \( t = 2 \) depends on whether they are informed about the seller’s type. First, suppose that banks are informed. In this case, L-sellers do not receive a new loan because they will produce nothing. By contrast, H-sellers receive financing if the bank can recover its unit cost of investment while giving H-sellers their outside option \( \lambda \theta \), by setting the repayment on the second loan to \( i^* = (1 - \lambda) \theta \). Hence we assume,

\[
(1 - \lambda) \theta > 1.
\]

When banks are uninformed, we assume the level of adverse selection is high enough that banks do not want to invest with sellers.
**Assumption 1.** $1/q > (1 - \lambda)\theta > 1$.

If adverse selection is low, it is profitable for banks to lend to sellers of unknown type in the second stage. We relegate the analysis of this case to Appendix B.3 because it is tedious and the results are unchanged.

Notice that Assumption 1 also implies that banks finds it optimal to lend to H-sellers at $t = 2$ even if they defaulted on their first loan. In the same way that banks cannot commit to loan terms, they can also not commit to not extending a loan upon default. In Appendix B.1, we consider an alternative setup where banks can commit to not extending a loan upon default, and show that it leads to the same trade-offs among the deposits and cash.

### 3.2 Bargaining between buyers and sellers at $t = 1$

In solving for the bargaining solution between buyers and sellers, we treat sellers and banks as a coalition.\(^{13}\) Once the negotiation is concluded, sellers and banks can decide on how to share the joint surplus. Recall that we assume that sellers abscond with a fraction $\lambda$ of the production and exit the economy if bargaining fails.

To determine the joint surplus from trade, we need to condition on banks’ lending decision at $t = 2$. If a loan is extended, H-sellers will generate an additional payoff $\theta - 1$ for the bank/seller coalition. To this end, let $p(m, k)$ be the bilateral price in meeting $m$ conditional on the bank’s future lending decision $k \in \{0, 1\}$.\(^{14}\) Assumption 1 implies that no loan is extended to L-sellers, so the continuation

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\(^{13}\)See Petrosky-Nadeau and Wasmer (2017) for this approach as well as other types of solution to solving bargaining problems involving three parties.

\(^{14}\)Since any repayment $r$ splits the surplus between the bank and the seller, it does not enter the bargaining solution.
payoff $\Delta(m, k)$ earned by the seller/bank coalition at $t = 3$ is given by

$$
\Delta(m, k) = \begin{cases}
\theta - 1 & \text{if } (m, k) = ((H, b), 1), \\
0 & \text{otherwise},
\end{cases}
$$

If the buyer and seller agree to trade at $p(m, k)$, the seller/bank coalition earns $p(m, k) - 1 + \Delta(m, k)$. By contrast, without trade, the seller walks away with his outside option and obtains utility $\lambda$. Since the bank has sunk its unit investment, the joint payoff is $\lambda - 1$. Combining the previous two equations, the joint surplus of the seller/bank coalition is

$$
p(m, k) - \lambda + \Delta(m, k).
$$

Since buyers have deep pockets, their surplus from trade is $u(m) - p(m, k)$, where $u(m) = u_H$ for $m = (H, H)$ and $u(m) = u_L$ otherwise. The bilateral price is then given by the Nash bargaining solution\textsuperscript{15}

$$
p(m, k) = (1 - \sigma)u(m) + \sigma \lambda - \sigma \Delta(m, k).
$$

The first term depends on the meeting $m$, while the last term depends both on the meeting $m$ and the bank’s decision $k$. First, H-buyers value quality, which implies a higher price in $(H, H)$-meetings. Second, their bargaining power allows buyers to extract a fraction $\sigma$ of the continuation surplus $\Delta(m, k)$ from $(H, H)$-meetings that are followed by continuation financing ($k = 1$). Intuitively, the H-seller/bank coalition is willing to cede part of it because it cannot be reaped if trade breaks down. Since L-sellers never receive re-financing, the full set of

\textsuperscript{15}Formally, $p(m, k)$ solves $\max [u(m) - p(m, k)]^\sigma \max [p(m, k) - \lambda + \Delta(m, k)]^{1-\sigma}$. 

12
possible equilibrium prices is given by

\[
p(m, k) = \begin{cases} 
  p_{HH} \equiv (1 - \sigma)u_H + \sigma \lambda - \sigma (\theta - 1) & \text{if } (m, k) = ((H, H), 1), \\
  \tilde{p}_{HH} \equiv (1 - \sigma)u_H + \sigma \lambda & \text{if } (m, k) = ((H, H), 0), \\
  p_{HL} \equiv (1 - \sigma)u_L + \sigma \lambda - \sigma (\theta - 1) & \text{if } (m, k) = ((H, L), 1), \\
  \tilde{p}_{HL} \equiv (1 - \sigma)u_L + \sigma \lambda & \text{if } (m, k) = ((H, L), 0), \\
  p_{Lb} \equiv (1 - \sigma)u_L + \sigma \lambda & \text{if } (m, k) = ((L, b), 0). 
\end{cases}
\]

Furthermore, we assume the following.

**Assumption 2.** \((1 - \sigma) (u_H - u_L) > \sigma (\theta - 1)\).

This assumption implies that the surplus which H-sellers can extract from H-buyers exceeds the surplus that L-sellers can extract from any buyer. Intuitively, it is satisfied if H-buyers do not have much bargaining power \(((1 - \sigma)/\sigma \text{ is high})\) relative to what they bring to the negotiation table \(((\theta - 1)/(u_H - u_L) \text{ is low})\).

We thus have \(p_{HH} > p_{Lb} > p_{HL}\).

Finally, we also assume that the gains from trade for the bank-seller pair are higher in the first production stage than in the second one. This renders the information extraction problem non-trivial. More specifically, it ensures that H-sellers generate sufficient sales in (H,L)-meetings to allow for full separation.

**Assumption 3.** \((1 - \sigma)u_L + \sigma (1 + \lambda) > (1 + \sigma)\theta\)

### 3.3 Loan repayment at \(t = 1\)

We turn to the loan repayment at \(t = 1\). Whenever sellers accept payment in bank deposits (under the OFF-D or ON-D schemes), the bank directly observes the sellers realized meeting and can set the interest rate accordingly. However, this is not true when sales are settled in cash under the OFF-C scheme. In this case, the bank can only elicit the information about the meeting through screening by offering a menu of contracts.
Settlement in cash. We first consider the OFF-C scheme. Notice that the bank is interested in learning both the type of the seller (to choose refinancing appropriately) as well as the sales price (to set the interest rate as high as possible). A direct mechanism revealing only the seller’s type could fail to achieve the second goal because HL-sellers generate lower sales than HH-sellers. Therefore, a bank would like to know both the seller’s and the buyer’s type. Since L-sellers always generate the same sales, the bank is only interested in learning which of the following three meetings took place: 
\((H, H)\), \((H, L)\), or \((L, b)\). Let \(\mathcal{M}\) denote this set of meetings. To elicit information, the bank can use different contract menus \(\{(r_m, k_m)\}_{m \in \mathcal{M}}\).

A separating contract is a list \(\{(r^S_m, k^S_m)\}_{m \in \mathcal{M}}\) such that 
\[ r^{S}_{HH} = r^{S}_{HL} \neq r^{S}_{Lb}, \]
and 
\[ k^{S}_{HH} = k^{S}_{HL} = 1 > k^{S}_{Lb} = 0. \]
This contract allows the bank to separate all H-sellers from L-sellers, and thus enables it to extend a second loan to all H-sellers at \(t = 2\).

A partially pooling contract is a list \(\{(r^P_m, k^P_m)\}_{m \in \mathcal{M}}\) such that 
\[ r^{P}_{HH} \neq r^{P}_{HL} = r^{P}_{Lb}, \]
and 
\[ k^{P}_{HH} = 1 > k^{P}_{HL} = k^{P}_{Lb} = 0. \]
Under this contract, the bank cannot distinguish HL-sellers from L-sellers. Accordingly, it will only be able to lend to HH-sellers at \(t = 2\) (using Assumption 1).

Finally, a pooling contract consists of a single interest rate \(r_m = \bar{r}\) offered to all sellers and \(k_m = \bar{k} = 0\) for any \(m \in \mathcal{M}\). This implies that the bank cannot distinguish among different types of sellers, and therefore will not lend at \(t = 2\). The following Lemma characterizes the aforementioned contract menus.

Lemma 1. Suppose that sellers choose the OFF-C trading scheme. Then,

i) the separating contract has 
\[ r^{S}_{Lb} = (1 - \lambda)p_{HL} \text{ and } r^{S}_{HH} = r^{S}_{HL} = r^{S}_{Lb} + \lambda \theta, \]
ii) the partially pooling contract has 
\[ r^{P}_{Lb} = r^{P}_{HL} = (1 - \lambda)p_{Lb} \text{ and } r^{P}_{HH} = r^{P}_{Lb} + \lambda \theta, \]
iii) and the bank never offers a pooling contract.

Proof. All Proofs are in the Appendix. ■

Lemma 1 provides several useful insights. First, under the separating con-
tract, the participation constraint of L-sellers is slack because sales prices are endogenous to the bank’s refinancing decision. Since HL-sellers receive continuation financing, the buyers they meet are able to extract part of the additional surplus generated in the future, so that that $p_{HL} < p_{Lb}$. Therefore, to ensure participation by HL-sellers, the bank must set $r_{Lb} < (1 - \lambda)p_{Lb}$.

Second, by contrast, the participation constraint of L-sellers binds under the partially pooling contract. Since HL-sellers receive no continuation financing, they generate the same sales as L-sellers, $p_{HL} = p_{Lb}$, because L-buyers are unable to extract more from H-sellers than from L-sellers.

Third, for either contract menu, the incentive compatibility constraint of HH-sellers binds so that $r_{HH} = r_{Lb} + \lambda \theta$. Intuitively, the maximum “spread” the bank can charge is $\lambda \theta$, because otherwise HH-sellers would have an incentive to pretend being L-sellers in which case they would not received the second loan.

The bank chooses the contract that maximizes its expected profits, which are given by its interest income minus the unit funding costs. In Section 3.1, we have already shown that continuation loans to H-sellers yield a net profit of $(1 - \lambda)\theta - 1$ for the bank. Moreover, Lemma 1 implies that successfully identified H-sellers pay an additional $\lambda \theta$ on the first loan, so that the bank effectively reaps the entire surplus from the second loan, $\theta - 1$. Accordingly, using the separating contract, the bank earns

$$B^S_{OFF-C} = r^S_{Lb} - 1 + q(\theta - 1),$$

while the partially pooling contract yields

$$B^P_{OFF-C} = r^P_{Lb} - 1 + q^2(\theta - 1).$$

Using the interest rates from Lemma 1 and the expressions for the prices from equation (1), we directly obtain the following result.

**Lemma 2.** Suppose that sellers choose the OFF-C trading scheme. Then, the
The bank offers a separating contract if and only if

\[ q(1 - q) \geq \sigma(1 - \lambda). \]  \hspace{1cm} (4)

Equation (4) illustrates the bank’s trade-off between the costs and benefits of separation. The LHS represents the bank’s benefit from separating the measure \( q(1 - q) \) of HL-sellers from L-sellers, which enables the bank to reap the extra surplus \( \theta - 1 \) from all H-sellers, not just HH-sellers.

The RHS illustrates the cost of separation in terms of foregone interest income, \( r_{Lb}^P - r_{Lb}^S = (1 - \lambda)\sigma(\theta - 1) \). Under the separating contract, HL-sellers generate lower sales than L-sellers (\( p_{HL} < p_{Lb} \)) because part of the continuation surplus is ceded to buyers in bilateral bargaining. Accordingly, \( r_{Lb}^S \) is pinned down exclusively by the participation constraint of HL-sellers, whereas L-sellers’ participation constraint is slack. This is not the case under the partially pooling contract, where HL-sellers receive no continuation financing and thus generate the same sales as L-sellers (\( p_{Lb} = \tilde{p}_{HL} \)). This enables the bank to raise \( r_{Lb}^P \), and thus increase its interest income relative to the separating contract.

Settlement in deposits. Now suppose the seller chooses settlement in deposits (either under the OFF-D or ON-D scheme). In this situation, the bank perfectly observes the seller’s type, so that the contract does not have to satisfy any incentive constraints for truthful reporting. Accordingly, all interest rates are pinned down by the relevant participation constraints, which include the cost \( e \) that sellers incur when forging their accounts. Since the bank is perfectly informed, all H-sellers get refinanced. This implies that the possible price realizations in the bargaining stage are \( p_{HH}, p_{HL} \) and \( p_{Lb} \).

**Lemma 3.** Suppose that sellers choose settlement in deposits (either OFF-D or ON-D). Then the bank optimally charges

\[ r_m^O = (1 - \lambda)p_m + 1_{m=(H,b)}\lambda \theta + e, \]  \hspace{1cm} (5)
where \( 1_{[\cdot]} \) denotes the indicator function.

The only difference between the OFF-D and ON-D schemes is that \( r_{HL}^D \) only arises under the OFF-D scheme. With online distribution, there are no \((H, L)\)-meetings due to perfect matching, so that the bank only sets \( r_{HH}^D \) and \( r_{Lb}^D \).

We can now determine the seller’s choice of trading scheme.

### 3.4 Seller’s choice of trading scheme at \( t = 0 \)

At \( t = 0 \), sellers choose a trading scheme to maximize expected profits. These are given by sales minus interest payment, \( p_m - r_m \), plus the benefits from obtaining continuation financing, where the expectation is taken over all possible meetings \( m \in \mathcal{M} \), and sellers take the bank’s choice of contract menu \( r_m \) and the associated refinancing decision \( k_m \) as given.

Under the partially pooling contract, only HH-sellers get refinanced, so that prices are \( p_{HH} > \tilde{p}_{HL} = p_{Lb} \). We can then write sellers’ expected profits as

\[
S_{OFF-C}^P = q^2 (p_{HH} - r_{HH}^P + \lambda \theta) + q(1 - q) (\tilde{p}_{HL} - r_{HL}^P) + (1 - q) (p_{Lb} - r_{Lb}^P)
= q^2 [\lambda p_{HH} + (1 - \lambda)(p_{HH} - p_{Lb})] + q(1 - q)\lambda \tilde{p}_{HL} + (1 - q)\lambda p_{Lb},
\]

where we have used the fact that continuation financing (here obtained by HH-sellers) generates an extra profit of \( \theta - i^* = \lambda \theta \). The second line illustrates that, in equilibrium, HL-sellers and all L-sellers just obtain their reservation payoff (a fraction \( \lambda \) of their sales), whereas HH-sellers additionally capture an informational rent equal to \( (1 - \lambda)(p_{HH} - p_{Lb}) \).

Under the separating contract, prices are \( p_{HH} > p_{Lb} > p_{HL} \) (by Assumption
so sellers’ expected profits are

\[ S_{OFF-C}^{S} = q^2 \left[ \lambda p_{HH} + (1 - \lambda)(p_{HH} - p_{HL}) \right] + q(1 - q)\lambda p_{HL} + (1 - q) \left[ \lambda p_{Lb} + (1 - \lambda)(p_{Lb} - p_{HL}) \right]. \]  

(7)

Compared to the partially pooling contract, HL-sellers generate lower sales because they receive continuation financing and thus need to cede some surplus to L-buyers. Since their participation constraint is the only one that binds, both HH-sellers and L-sellers obtain an informational rent equal to \((1 - \lambda)(p_{HH} - p_{HL})\) and \((1 - \lambda)(p_{Lb} - p_{HL})\) respectively.

Finally, expected profits under the ON-D and OFF-D schemes are

\[ S_{ON-D} = q\lambda p_{HH} + (1 - q)\lambda p_{Lb} - (e - q\lambda\theta) \]  

(8)

\[ S_{OFF-D} = q^2\lambda p_{HH} + q(1 - q)\lambda p_{HL} + (1 - q)\lambda p_{Lb} - (e - q\lambda\theta) \]  

(9)

When payments are settled in deposits, all sellers receive exactly their reservation utility, plus a term that represents the cost of forging their accounts minus the benefit from strategically defaulting on the first loan.\(^{16}\) The following assumption provides a sufficient condition to rule out such strategic default.

**Assumption 4.** \(e \geq q\lambda\theta.\)

It is immediate that \(S_{ON-D} > S_{OFF-D},\) so sellers never choose OFF-D scheme. Then straightforward calculations lead to the following result.

**Proposition 1. (Equilibrium in the baseline model)**

1. For \(\sigma(1 - \lambda) \geq q(1 - q),\) banks offer a partially pooling contract under the OFF-C scheme. In this case, sellers distribute their goods online if \(q(\lambda - q)(1 - \sigma)(u_H - u_L) - (e - q\lambda\theta) \geq q(\lambda - q)\sigma(\theta - 1),\) and offline otherwise.

\(^{16}\)With deposits, the bank learns sellers’ type independently of the loan repayment. Accordingly, H-sellers can in principle default on their first loan and still obtain continuation financing at \(t = 2,\) since the bank will find the extension of a new loan optimal. With cash, this cannot happen as the bank only learns the seller’s type through the repayment.
2. For $\sigma(1 - \lambda) < q(1 - q)$, banks offer a separating contract under the OFF-C scheme. In this case, sellers distribute their goods online if $q(\lambda - q)(1 - \sigma)(u_H - u_L) - (e - q\lambda\theta) \geq (1 - q)(1 - \lambda)\sigma(\theta - 1)$, and offline otherwise.

3. All online sales are settled in deposits (by assumption).

Figure 2 illustrates the equilibrium by highlighting the relevant regions from Proposition 1 in the $(\lambda, q)$-space. The solid black curve defined by $\sigma(1 - \lambda) = q(1 - q)$ delineates the regions of the parameter space for which the bank uses a separating contract (above) or a partially pooling contract (below).

When choosing among trading schemes, sellers trade off the efficiency gains associated with online distribution and the informational rents (i.e. the gains from preserving privacy) that they can earn from the use of cash when selling offline. A high $\lambda$ enables sellers to reap a higher share of sales, and at the same time implies lower rents from contracting (see equations (6) and (7)). Accordingly, sellers only opt for the ON-D scheme if $\lambda$ is sufficiently high. The relationship between sellers’ choice and $q$ is more complex because the distribution of rents across sellers differs between the separating or a partially pooling contract.

First, consider the region below the bold line, where the bank offers a partially pooling contract under the OFF-C scheme. Ignoring the term $e - q\lambda\theta$, we can write the difference $S_{ON-D} - S_{OFF-C}^P$ as

$$q(1 - q)\lambda(p_{HH} - \tilde{p}_{HL}) - q^2(1 - \lambda)(p_{HH} - \tilde{p}_{HL}).$$

The first term represents the efficiency gain. Under the ON-D scheme, $(H, L)$-meetings no longer occur, which increases sales from $\tilde{p}_{HL}$ to $p_{HH}$ for a fraction $q(1 - q)$ of all meetings. Sellers reap a share $\lambda$ of these gains. The second term represents the cost. Under the ON-D scheme, banks learn sellers’ types for free, so that HH-sellers no longer earn the informational rent $(1 - \lambda)(p_{HH} - \tilde{p}_{HL})$ (realized with probability $q^2$). Equation (10) is positive if $\lambda > q$, which is represented by the straight line in the lower left corner of Figure 2.
The trade-off is qualitatively the same for the parameter region above the solid line, where the bank offers a separating contract under the OFF-C scheme. Again ignoring \( e - q\lambda\theta \), the difference \( S_{ON-D} - S_{OFF-C}^S \) can be expressed as

\[
q(1 - q)\lambda(p_{HH} - p_{HL}) - q^2(1 - \lambda)(p_{HH} - p_{HL}) - (1 - q)(1 - \lambda)(p_{Lb} - p_{HL}).
\] (11)

Relative to the partially pooling contract, HL-sellers obtain lower sales under separation (\( p_{HL} < \tilde{p}_{HL} \)). Therefore, a switch towards online distribution generates relatively larger efficiency gains. At the same time, sellers also forego more informational rents when opting for online distribution because both HHI-sellers and L-sellers are able to extract more than their reservation payoff under the OFF-C scheme. Accordingly, online distribution is particularly attractive for intermediate values of \( q \) because offline distribution leads to a fraction \( q(1 - q) \) of inefficient \((H, L)\)-meetings.

**Equilibrium with low adverse selection.** Our derivation of the equilibrium was based on the assumption that adverse selection is sufficiently high to render uninformed lending unprofitable (see Assumption 1). In Appendix B.3, we show that precisely the same equilibrium obtains when adverse selection is low, or \( q(1 - \lambda)\theta > 1 \). Intuitively, a pooling contract prevents the bank from fully appropriating the gains arising from the continuation investment through the interest rate on the first loan. Accordingly, a contract that reveals some information to the bank yields a strictly higher payoff. This result is already reflected in Figure 2, which spans the parameter space for both high and low adverse selection.
Figure 2: Equilibrium map in \((\lambda, q)\)-space.

Notes: In all figures we use the following parameters that satisfy Assumptions 2 and 3:
\[
\sigma = 1/3, \quad \lambda_P = 0.05, \quad \theta = 4, \quad u_H = 12, \quad u_L = 8.2.
\]
Also \(e\) is such that 
\[
e = (1 + 0.025)q\lambda\theta \]
such that Assumption 4 is always satisfied. The range of \(\lambda\) is such that the constraint \((1 - \lambda)\theta > 1\) of Assumption 1 is satisfied. The figures shows the solution under both high adverse selection \((q(1 - \lambda)\theta < 1)\) and low adverse selection \((q(1 - \lambda)\theta > 1)\) analyzed in the Appendix.

4 Central bank digital currency

In this section, we expand the set of payment instruments by introducing a central bank digital currency. We think of CBDC as an electronic version of cash. In our context, this means that CBDC allows sellers to conduct online sales without revealing their type to the bank. Accordingly, sellers can now also choose an online-CBDC trading scheme (ON-CBDC).\(^\text{17}\)

Since online distribution implies perfect matching, the bank’s choice is limited to a separating and a pooling contract. However, the pooling contract does not allow the bank to extract any of the surplus that arises from continuation investment. Accordingly, it always opts for separation.

\(^\text{17}\)Note that an offline-CBDC scheme is the same as the OFF-C scheme, so we do not need to consider it separately.
Lemma 4. If sellers choose the ON-CBDC trading scheme, the bank always uses a separating contract with \( r_{Lb} = (1 - \lambda) p_{Lb} \) and \( r_{HH} = r_{Lb} + \lambda \theta \).

Under the ON-CBDC scheme, bilaterally negotiated prices are \( p_{HH} \) and \( p_{Lb} \). Using the contract in Lemma 4, it follows that sellers’ expected payoff is

\[
S_{ON-CBDC} = q [\lambda p_{HH} - (1 - \lambda) (p_{HH} - p_{Lb})] + (1 - q) \lambda p_{Lb}.
\] (12)

Comparison with equation (8) shows that \( S_{ON-CBDC} > S_{ON-D} \), and hence CBDC fully displaces deposits. The separating contract enables the bank to appropriate the continuation surplus, but leaves all the gains from more efficient matching to the seller. With deposits, some of these gains also go to the bank, making the seller strictly worse off. Further comparison of equations (6), (7) and (12) leads to the following result.

Proposition 2. (Equilibrium with CBDC)

1. For \( \sigma(1 - \lambda) \geq q(1 - q) \), banks offer a partially pooling contract under the OFF-C scheme. In this case, sellers always distribute their goods online.
2. For \( \sigma(1 - \lambda) < q(1 - q) \), banks offer a separating contract under the OFF-C scheme. In this case, sellers distribute their goods online if \( q(1 - q)(1 - \sigma)(u_H - u_L) \geq (1 - \lambda) \sigma(\theta - 1) \), and offline otherwise.
3. All online sales are settled in CBDC.

Comparing Propositions 1 and 2 reveals that the introduction of CBDC leads to an increase in online sales. This is shown by Figure 3, which plots the equilibrium under CBDC in the \((\lambda, q)\)-space (overlaying the depiction of the equilibrium with only cash and deposits shown of Figure 2).

Essentially, digital cash enables sellers to capture the best of both worlds. They can reap some of the efficiency gains of online distribution, and at the same time earn some informational rents related to remaining anonymous towards the bank. For \( \sigma(1 - \lambda) \geq q(1 - q) \), sellers now always opt for the ON-CBDC scheme, but otherwise, there are parameter combinations for which they still use cash. To
understand why this is the case, note that we can write

$$S_{\text{ON-CBDC}} - S_{\text{OFF-C}}^{S} = q(1-q) [\lambda(p_{HH} - p_{HL}) + (1 - \lambda)(p_{HH} - p_{Lb})]$$

$$- q^2(1-\lambda)(p_{Lb} - p_{HL}) - (1-q)(1-\lambda)(p_{Lb} - p_{HL})$$

(13)

The first term represents the benefits of better matching with CBDC. As a consequence of online distribution, all H-sellers meet H-buyers, there are no more (H, L)-meetings. Therefore the sales of those H-sellers who would have been HL-sellers with offline distribution increase by $p_{HH} - p_{HL}$. Moreover, they now also earn an informational rent as a consequence of the separating contract offered by the bank under CBDC, which is represented by the second term.

The remaining two terms represent the costs of adopting CBDC. Under the OFF-C scheme with a separating contract, the lowest interest rate $r_{Lb}^S$ is pinned down by the participation constraint of HL-sellers. With CBDC, HL-sellers no longer exist, and the lowest interest rate is pinned down by L-sellers participation constraint. As a consequence, the informational rents earned by HH-sellers decline, and those for L-sellers disappear entirely.

Ultimately, the sign of equation (13) is ambiguous and depends on parameters. Thus, an equilibrium where cash is used continues to exist under CBDC.

To summarize, a switch from offline to online sales improves welfare through two channels. First, the matching of buyers and sellers becomes more efficient, which means that utility $u_H$ is reaped more frequently. Second, since banks always use a separating contract under CBDC, they also provide continuation financing to all H-sellers. This is not the case under the OFF-C scheme with the partially pooling contract, where only HH-sellers are granted a second loan. Accordingly, for $\sigma(1-\lambda) \geq q(1-q)$, CBDC also increases welfare because it allows the continuation surplus $\theta - 1$ to be reaped for a wider range of parameters.
5 Digital platforms with financial services

So far, we have been silent about the way online sales are conducted. In this section, we consider a richer environment in which online sales occur through a digital platform. We first study the case where the platform can also lend to sellers, and then study a model in which the incumbent platform faces competition from a potential entrant.

5.1 Competition in the loan market

Here we assume that the platform can lend to the seller at $t = 2$. Moreover, it can provide a digital token as means of payment at $t = 0$, giving rise to an online-token (ON-T) trading scheme. However, we assume that banks remain monopolists for the first loan.\textsuperscript{18} The platform has the same fundings costs as the bank.

\textsuperscript{18}This can be rationalized by assuming that banks, unlike platforms, are able to resolve an initial adverse selection problem. Suppose that there are productive and unproductive sellers seeking to borrow at $t = 0$. Unproductive sellers never produce anything but consume the loan, while productive sellers become H-sellers with probability $q$ and L-sellers otherwise. The bank has a screening technology to determine who is productive or who is not, which enables
Clearly, the distribution of information between the bank and the platform is critical for competition in the market for continuation loans. We assume that the platform learns the seller’s type only if he uses tokens to settle his online transactions. In Appendix B.2, we study an extension of the model where the platform also derives information from observing the sales it intermediates. We show all our results, and in particular sellers’ choice between tokens and CBDC, are unchanged as long as tokens provide positive, but arbitrarily small informational value.

We assume that the platform and the bank engage in Bertrand competition at \( t = 2 \) if both lenders have the same information. Let \( s = 1 - \frac{1}{\theta} \) denote the share of the surplus \( \theta \) appropriated by the seller in this case.\(^{19}\) If there is no competition in the lending market at \( t = 2 \), we assume that the seller can extract a share \( \lambda_P \) from his sales at \( t = 3 \) when borrowing from the platform, and a share \( \lambda \) when borrowing from the bank. \( \lambda_P \) could be equal to \( \lambda \) but we also allow for different values. In line with Assumption 1, we impose \( \frac{1}{q} > (1 - \lambda_P)\theta > 1 \).

**Settlement in deposits.** To start, suppose that sellers use the platform and choose deposits as means of payment. This implies that only the bank knows the sellers’ type and the platform does not lend. Accordingly, the bank is a monopolist as in Section 3 and sellers obtain

\[
S_{ON-D}^C = S_{ON-D}
\]

where the superscript \( C \) denotes competition in the lending market.

**Settlement in CBDC.** Next, suppose the seller uses (anonymous) CBDC. This implies that neither the platform nor the bank can learn its type from his payments activity. Since the platform cannot lend, the analysis is the same as in Section 4.

\(^{19}\)Lenders net profit is \((1 - s)\theta - 1\), which must be equal to zero under Bertrand competition.
The bank always uses a separating contract, and the seller’s payoff is given by

$$S_{ON-CBDC}^C = S_{ON-CBDC}$$  \hspace{1cm} (15)$$

**Settlement in tokens.** Finally, suppose that the seller uses the platform’s tokens as means of payment (the ON-T scheme). Thus, the platform learns the seller’s type from his payment activity, while the bank can only acquire information through screening. The following Lemma summarizes the bank’s choice of lending contract in this case.

**Lemma 5.** Suppose that sellers choose the ON-T trading scheme. Then, for

$$\frac{1 + \lambda}{1 - \lambda_P} > \theta$$  \hspace{1cm} (16)$$

the bank offers a separating contract with $$r_{Lb} = (1 - \lambda)p_{Lb}$$ and $$r_{HH} = r_{Lb} + (s - \lambda_P)\theta$$. Otherwise, the bank offers a pooling contract with $$\bar{r} = (1 - \lambda)p_{Lb}$$.

Lemma 5 states that the bank does not always opt for separation when sellers choose settlement in tokens—unlike under the ON-CBDC scheme. While the bank would still like to achieve separation, it is not always feasible. This result arises because the use of tokens implies that the platform is informed and thus always willing to lend to H-sellers at $$t = 2$$. The presence of a competing informed lender at $$t = 2$$ alters H-sellers’ incentives to mimic the behaviour of L-sellers towards the bank, and can therefore limit the bank’s ability to elicit information.

Under the separating contract, the bank is also informed, so that H-sellers can reap the competitive surplus $$s\theta$$ from the second loan upon repaying $$r_{HH}$$ at $$t = 1$$. Incentive compatibility then requires that they must prefer truthful reporting to lying. Pretending to be an L-seller, they would only repay $$r_{Lb}$$, but the bank would not learn their type. Accordingly, the platform would act as a monopolist at $$t = 2$$, and only leave sellers with their outside option $$\lambda_p\theta$$. The spread in the lending rate must therefore satisfy $$(s - \lambda_p)\theta \geq r_{HH} - r_{Lb}.$$
The incentives for L-sellers are identical to the case without the platform because an informed lender will never grant them a loan. Thus, as before, incentive compatibility dictates that the cost of lying must exceed the benefit from absconding with the continuation loan, \( r_{HH} - r_{Lb} \geq \lambda \). Taken together, a separating contract requires that both types of sellers report truthfully. This is only feasible if \((s - \lambda P)\theta > \lambda\), which, using the expression for \(s\), can be simplified to expression (16).

Interestingly, sellers’ expected profits are the same for both types of contracts. In either case, they earn

\[
S_{ON-T}^C = q [\lambda p_{HH} + (1 - \lambda)(p_{HH} - p_{Lb}) + \lambda P\theta] + (1 - q)\lambda P_{Lb}.
\] (17)

To gain intuition for this result, note that the H-seller’s surplus from competition in the lending market between the bank and the platform is equal to \((s - \lambda P)\theta\). Lemma 5 shows that this is exactly equal to the difference between the high interest rate in the separating equilibrium and the pooling rate, \(r_{HH} - \bar{r}\).

While the type of lending contract for the first loan does not affect the seller’s payoff, it determines the way profits are allocated between the bank and the platform. When the separating contract is used, there is perfect competition for the second loan, and the platform makes zero profits and the entire surplus goes to the bank. By contrast, if the pooling contract is used when separation is infeasible, the platform is a monopolist lender for the continuation loan and it makes positive profits.

Comparing equations (14), (17), and (15), we directly see that sellers always prefer tokens over CBDC or deposits because the use of tokens enable competition (since the bank elicits information via the separating contract) while the use of CBDC suppresses it (since the platform remains uninformed). We then can thus conclude the following.

**Proposition 3. (Equilibrium with a digital platform)**
1. For $\sigma (1 - \lambda) \geq q(1 - q)$, banks offer the partially pooling contract of the OFF-C scheme. In this case, sellers always distribute their goods online.

2. For $\sigma (1 - \lambda) < q(1 - q)$, banks offer the separating contract of the OFF-C scheme. In this case, sellers distribute their goods online if $q(1 - q)(1 - \sigma)(u_H - u_L) \geq (1 - \lambda)\sigma (\theta - 1) - q\lambda P\theta$, and offline otherwise.

3. All online sales are settled in tokens—even when a CBDC is available.

Figure 4 shows the equilibrium map in the $(\lambda, q)$-space when sellers can use the platform’s tokens. Relative to CBDC, the use of tokens expands the set of parameters for which merchants opt for online distribution. Intuitively, increased competition in the credit market ensures that sellers are able to reap part of the extra surplus $\theta - 1$ that is generated through informed lending at $t = 2$. This helps to align private incentives with social welfare.

5.2 Platform innovation

Digital platforms are often blamed for anticompetitive practices. One example in this direction is the concept of a “walled garden,” which aims to lock in consumers by limiting interoperability with other platforms. To analyze this issue, we modify
our setup as follows. Suppose that a second platform (the “entrant”) is set up at 
\( t = 2 \) with probability \( \pi \). The new platform offers a better matching technology 
which enables sellers to generate a payoff \( \hat{\theta} > \theta \) with a second loan. Otherwise, 
the entrant is identical to the incumbent, it can also grant loans and issue tokens 
as payment means, and faces a unit funding cost.

The incumbent is a walled garden in the sense that sellers will not learn 
about the emergence of the competitor platform if they use tokens as means of payment. When using deposits or CBDC, the seller learns at \( t = 2 \) that a new 
platform has come in operation only after repaying the initial loan to the bank.

We denote ex-ante expected productivity by \( \tilde{\theta} \equiv \pi \hat{\theta} + (1 - \pi)\theta \). To keep 
matters simple, we adjust Assumptions 1 - 4 to reflect the extended setup.

**Assumption 1’.** \( 1/q > (1 - \lambda)\hat{\theta} \) and \( (1 - \lambda)\theta > 1 \).

**Assumption 2’.** \( (1 - \sigma)(u_H - u_L) > \sigma(\tilde{\theta} - 1) \).

**Assumption 3’.** \( (1 - \sigma)u_L + \sigma(1 + \lambda) > (1 + \sigma)\tilde{\theta} \)

**Assumption 4’.** \( e \geq q\lambda\tilde{\theta} \).

We assume that the bank can compete with platforms, and platforms with 
identical information compete with each other. Bertrand competition implies that 
the seller appropriates the entire surplus net of funding costs, \( \theta' - 1 \), with \( \theta' \in \{\theta, \hat{\theta}\} \).

As before, the incumbent platform only learns the seller’s type if he uses 
it’s token as means of payment at \( t = 1 \). In Appendix B.2, we consider the case 
where the platform also learns from observing the sales it intermediates. As long 
as tokens provide sufficient incremental information, our results are unchanged.

If the seller uses the incumbent platform’s token, he does not learn about 
the existence of the new platform, and his payoff is as in the case with a single 
platform studied above,

\[
S_{ON-T}^{PC} = S_{ON-T}^C.
\]
where $PC$ stands for platform competition.

Now suppose instead that the seller uses deposits. This implies that he learns about the new platform, and H-sellers generate continuation surplus $\tilde{\theta}$ in expectation. Therefore, the price in $(H, H)$ meetings, denoted by $\hat{p}_{HH}$, now reflects the increased expected productivity $\hat{\theta}$, and is thus given by

$$\hat{p}_{HH} = (1 - \sigma)u_H + \sigma\lambda - \sigma(\hat{\theta} - 1)$$

By contrast, the price $p_{Lb}$ from equation (1) continues to apply in $(L, L)$-meetings. Since none of the two platforms know the seller’s type, the bank is a monopolist. Accounting for the increased productivity, the sellers’ payoff using deposits is

$$S_{ON-D}^{PC} = q\lambda\hat{p}_{HH} + (1 - q)\lambda p_{Lb} + q\lambda \hat{\theta} - e.$$

Finally, suppose that the seller uses CBDC. In this case, neither the bank nor the platform learn the seller’s type, but the seller learns about the emergence of the new platform. Accordingly, the payoff under CBDC is

$$S_{ON-CBDC}^{PC} = q[\lambda\hat{p}_{HH} + (1 - \lambda)(\hat{p}_{HH} - p_{Lb})] + (1 - q)\lambda p_{Lb}$$

It directly follows from Assumption 4’ that $S_{ON-CBDC}^{PC} > S_{ON-D}^{PC}$ and deposits are thus never used. Moreover, direct calculations reveal that $S_{ON-T}^{PC} > S_{ON-CBDC}^{PC}$, and thus tokens remain the payment method of choice for sellers.

**Proposition 4. (Equilibrium with platform innovation)**

The equilibrium with platform innovation is the same as the equilibrium with a single digital platform characterized in Proposition 3. All online sales take place on the incumbent platform and are settled with tokens.

The seller essentially opts for the lesser of two evils. If he uses the incumbent platform’s tokens, he does not learn about the entrant platform. This allows him to limit the bank’s market power, but at the same time prevents the realization of
potential efficiency gains associated with platform entry. By contrast, if he uses deposits, he learns about the entrant, but faces a monopoly bank. While this increases investment efficiency, all the additional surplus is appropriated by the bank through the interest rate on the first loan. Accordingly, the seller is better off with tokens. Since CBDC eliminates competition in lending, it is not an attractive alternative.

6 Data sharing through CBDC

As the previous sections highlight, sellers can choose which financier gets informed by opting for the right payment instrument. Leaving contractual arrangements aside, cash or CBDC leave all creditors uninformed. In this section, we expand the features of CBDC and assume it is designed such that sellers can control the information revealed to any lenders, at any point in time. This is consistent with a broader concept of privacy that goes beyond the dimension of anonymity, as summarized succinctly by Acquisti et al. (2016): “Privacy is not the opposite of sharing—rather it is control over sharing.”

We first consider the previous model where the bank competes with a digital platform for the continuation loan. Then, we additionally consider the model with the more efficient entrant platform, which also allows us to study the effects of data-sharing on inter-platform competition.

6.1 Loan competition and data sharing

The ability to share data through CBDC has profound consequences for the equilibrium in the lending market at $t = 2$. The seller has no incentive to reveal his type before repayment because the bank cannot commit to the contract terms. However, H-sellers have an incentive to reveal their type after the repayment because it enables them to introduce perfect competition between the bank and the
platform for the continuation loan. Given Assumption 1, the bank will find it opti-
mal to compete for such a loan, and H-sellers will obtain $s\theta$ from the continuation
investment. Formally, if the bank uses a separating contract, the ICs read

\[
p_{HH} - r_{HH} + s\theta \geq p_{HH} - r_{Lb} + s\theta
\]
\[
p_{Lb} - r_{Lb} \geq p_{Lb} - r_{HH} + \lambda
\]

which implies $r_{Lb} \geq r_{HH} \geq r_{Lb} + \lambda$, a contradiction. Hence a separating contract
is not feasible, and the bank can only offer a pooling contract with the interest
rate $\bar{r}$ given by equation (33). Therefore, seller’s ex-ante expected payoff is given
by

\[
S^C_{ON-CBDC^*} = q[\lambda p_{HH} + (1 - \lambda)(p_{HH} - p_{Lb}) + s\theta] + (1 - q)\lambda p_{Lb}, \quad (19)
\]

where the asterisk indicates that the CBDC allows for data-sharing. Since $s\theta =
(\theta - 1)$ and $s > \lambda_p$, a comparison with equation (18) reveals that $S^C_{ON-CBDC^*} >
S^C_{ON-T}$. We then can conclude the following.

**Proposition 5. (Equilibrium with a digital platform and data sharing via CBDC)**

Sellers always distribute their goods online. All online sales are settled in CBDC.

### 6.2 Platform competition and data sharing

We now turn to analyze the implications of data sharing for platform competition.
Suppose the seller uses CBDC, which implies that he becomes aware of the new
platform and sales prices are given by $\hat{p}_{HH}$ and $p_{Lb}$. Since H-sellers can reveal
their type after repayment of the first loan, only a pooling contract is feasible,
and Assumption 2’ implies that $\hat{p}_{HH} > p_{Lb}$, so that $\bar{r} = (1 - \lambda)p_{Lb}$. The seller’s
expected payoff under CBDC with data sharing is then equal to

\begin{align}
S_{ON-CBDC}^{PC} &= q \left[ \lambda \hat{\phi}_{HH} + (1 - \lambda)(\hat{\phi}_{HH} - p_{LB}) + (\bar{\theta} - 1) \right] + (1 - q)\lambda p_{LB} \\
&= S_{ON-CBDC}^{PC} + q(\bar{\theta} - 1) \quad (20) \\
&= S_{ON-CBDC}^{C} + q(1 - \sigma)(\bar{\theta} - \theta). \quad (21)
\end{align}

The last term in equation (20), \( q(\bar{\theta} - 1) \), captures the additional benefit of competition that data sharing provides relative to an environment where CBDC only allows sellers to hide their type. Similarly, the term \( q(1 - \sigma)(\bar{\theta} - \theta) \) in (21) captures the additional benefit of platform innovation that data sharing allows to reap relative to an environment with only a single platform. Since payoffs under deposits and tokens are identical to those in Section 5.2, we can directly conclude the following.

**Proposition 6. (Equilibrium with platform competition and data sharing via CBDC)**

Sellers always distribute their goods online, and use the entrant platform whenever available. All online sales are settled in CBDC.

If follows from Proposition 6 that a CBDC with data sharing capabilities achieves the first best allocation in the sense that (1) all sellers use the more efficient online platform technology at \( t = 1 \), (2) all H-sellers get a second loan, and (3) all H-sellers use the most efficient platform at \( t = 3 \).

7 Conclusion

We analyzed how digital privacy concerns give rise to the need for a payment instrument that permits competition through allowing selective data sharing. Our findings have important implications for the design of CBDC. In particular, CBDC may only become successful if it facilitates data sharing. While private means
of payment may in principle also provide such functionalities, incentives for the monopolization of data access may be too strong. However, absent data-sharing, private payment instruments such as digital tokens issued by platforms may crowd out CBDC, and also threaten the role of deposits as payment instrument in the digital sphere. As we have shown, sellers always prefer to use these tokens to deposits when they are available because they can then escape banks’ capture. In other words, in our environment disintermediation takes place because the banking sector is not competitive and platform tokens discipline banks into competition.

We have left unspecified the details of how financiers can learn by inspecting payment flows. Further investigation in this direction may give interesting insights. Also, we have not considered how data generated on a platform can be used to improve future sales, (i.e. how trading on the platform at $t = 1$ may lead to better trading at $t = 3$). These are important topics that we leave for future research.
References


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He, Z., J. Huang, and J. Zhou (2021). Open Banking: Credit Market Competition When Borrowers own the data.


A Proofs

A.1 Proof of Lemma 1

Suppose sellers choose the offline-cash (OFF-C) trading scheme. First, under the pooling contract, the bank does not learn sellers’ type. Hence, no loan will be extended at $t = 2$, and bilateral prices are $\hat{p}_{HH}$, $\hat{p}_{HL}$, and $p_{Lb}$. Given that $\hat{p}_{HH} > \hat{p}_{HL} = p_{Lb}$, profit maximization implies that the participation constraints (PCs) of HL-sellers and L-sellers bind, so that

$$\bar{r} = (1 - \lambda)p_{Lb}$$  \hspace{1cm} (22)

As a result, the bank earns under the pooling contract

$$B_{\text{OFF-C}}^{\text{pooling}} = \bar{r} - 1 = (1 - \lambda)p_{Lb} - 1.$$  \hspace{1cm} (23)

Second, under the separating contract, the bank lends to all H-sellers and bilateral prices are $p_{HH}$, $p_{HL}$, and $p_{Lb}$. The contract has to satisfy the following three incentive constraints (IC) for the ex-post types HH, HL, and L:

$$p_{HH} - r_{HH} + \lambda \theta \geq p_{HH} + \max\{-r_{HL} + \lambda \theta, -r_{Lb}\},$$

$$p_{HL} - r_{HL} + \lambda \theta \geq p_{HL} + \max\{-r_{HH} + \lambda \theta, -r_{Lb}\},$$

$$p_{Lb} - r_{Lb} \geq p_{Lb} + \max\{-r_{HH} + \lambda, -r_{HL} + \lambda\}.$$  

Combining the first two incentive constraints yields $r_{HH} = r_{HL}$. Note that an L-seller can only pretend to be an H-seller if his sales are sufficient to pay the high interest rate. This is feasible because Assumption 3 implies $p_{Lb} > r_{HH}$. The ICs can thus be combined to $\lambda \theta \geq r_{HH} - r_{Lb} \geq \lambda$, and profit-maximization then yields

$$r_{HH} = r_{HL} = r_{Lb} + \lambda \theta.$$  \hspace{1cm} (24)
The separating contract must also satisfy the following three PCs:

\[ p_{HH} - r_{HH} + \lambda \theta \geq \lambda p_{HH}, \]
\[ p_{HL} - r_{HL} + \lambda \theta \geq \lambda p_{HL}, \]
\[ p_{Lb} - r_{Lb} \geq \lambda p_{Lb}. \]

Substituting interest rates \( r_{HH} \) and \( r_{HL} \) from (24) and using the ordering \( p_{HH} > p_{Lb} > p_{HL} \) then leads us to conclude that

\[ r_{S_{Lb}} = (1 - \lambda)p_{HL} \]  
(25)

in separating equilibrium. Hence, \( r_{S_{HH}} = r_{S_{HL}} = (1 - \lambda)p_{HL} + \lambda \theta \). Thus, the separating contract yields an expected bank profit of

\[
B_{OFF-C}^S = q^2(r_{HH}^S - 1) + q(1 - q)(r_{HL}^S - 1) + (1 - q)(r_{Lb}^S - 1) + q[(1 - \lambda)\theta - 1] \\
= (1 - \lambda)p_{HL} - 1 + q(\theta - 1),
\]  
(26)

where the term \((1 - \lambda)\theta - 1\) represents the bank’s share of the surplus from the continuation loan granted at \( t = 2 \).

Third, consider the partially pooling contract, under which the bank only extends continuation finance to HH-sellers. Bilateral prices are then given by \( p_{HH}, \hat{p}_{HL} \) and \( p_{Lb} \), so the simplified ICs read

\[ p_{HH} - r_{HH} + \lambda \theta \geq p_{HH} - r_{Lb}, \]
\[ \hat{p}_{HL} - r_{Lb} \geq \hat{p}_{HL} - r_{HH} + \lambda \theta, \]
\[ p_{Lb} - r_{Lb} \geq p_{Lb} - r_{HH} + \lambda. \]

Since \( \hat{p}_{HL} = p_{Lb} \), we directly obtain

\[ r_{HH} - r_{Lb} = \lambda \theta. \]  
(27)
The PCs are
\[ p_{HH} − r_{HH} + λθ \geq λp_{HH} \]
\[ \tilde{p}_{HL} − r_{Lb} \geq λ\tilde{p}_{HL} \]
\[ p_{Lb} − r_{Lb} \geq λp_{Lb} \]

which, using \( \tilde{p}_{HL} = p_{Lb} \) again, yields
\[ r_{Lb}^{P} = (1 − λ)p_{Lb} = r_{HL}^{P}. \] (28)

Thus, expected bank profits under partially pooling are
\[ B_{OFF−C}^{P} = q^{2}(r_{HH}^{P} − 1) + (1 − q^{2})(r_{Lb}^{P} − 1) + q^{2}[(1 − λ)θ − 1] \]
\[ = (1 − λ)p_{Lb} − 1 + q^{2}(θ − 1). \] (29)

Direct inspection reveals that \( B_{OFF−C}^{P} > B_{OFF−C}^{pooling} \), so the pooling contract is never optimal.

**A.2 Proof of Lemma 2**

The Lemma follows directly from comparing the equations for \( B_{OFF−C}^{S} \) and \( B_{OFF−C}^{P} \), evaluated at the interest rates given in Lemma 1 and using the expression for the prices in equation (1).
A.3 Proof of Lemma 3

When deposits are used, the bank learns the type of the seller. Thus, no ICs are needed and the relevant PCs are

\[
\begin{align*}
    p_{HH} - r_{HH}^D + \lambda \theta &\geq \lambda p_{HH} - e + \lambda \theta \\
    p_{HL} - r_{HL}^D + \lambda \theta &\geq \lambda p_{HL} - e + \lambda \theta \\
    p_{Lb} - r_{Lb}^D &\geq \lambda p_{Lb} - e,
\end{align*}
\]

Profit maximization implies that each of these PCs bind, resulting in the interest rate stated in the lemma.

A.4 Proof of Lemma 4

Since there are only two types of matches with online sales, the bank’s choice is limited to a separating and a pooling contract. The separating contract with CBDC has to satisfy the following two ICs

\[
\begin{align*}
    p_{HH} - r_{HH} + \lambda \theta &\geq p_{HH} - r_{Lb} \\
    p_{Lb} - r_{Lb} &\geq p_{Lb} - r_{HH} + \lambda,
\end{align*}
\]

which together with profit-maximization yields

\[ r_{Lb} = r_{HH} - \lambda \theta \]

The two PCs read

\[
\begin{align*}
    (1 - \lambda)p_{HH} &\geq r_{Lb} \\
    (1 - \lambda)p_{Lb} &\geq r_{Lb}
\end{align*}
\]
Since $p_{HH} > p_{Lb}$, only the PC for L-sellers binds, so that

$$r_{Lb} = (1 - \lambda)p_{Lb}$$

The bankers’ expected payoff with the separating CBDC contract is

$$B_{ON-CBDC}^{separating} = q[r_{HH} + (1 - \lambda)\theta - 1] + (1 - q)r_{Lb} - 1$$

$$= (1 - \lambda)p_{Lb} + q(\theta - 1) - 1$$

Next, consider the pooling equilibrium. Since, $p_{HH} > p_{Lb}$, the pooling rate is

$$\bar{r} = (1 - \lambda)p_{Lb}$$

and the banker’s payoff is

$$B_{ON-CBDC}^{pooling} = (1 - \lambda)p_{Lb} - 1$$

Since $B_{ON-CBDC}^{separating} > B_{ON-CBDC}^{pooling}$, the bank will use the separating contract when sellers select the ON-CBDC trading scheme.

### A.5 Proof of Lemma 5

The separating contract has to satisfy the following two ICs

$$p_{HH} - r_{HH} + s\theta \geq p_{HH} - r_{Lb} + \lambda_p\theta$$

$$p_{Lb} - r_{Lb} \geq p_{Lb} - r_{HH} + \lambda.$$  

When an H-seller pretends to be an L-seller, he forgoes the competitive surplus $s\theta$ and instead obtains $\lambda_p\theta$ by borrowing from the (monopoly) platform. Similarly, an L-seller can obtain $\lambda$ when pretending to be an H-seller through absconding.
with the continuation loan. Combining both inequalities, we get

$$(s - \lambda_P) \theta \geq r_{HH} - r_{Lb} \geq \lambda$$  \hspace{1cm} (30)$$

Interestingly, while the separating contract was always feasible without competition, a separating contract is now no longer feasible if $\lambda > (s - \lambda_P) \theta$, or $\frac{1+\lambda}{1-\lambda_P} > \theta$. In this case, L-sellers derive a higher benefit from pretending to be H-sellers than H-sellers themselves.

A separating contract also has to satisfy the PCs, which read

$$p_{HH} - r_{HH} + s\theta \geq \lambda p_{HH} + \lambda_P \theta$$
$$p_{Lb} - r_{Lb} \geq \lambda p_{Lb}$$

Given $r_{Lb}$ and assuming feasibility ($\theta < \frac{1+\lambda}{1-\lambda_P}$), the profit-maximizing bank will set

$$r_{HH} = r_{Lb} + (s - \lambda_P) \theta$$  \hspace{1cm} (31)$$

Substitution into the PCs together with $p_H > p_{Lb}$ from Assumption 2 then implies

$$r_{Lb} = (1 - \lambda) p_{Lb}$$  \hspace{1cm} (32)$$

Alternatively, the bank can offer a pooling contract where all borrowers pay the same rate.\textsuperscript{20} Since this contract only reflects the PCs, we directly get

$$\bar{r} = (1 - \lambda)p_{Lb}$$  \hspace{1cm} (33)$$

Banks’ choice regarding contract terms is determined by profit maximization.\textsuperscript{20} Notice that there can be no partially pooling contract because there are only two types of meetings, $(H, H)$ and $(L, L)$.

\textsuperscript{20}
The separating contract yields

$$B_{ON-T}^{separating,C} = q \left(r_{HH} - 1 + \frac{1}{2} \{(1 - s)\theta - 1\}\right) + (1 - q)(r_{Lb} - 1)$$

$$= (1 - \lambda)p_{Lb} + q(s - \lambda P)\theta - 1,$$

while the pooling contracts leads to

$$B_{ON-T}^{pooling,C} = (1 - \lambda)p_{Lb} - 1. \quad (34)$$

We can directly observe that $B_{ON-T}^{separating,C} > B_{ON-T}^{pooling,C}$. This implies that the bank will offer a separating contract whenever feasible, and a pooling contract otherwise.

### B Additional results

#### B.1 Commitment to punish upon default

We have assumed that the bank cannot commit to punish the seller if he defaults on the loan. While this is in line with the bank also not being able to commit to the loan terms, we here consider the alternative case where the bank can commit to such a punishment. To keep matters simple, we drop the assumption that absconding under deposits generates an additional fixed cost of $e$.

If the bank can commit to not extending a loan upon default, H-sellers must repay their loan in the case deposits are used. Consider the OFF-D trading scheme. The PCs become

$$p_{HH} - r_{HH}^{d} + \lambda\theta \geq \lambda p_{HH}$$

$$p_{HL} - r_{HL}^{d} + \lambda\theta \geq \lambda p_{HL}$$

$$p_{Lb} - r_{Lb}^{d} \geq \lambda p_{Lb},$$
which can be solved for the interest rates

\[
\begin{align*}
    r^d_{HH} &= (1 - \lambda)p_{HH} + \lambda \theta \\
    r^d_{HL} &= (1 - \lambda)p_{HL} + \lambda \theta \\
    r^d_{Lb} &= (1 - \lambda)p_{Lb}
\end{align*}
\] (35)

Following exactly the same logic, interest rates for the ON-D scheme are given by (35) and (36). Straightforward computations then show that sellers’ expected profit from both schemes is given by

\[
S_{OFF-D} = q^2 \lambda p_{HH} + q(1 - q)\lambda p_{HL} + (1 - q)\lambda p_{Lb}
\]

and

\[
S_{ON-D} = q\lambda p_{HH} + (1 - q)\lambda p_{Lb}
\]

Since the ability to commit does not affect payoffs when sales are settled in cash (in case of default the bank learns nothing, and thus does not lend), they are still given by equations (6) and (7) in the main text. It can be seen readily that

\[
\min\{S_{separating}^{OFF-D}, S_{partpooling}^{OFF-C}\} > S_{OFF-D},
\]

so deposits are never used to settle offline sales. We then obtain the following result, which corresponds to Proposition 1 for the case where \(e = q\lambda \theta\).

**Proposition 7. (Equilibrium with commitment to punish upon default)**

1. For \(\sigma(1 - \lambda) \geq q(1 - q)\), banks offer a partially pooling contract under the OFF-C scheme. In this case, sellers distribute their goods online if \(\lambda \geq q\), and offline otherwise.
2. For \(\sigma(1 - \lambda) < q(1 - q)\), banks offer a separating contract under the OFF-C scheme. In this case, sellers distribute their goods online if \(q(\lambda - q)(1 - \sigma)(u_H - u_L) \geq (1 - q)(1 - \lambda)\sigma(\theta - 1)\), and offline otherwise.
3. All online sales are settled in deposits (by assumption).
B.2 A more informed platform

In this section, we relax the assumption that payment tokens are the only source of information for the platform. Instead, we assume that the platform receives a perfect signal about sellers’ type with probability $\xi$, while it remains uninformed with probability $1 - \xi$ (so the main text corresponds to $\xi = 0$). Moreover, to simplify the exposition, we also assume that the bank observes whether the platform has received a signal or not.\(^\text{21}\)

B.2.1 Lending market competition

Suppose that sellers opt for CBDC. If the bank chooses to become informed through a separating contract, it will compete with the platform with probability $\xi$, and otherwise act as a monopolist. Accordingly, this allows H-sellers to reap an expected surplus of $s^*\theta$, where $s^* = \xi s + (1 - \xi)\lambda < s$. The separating contract thus has to satisfy the following ICs

\[
\begin{align*}
    p_{HH} - r_{HH} + s^*\theta & \geq p_{HH} - r_{Lb} + \xi \lambda \theta \\
    p_{Lb} - r_{Lb}^+ & \geq p_{Lb} - r_{HH} + \lambda,
\end{align*}
\]

This implies

\[
(s^* - \xi \lambda)\theta \geq r_{HH} - r_{Lb} \geq \lambda
\]

Moreover, L-sellers’ PC yields

\[
r_{Lb} = (1 - \lambda)p_{Lb}
\]

\(^{21}\)If the bank does not know whether she faces an informed or uninformed competitor in the lending market, solving for the equilibrium would be considerably more complex.
We henceforth assume that \((s^* - \xi \lambda_p) \theta > \lambda\), so a separating contract is feasible.\(^{22}\) Profit-maximization by the bank then implies

\[
r_{HH} = r_{Lb} + (s^* - \xi \lambda_p) \theta
\]

Note that a pooling contract would yield lower bank profits because it prevents the bank from charging higher interest rates from from H-sellers and extract the continuation surplus. Sellers’ payoff is given by

\[
S^*_C \equiv S_{CBDC-ON} = q[\lambda p_{HH} + (1 - \lambda)(p_{HH} - p_{Lb}) + \xi \lambda_p \theta] + (1 - q)\lambda p_{Lb}
\]

The existence of the platform limits the surplus the bank can extract by providing an alternative source of financing for the second loan. Anything beyond what sellers can obtain from a monopoly platform \((\xi \lambda_p \theta)\) is appropriated by the bank. Notice that we have \(S^*_C \equiv S_{CBDC-ON} = S_{CBDC}\) as \(\xi = 0\), which corresponds to the main text. As \(\xi \to 1\), the informational value of tokens diminishes, so \(S^*_C \equiv S_{CBDC-ON} \to S^*_C \equiv S_{ON-T}\).

Notice that \(S^*_C \equiv S_{ON-T} = S^*_C \equiv S_{ON-T}\), i.e. the platform is perfectly informed when tokens are used independently of what the platform knows without. Accordingly, sellers prefer tokens to CBDC whenever \(\xi < 1\).

Finally, consider the case where sellers opt for deposits as means of payments. With probability \(\xi\), the bank and the platform are informed, leading to perfect competition. By contrast, the bank is a monopolist with probability \(1 - \xi\). Thus, sellers earn

\[
S^*_C \equiv S_{ON-D} = q \lambda p_{HH} + (1 - q) \lambda p_{Lb} - (e - q s^* \theta)
\]

and so sellers would prefer tokens over deposits whenever

\[
q(1 - \lambda)(p_{HH} - p_{Lb}) > q(s^* - \lambda p_\theta) - e.
\]

\(^{22}\)The analysis for the case when the separating contract is not feasible is slightly more tedious, and available upon request. It does not deliver any further insights, since sufficiently low values of \(\xi\) lead to the same conclusions.
Note that the LHS is always positive, so a sufficient condition the above inequality to hold is that the RHS is non-negative. Since \( e \geq q \lambda \theta \) by assumption, this is always the case for \[ \frac{\lambda_p}{s - \lambda} \geq \xi. \] (37)

### B.2.2 Platform innovation

Now consider the case of platform innovation. When sales are settled with tokens, the seller does not learn about the new platform, and the resulting payoff is the same as without the platform \( S^{PC*}_{ON-T} = S^{C*}_{ON-T} = S^{C}_{ON-T} \).

When CBDC is used instead, the seller does learn about the new platform. Substituting expected productivity \( \tilde{\theta} \) into the payoffs from the previous subsection, we get

\[
S^{PC*}_{ON-CBDC} = q \left[ \lambda \tilde{\rho}_{HH} + (1 - \lambda) (\rho_{HH} - p_{Lb}) + q \lambda_p \tilde{\theta} \right] + (1 - q) p_{Lb},
\]

Sellers thus prefer tokens to CBDC whenever \( S^{PC*}_{ON-T} > S^{PC}_{ON-CBDC} \), or

\[ (\sigma - \lambda_p \xi)(\tilde{\theta} - \theta) + \lambda_p \theta (1 - \xi) > 0 \]

Since the second term is always positive, a sufficient condition for the inequality to hold is

\[ \frac{\sigma}{\lambda_p} \geq \xi. \] (38)

The use of deposits also enable sellers to learn about the entrant. Sellers obtain

\[ S^{PC}_{ON-D} = q \lambda \tilde{\rho}_{HH} + (1 - q) \lambda p_{Lb} - (e - q \tilde{s}^* \tilde{\theta}) \]

where \( \tilde{s} = 1 - \tilde{\theta}^{-1} \) and \( \tilde{s}^* = \xi \tilde{s} + (1 - \xi) \lambda \). Accordingly, tokens are preferred to deposits whenever

\[
q(1 - \lambda) (p_{HH} - p_{Lb}) + \lambda (p - p)] > q(\tilde{s}^* \tilde{\theta} - \lambda_p \theta) - e
\]
The LHS is always positive, so this condition is satisfied if the RHS is non-positive. Since $e \geq q\lambda \theta$ by assumption, this is always the case for $(\lambda_p + \lambda)\theta \geq \tilde{s}'\tilde{\theta}$, or

$$\frac{\lambda_p\theta - \lambda(\tilde{\theta} - \theta)}{(\tilde{s} - \lambda)\tilde{\theta}} \geq \xi. \quad (39)$$

Finally, a CBDC with data sharing leads to the same payoffs as in the main text. Hence it would be the payment instrument chosen by sellers.

**B.3 Low adverse selection**

In this section, we analyze the case where adverse selection is low and uninformed lending is profitable. Formally, this corresponds to $q(1 - \lambda)\theta > 1$. We have to consider the following cases.

1. The bank uses a pooling contract and lends to all sellers at $t = 2$.

2. The bank uses a partially pooling contract that separates H-sellers in (H,H)-meetings, but pools L sellers and H sellers in (H,L)-meetings, and only lends to the first set of H-sellers.

3. The bank uses a separating contract and only lends to all H-sellers at $t = 2$.

Note that the first contract differs from the pooling contract in the main text, since it is now profitable to lend to all sellers when using a pooling contract. The remaining two contracts are identical to the ones studied in the main text. Accordingly, the banks’ payoffs are given by equations 29) and (26), respectively.

**Pooling contract with lending to all sellers at $t = 2$.** When the bank

\footnote{Notice that lending to all sellers would violate incentive compatibility. H-Sellers in (H,H)-meetings would want to pretend to be H-sellers in (H,L)-meetings and enjoy a lower interest rate, but still receive continuation financing.}
lends to all sellers at \( t = 2 \), bilateral prices are given by

\[
p(m) = \begin{cases} 
  p_{HH} & \equiv (1 - \sigma)u_H + \sigma \lambda - \sigma (\theta - 1) \\
  p_{HL} & \equiv (1 - \sigma)u_L + \sigma \lambda - \sigma (\theta - 1) \\
  \hat{p}_{Lb} & \equiv (1 - \sigma)u_L - \sigma
\end{cases}
\]

Here, \( \hat{p}_{Lb} \) now accounts for the fact that L-sellers will generate a payoff of \( \lambda - 1 < 0 \) for the bank/seller coalition (sellers will get a loan of 1 but abscond to obtain \( \lambda \)). Note that

\[
p_{HL} - \hat{p}_{Lb} = \sigma [1 + \lambda - (\theta - 1)]
\]

which can be positive or negative. We thus have two cases to analyze, in which either the PC of HL-sellers binds (for \( 1 + \lambda < \theta - 1 \)) or the one of L-sellers binds (for \( 1 + \lambda > \theta - 1 \)).

First, suppose \( \min(\hat{p}_{Lb}, p_{HH}) > p_{HL} \), so the PC of HL-sellers will bind, and the pooling rate is \( \bar{r} = (1 - \lambda)p_{HL} \). Thus, bank profits are

\[
B^*_{HL} = (1 - \lambda)p_{HL} - 1 + [q \theta (1 - \lambda) - 1] \\
= (1 - \lambda)[(1 - \sigma)u_L + \sigma \lambda] + (q - \sigma)(1 - \lambda) (\theta - 1) - (1 - q(1 - \lambda)) - 1
\]

(40)

Next, suppose \( \min(p_{HL}, p_{HH}) > p_{Lb} \), so that the PC of L-sellers binds, and the pooling rate is \( \bar{r} = (1 - \lambda)p_{Lb} \). Then, bank profits are

\[
B^*_L = (1 - \lambda)p_{Lb} - 1 + [q \theta (1 - \lambda) - 1] \\
= (1 - \lambda)[(1 - \sigma)u_L + \sigma \lambda] + (1 - \lambda) [q \theta - \sigma (1 + \lambda)] - 1 - 1.
\]

(41)

**Banks’ contract choice.** Recall that the choice between separating and partially pooling contracts is governed by inequality (4). Straightforward algebra
reveals that

\[ B_{HL}^* - B_{partpool}^{OFF-C} = (\theta - 1) \left[ (q(1 - q) - \sigma(1 - \lambda) - q\lambda) - (1 - q(1 - \lambda)) \right] < 0 \]

where the inequality follows from the fact that \( 1 - q(1 - \lambda) > 0 \) and \( q(1 - q) \leq \sigma(1 - \lambda) \) in any equilibrium with partial pooling. Moreover, we have

\[ B_{HL}^* - B_{separating}^{OFF-C} = (\theta - 1) [q(1 - \lambda) - q] - (1 - q(1 - \lambda)) < 0. \]

Together, this implies that We thus have \( \min\{B_{separating}^{OFF-C}, B_{partpool}^{OFF-C}\} > B_{HL}^* \), and banks never lends to all sellers in equilibrium when the IC of HL-sellers binds.

Now, consider the case where L-sellers’ PC binds, so that \( 1 + \lambda > \theta - 1 \). Direct calculations reveal

\[ B_{Lb}^* - B_{separating}^{OFF-C} = -\lambda q\theta - \sigma(1 - \lambda) [(1 + \lambda) - (\theta - 1)] - 1 + q < 0 \]

and

\[ B_{Lb}^* - B_{partpool}^{OFF-C} = (1 - \lambda) [q\theta - \sigma(1 + \lambda)] - 1 - q^2[\theta - 1] + 1 \\
= (-\lambda q\theta + q - 1) + [q(1 - q) - (1 - \lambda)\sigma][\theta - 1] \\
+ (1 - \lambda)\sigma[\theta - 1 - (1 + \lambda)] < 0 \]

The second inequality is established as follows. The first term is obviously negative, the second term is negative because we must have \( (1 - \lambda)\sigma \geq q(1 - q) \) whenever the bank uses a partially pooling contract, and the third term is negative because the incentive constraint of L-sellers only binds for \( 1 + \lambda > \theta - 1 \).

Thus, \( \min\{B_{separating}^{OFF-C}, B_{partpool}^{OFF-C}\} > B_{Lb}^* \), and banks never lend to all sellers when L-sellers PC is binding.

To conclude, pooling is never optimal even if adverse selection is sufficiently low to render uninformed lending profitable. Intuitively, separation (either full or partial) allows banks reduce future losses from inefficient investment by L-sellers,
and thus leads to higher bank profits.

C Micro-foundations for payments

In this section, we sketch the setup of a model with micro-foundations for payments, in the spirit of the new-monetarist literature. Time is discrete and continues forever. Each period has 4 subperiods ($s = 0, 1, 2, 3$ as in the model). There is a continuum of buyers, sellers, banks, and a platform. Buyers are infinitely lived, while sellers, bankers and the platform live for one period. There are two types of sellers and buyers, H and L. Taking the viewpoint of sellers, we refer to subperiods 0 (1) and 2 (3) as the first and second investment (production) stage.

There are two goods: goods that buyers can produce (B-goods) and goods that sellers can produce (S-goods). Buyers produce B-goods in the two investment stages, while sellers produce S-goods in the two production stages. Goods are not storable, once produced they have to be consumed.

Buyers produce B-goods at will by incurring a linear cost of production. Sellers need to invest one unit of the B-good in the first investment stage to produce 1 unit of the S-good in the first production stage, and H sellers need to invest one unit of the B-good in the second investment stage to produce 1 unit of the S-good in the second production stage. L-sellers cannot produce in the second production stage. All agents derive a linear utility from consuming B-goods. In the first production stage, buyers of type $i$ derive utility $u_{ij}$ from consuming one unit of the S-good produced by a seller of type $j$. We set $u_{HH} \equiv u_{H}$ and $u_{ij} \equiv u_{L} < u_{H}$ for all $ij \neq HH$. Buyers of any type derive one util from consuming one unit of the S-good produced in the second production stage.

The utility of buyers of type $i$ when they produce $y_{s}$ units of B-goods in subperiod $s = 0, 2$, consume $x_{s}$ units of B goods in subperiod $s = 0, 2$ and consume $c_{ij} \in \{0, 1\}$ units of the S-good in the first production stage produced by seller $j$
and $c_3$ units in the second production stage is

$$U_i(y_0, x_0, c_{1j}, y_2, x_2, c_3) = x_0 - y_0 + \sum_j c_{1j} u_{ij} + x_1 - y_1 + c_3,$$  \hspace{1cm} (42)

where we removed the index $j$ in $c_3$ as it is inconsequential.

The utility of sellers of type $j$ when they invest $y_s \in \{0, 1\}$ units of the B-good in subperiod $s = 0, 2$, consume $x_s$ units of B-good in subperiod $s = 0, 2$, and consume $c_s$ units of the S-good in subperiod $s = 1, 3$ is

$$V_j(y_0, x_0, c_1, y_2, x_2, c_3) = x_0 + x_2 + \lambda(c_1 + c_3)$$ \hspace{1cm} (43)

with $\lambda < 1$ (alternatively, sellers face a liquidation cost when consuming their production). Notice that investment and production is costless for sellers. Finally, banks and platform owners have the same utility as sellers, but with $\lambda = 1$.

Sellers and buyers don’t trust each other, so sellers (buyers) need a means of payment to buy B-goods (S-goods). Banks and the platform are trustworthy and can issue IOUs (bank deposits and tokens respectively). A bank deposit or a token issued in the investment stage is a promise to one unit of the B-good in the next production stage. There is also cash (and CBDC) provided by the monetary authority. The stock of cash (and CBDC) in period $t$ is $M_t$ and the monetary authority makes cash inexpensive to use by running the Friedman rule. The timing within each period is as follows.

**First investment stage** ($s = 0$). Buyers acquire cash/bank deposits/tokens, banks acquire B-goods, sellers get investment from banks and invest directly (so they cannot consume the B-good that is supposed to be invested).

**First production stage** ($s = 1$). Sellers choose trading venue, buyers follow (possibly exchanging cash for deposits/tokens). Sellers’ learn their types, trade (production and consumption) occurs through bargaining and a payment is exchanged.
**Second investment stage** \((s = 2)\). Sellers buy B-goods with acquired means of payment, consume/repay loans. Sellers can only run away with a fraction \(\lambda\) of their sales. Banks/platform settle claims on deposits/tokens previously issued. Buyers acquire cash/means of payment with cash. Bank/platform lends to known H-sellers.

**Second production stage** \((s = 3)\). H-sellers produce \(\theta\) units of the S-good, and they sell their goods for an equivalent of \(\theta\) to buyers (their reservation price), deposits or tokens are exchanged. Sellers repay their loans or run away with a fraction \(\lambda\) of their sales. Sellers, banks, and the platform consume and are replaced by a new cohort of sellers, banks and platform.\(^{24}\)

**Buyers’ problem**

Let \(z_0, d_0, e_0\) and \(\tau_0\) be the real amounts (as measured in terms of B-goods) of cash \((z_0)\), bank deposits \((d_0)\), CBDC \((e_0)\) and tokens \((\tau_0)\), that buyers demand in the first investment stage. The problem of buyers is

\[
V_0(m_0) = \max_{y_0, x_0, z_0, d_0, e_0, \tau_0} -y_0 + x_0 + E_v V_1^v(z_0, d_0, e_0, \tau_0)
\]

\[
s.t. \quad y_0 + m_0 = x_0 + z_0 + d_0 + e_0 + \tau_0
\]

where the expectations operator is taken over sellers’ choice of trading venue, which buyers perfectly anticipate in equilibrium. Substituting the budget constraint,

\[
V_0(m_0) = \max_{z_0, d_0, e_0, \tau_0} m_0 - (z_0 + d_0 + e_0 + \tau_0) + E_v V_1^v(z_0, d_0, e_0, \tau_0)
\]

\[
= m_0 + V_0(0)
\]

and \(V_0\) is quasilinear as in Lagos and Wright (2001).

Given sellers’ choice of trading venue of \((v = z, d, e, \tau)\) where we abuse nota-

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\(^{24}\)Alternatively, one can assume that sellers change type each period, and that banks and platforms distribute their equity at the end of each period.
tion and use $v = z$ to denote offline-cash, $d$ to denote online-deposit, etc.), buyers solve

$$V^v_1(z_0, d_0, e_0, \tau_0) = \max_{c_{1j} \in \{0, 1\}} E_J \{ c_{1j} u_{ij} + V_2(m_2) \}$$

s.t. $p c_{1j} \leq v_0$

$$m_2 = z_0 + d_0 + e_0 + \tau_0 - p c_{1j}$$

where $E_J$ is the expectation over meeting a seller of type $j \in J$. In the second investment stage, only the total real value of the portfolio of payment instruments matters, so we can use $m_2$ as the state variable (the budget constraint ensures this amount is positive). $p$ solves the bargaining problem (which we solve after we have defined the problem of sellers).

The value for buyers of entering the second investment stage with a portfolio of payment instruments worth $m_2$ is

$$V_2(m_2) = \max_{y_2, x_2, z_2, d_2, e_2, \tau_2} -y_2 + x_2 + V_3(z_2, d_2, e_2, \tau_2)$$

s.t. $y_2 + m_2 = x_2 + (z_2 + d_2 + e_2 + \tau_2)$

Notice that in the second investment stage, buyers redeem their portfolio and so they can get the equivalent in B-goods (or carry over the balance to the second production stage). Hence

$$V_2(m_2) = \max_{z_2, d_2, e_2, \tau_2} m_2 - (z_2 + d_2 + e_2 + \tau_2) + V_3(z_2, d_2, e_2, \tau_2)$$

Finally,

$$V_3(z_2, d_2, e_2, \tau_2) = \max_{c_3} c_3 + \beta V_0((z_2 + d_2 + e_2 + \tau_2 - c_3 p_3)(1 + \rho))$$

$$= \max_{c_3} c_3 (1 - p_3) + \beta V_0((z_2 + d_2 + e_2 + \tau_2)(1 + \rho))$$

$$= \max_{c_3} c_3 (1 - p_3) + (z_2 + d_2 + e_2 + \tau_2) + \beta V_0(0)$$
subject to

\[ p_3c_3 \leq z_2 + d_2 + e_2 + \tau_2 \]

where \( p_3 \) is the price at \( s = 3 \). We have used the fact that the monetary authority implements the Friedman rule, so that the real rate of return on payment balances is \( 1/\beta \), so buyers have no cost of holding any means of payment and their budget constraint (44) never binds. Also, replacing the expression for \( V_3 \) into the expression for \( V_2 \), we can write

\[
V_2(m_2) = m_2 + \max_{c_3, z_2, d_2, e_2, \tau_2} c_3 (1 - p_3) + \beta V_0(0)
\]

s.t. \( p_3c_3 \leq z_2 + d_2 + e_2 + \tau_2 \).

Since buyers are never constrained (thanks to the Friedman rule), we must have \( p_3 = 1 \). Otherwise, buyers would demand an infinite quantity of the S-good and the market would not clear.

**Sellers’ problem**

Sellers are born at the start of each period with no endowment but only with their production technology. Their utility in the first investment stage is

\[
W_0 = \max_{y \in \{0, 1\}, v \in \{z, d, e, \tau\}} E_j W_{1, j}^v(y)
\]

where \( v \) is the choice of trading venue, \( y \in \{0, 1\} \) is the amount borrowed from the bank, and \( E \) is the expectation over types. Then, a seller of type \( j = H, L \) has the following payoff

\[
W_{1, j}^v(0) = 0 \quad \text{for all } v
\]

\[
W_{1, j}^v(1) = \max \{ E_i W_{2, j}(p_{ij}); \lambda \}
\]
since sellers can always consume production to obtain utility $\lambda$, and $p_{ij}$ is the price between buyer $i$ and seller $j$ for one unit of production. Finally,

$$W_{2,j}(p_{ij}) = \max_{\hat{p} \leq r(p_{ij})} p_{ij} - r(\hat{p}) + y_2(\hat{p}) W_{3,j}$$

where $\hat{p}$ is the announcement of seller to the bank. The bank refinances sellers with probability $y_2(\hat{p})$ in which case they get $W_{3,j}$, where

$$W_{3,L} = \lambda$$
$$W_{3,H} = \max \{ \theta p_3 - i, \lambda \theta \} = \max \{ \theta - i, \lambda \theta \}$$

so that $H$ sellers produce $\theta$, and either sell it at price $p_3 = 1$ to repay their debt $i$ or consume it for utility $\lambda \theta$.

**Banks’ problem**

Each bank is matched with one seller and issues deposits $d_{0}^B$ to buyers in $t = 0$ to maximize

$$\max_{d_{0}^B, y_2 \in \{0, 1\}} E[(d_{0}^B - 1) + r(\hat{p}) - d_{0}^B + y_2(\hat{p}) W_{B}(\hat{p})]$$

s.t. $d_{0}^B \geq 1$

The bank issues $d_{0}^B \geq 1$ deposits in the first investment stage, it invests 1 with the seller and consumes $d_{0}^B - 1$ (B-goods are not storable). After the first production stage, the seller repays $r(\hat{p}_{ij})$ and the bank redeems deposits. Given the information it obtains, the bank refinances the seller in the second investment stage or not, so that

$$W_{B}(\hat{p}) = \max_{d_{2}^B} \left( d_{2}^B - y_2(\hat{p}) \right) + 1_{\text{info } \hat{p} = \Pi_i(\theta)} - d_{2}^B,$$

If the bank knows the seller is L, the bank chooses $y_2(\hat{p}) = 0$ and $d_{2}^B = 0$. 57
Bargaining between buyers and sellers

We can now solve for the bargaining problem. Sellers maximizing the surplus of the seller/bank coalition, which is

\[ p_{ij} - r(\hat{p}) + y_2(\hat{p})W^j_d + r(\hat{p}) - d_0 + y_2(\hat{p})W^B(\hat{p}) = \]

\[ p_{ij} + y_2(\hat{p}) (W^j_d + W^B(\hat{p})) - d_0 = \]

\[ \begin{cases} 
  p_{ij} - d_0 & \text{if } \hat{p} \Rightarrow \text{L-seller or no info} \\
  p_{ij} - d_0 + \theta - 1 & \text{if } \hat{p} \Rightarrow \text{H-seller} 
\end{cases} \]

and the “outside option” is \( \lambda - d_0 \). Therefore, the bargaining between buyers and sellers is

\[ \max_{p_{ij}} [u_{ij} + V_2(z + d + e + \tau - p_{ij}) - V_2(z + d + e + \tau)]^\sigma [p_{ij} - d_0 + \mathbb{I}_{\text{info=H}}(\theta - 1) - (\lambda - d_0)]^{1-\sigma} \]

which simplifies to

\[ \max_{p_{ij}} [u_{ij} - p_{ij}]^\sigma [p_{ij} - \lambda + \mathbb{I}_{\text{info=H}}(\theta - 1)]^{1-\sigma} \]

due to the linearity of \( V_2 \), and is thus the same solution as in the paper.

The platform’s problem

The platform can issue tokens in the first investment stage (but it cannot fund sellers at that stage). The problem of the platform is

\[ \max_{\tau_0^P, z_0^P} \tau_0^P - z_0^P + E_J V^P(z_0^P, \tau_0^P; p_j) \]

\[ \text{s.t. } z_0^P \leq \tau_0^P, \]

where \( E_J \) is the expectation over sellers’ types when trading at price \( p_j \). The constraint reflects the fact that the platform can save the profit from selling its
tokens with cash.

In the second investment stage, the platform redeems its tokens and decides whether to fund a seller (given it observed the price $p_j$),

$$V^P(z^P_0, \tau^P_0; p(j)) = z^P_0 - \tau^P_0 + \max_{\tau^P_2, y^P_2 \in \{0, 1\}} (\tau^P_2 - y^P_2) + y^P_2 i(\theta) - \tau^P_2,$$

If the platform knows the seller is of type L, it chooses $y^P_2 = 0$ and $\tau^P_2 = 0$.

Market clearing

The markets payment means must clear at each stage of each period. We assume CBDC is purchased with cash. Notice that at the start of the first investment stage, buyers are holding the stock of cash and possibly CBDC. In the first investment stage, market clearing is

$$d_0 = d^B_0$$

$$\tau_0 = \tau^P_0$$

$$z_0 + e_0 + z^P_0 = \phi M = (z_2(t - 1) + e_2(t - 1)) \frac{1}{\beta}$$

where $\phi$ is the real value of money and $M$ is the nominal stock of money, and $e$ is the demand for CBDC. In the second investment stage, market clearing is

$$d_2 = d^B_2$$

$$\tau_2 = \tau^P_2$$

$$z_2 + e_2 = z_0 + e_0 + z^P_0$$

Here, only buyers demand cash, while buyers and the platform bring cash to the market. Because all payment instruments have the same rate of return (independent of their payment service), and thanks to the Friedman rule, all agents are indifferent as to which instrument they hold. Therefore, in this setup all the
analysis in the main text goes through.