Asset Encumbrance, Bank Funding and Fragility*

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Abstract

We propose a model of asset encumbrance by banks subject to rollover risk and study the consequences for fragility, funding costs, and prudential regulation. A bank's encumbrance choice trades off the benefit of expanding profitable investment, funded by cheap long-term senior secured debt, against the cost of greater fragility via runs on unsecured debt. We derive several testable implications about privately optimal encumbrance ratios. Deposit insurance or wholesale funding guarantees induce excessive encumbrance and fragility. To eliminate such risk-shifting incentives, policymakers can impose limits on asset encumbrance ex ante or Pigouvian taxes on encumbrance ex post. We use these normative implications to evaluate current policies in several jurisdictions.

Keywords: asset encumbrance, rollover risk, wholesale funding, fragility, runs, secured debt, unsecured debt, encumbrance limits, encumbrance surcharges.

JEL classifications: G01, G21, G28.

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1 Introduction

Banks attract secured funding by pledging assets on their balance sheet as collateral and encumbering, or ring-fencing, them so that they are unavailable to, and take precedent over the claims of, unsecured creditors in the event of a default. Following the Global Financial Crisis and the European sovereign debt crisis, many banks in Europe and the United States have become reliant on long-term senior secured debt, as investors have raced for safety and towards instruments that are excluded from writedowns in many resolution frameworks (Haldane, 2012; CGFS, 2013). The share of covered bonds in total gross bond issuance by euro area banks rose from 26% in 2007 to 42% in 2012 (Rixtel and Gasperini, 2013). The share of advances, which are akin to covered bonds, extended by the Federal Home Loan Bank (FHLB) system to large US banks rose from 23% in 2007 to 42% in 2017 (Gissler and Narajabad, 2017).[1]

Greater asset encumbrance levels raise positive and normative questions. Although senior secured debt instruments are extremely safe – no German covered bond has defaulted since 1901 and no FHLB advance has defaulted since 1932 – the relationship between asset encumbrance and rollover risk in unsecured debt markets is unclear. What determines a bank's encumbrance choice? How is this choice affected by policies aimed at reducing fragility, such as deposit insurance or wholesale funding guarantees? To the extent that such policies distort encumbrance choices, how should corrective measures be designed? And how do measures to limit asset encumbrance recently introduced in several jurisdictions fare against a normative benchmark?

[1]These collateralized debt instruments are distinct from securitization. In securitizations, the creditor is paid directly from the cash flow of the pledged asset. In default, the creditor has no recourse to the borrower's other assets. By contrast, with covered bonds and advances, creditors have recourse to the bank's unsecured assets in the event of default. Moreover, the bank is required to replenish impaired assets in the ring-fence to maintain its quality over time. A further distinction is that, with covered bond-type transactions, pledged assets remain on balance sheet for accounting purposes, whereas in securitizations, assets are transferred off-balance sheet to facilitate credit risk transfer and economize on capital requirements (Schwarcz, 2011; Acharya et al., 2013).

In this paper, we propose a model of asset encumbrance. Building on Rochet and Vives (2004) and Vives (2014), a representative bank is subject to rollover risk of unsecured debt. The bank has initial equity and seeks funding for profitable investments from a segmented funding market. Unsecured debt is issued to risk-neutral investors and secured debt is issued to risk-averse investors by encumbering assets in a bankruptcy-remote entity on the bank's balance sheet. When the balance sheet is subject to shocks, the effects are borne entirely by unsecured debt-holders, while secured debt-holders are insulated. The Modigliani-Miller theorem does not hold in our model. Costly liquidation of investment implies that the choice between debt and equity matters, while debt markets are segmented according to risk preferences.

Our analysis uses global games techniques (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005) to pin down a unique equilibrium. A run on unsecured debt occurs whenever the shock exceeds a threshold that depends on the value of unencumbered assets. We link the incidence of runs to the banks encumbrance choice and solve for the face values of secured and unsecured debt.

Asset encumbrance alters run dynamics by driving a wedge between the conditions for illiquidity and insolvency. If the bank prematurely liquidates assets to satisfy unsecured debt withdrawals, it can only use unencumbered assets since encumbered assets are pledged to secured debtholders. But if unsecured debt is rolled over, the bank can pay unsecured debtholders using residual encumbered assets once secured debtholders have been paid. This is possible because of the overcollateralization of the asset pool backing secured debt, which ensures that value of encumbered assets exceeds the face value of secured debt. While illiquidity only depends on unencumbered assets, insolvency depends on all assets. We show that the illiquidity condition is more binding than the insolvency condition if unsecured debt is cheap. Asset encumbrance can, therefore, make solvent banks illiquid and prone to runs. This result contrasts with Rochet and Vives (2004), where an illiquid bank is always insolvent.

Greater asset encumbrance ratios induce two opposing effects on fragility. The benefit of greater encumbrance is that it permits banks to raise a greater amount of cheap secured debt to finance profitable investment. The drawback is that too few unencumbered assets are available to meet unsecured debt withdrawals, exacerbating rollover risk. This latter effect becomes dominant if there is a large cost to recovering encumbered assets after bank failure. There is reason to believe these costs are large. Secured debtholders may have to assert their claims against unsecured debtholders in expensive and protracted legal proceedings (Ayotte and Gaon, 2011; Duffie and Skeel, 2012; Fleming and Sarkar, 2014). Access to critical infrastructure, such as risk management systems, may also be impeded in bankruptcy, reducing the realized value of encumbered assets (Bolton and Oehmke, 2016). Hence, more encumbrance heightens fragility. Consistent evidence includes Bennett et al. (2005), who show that more FHLB advances increase the default probability of US commercial banks, and Garcia et al. (2017), who show that greater covered bond issuance increases the default risk of European banks.

The model yields a rich set of testable implications that are consistent with existing evidence and can inform future empirical work. The bank's privately optimal encumbrance ratio balances marginal cost (greater fragility and a higher run probability) against the marginal benefit (greater profitability conditional on no run). Accordingly, greater encumbrance ratios arise when (a) investment is more profitable; (b) funding costs are lower; (c) the distribution of balance sheet shocks is more favorable; (d) unsecured debt is rolled over more frequently; (e) recovery costs on encumbered assets are lower; (f) liquidation values of investment are higher; (g) liquid reserves are higher, and (h) the risk premium of risk-neutral investors is lower. In addition, we find that the impact of bank capital on encumbrance ratios is ambiguous.

We turn to studying normative implications of asset encumbrance. In many advanced countries, public guarantee schemes for unsecured debtholders are an integral part of the financial system. While such privileges usually apply to retail depositors, they often extend to unsecured wholesale depositors during a financial crisis. In our model, the bank does not internalize the social cost of servicing the guarantee and so has an incentive to excessively encumber assets, which exacerbates financial fragility.[2]

Our model provides a natural framework with which to examine prudential safeguards against such excessive asset encumbrance. If the financial regulator observes asset encumbrance ex ante, limits on encumbrance ratios can achieve the social optimum. If encumbrance can only be observed ex post, however, risk-neutral Pigovian taxes on encumbrance ratios are also able to ensure the social optimum. A linear tax on encumbrance corrects the incentive of the bank to shift risk to the guarantee scheme, while rebating the revenue in lump-sum fashion ensures that there are sufficient resources to avoid excessive fragility. For taxes contingent on the face value of unsecured debt, we derive a closed-form solution for the optimal tax rate.

The normative implications of our model provide an analytical basis for regulatory measures on asset encumbrance taken in several jurisdictions. Some countries (e.g. Australia) have adopted a cap on encumbered assets similar to the analysis presented in this paper. In the United States, a cap is applied to the share of secured debt to total liabilities. Both measures are equivalent in our model. In Italy, encumbrance caps for banks are contingent on their capital ratios, with no limits for highly capitalized banks. Our analysis suggests that while such an approach may curb the incentive for lowly capitalized banks to encumber excessively, it does not reduce the incentives for highly capitalized institutions. Deposit insurance premia for

[2]Our model abstracts from deposit insurance premiums. In practice, these premiums are usually insensitive to encumbrance ratios, so banks have incentives to excessively encumber assets.

systemic Canadian banks partly reflect the extent to which their assets are encumbered (CDIC, 2017). This encumbrance surcharge can be considered as a form of the Pigovian taxation analysed in our paper.[3]

Our paper contributes to the literature on bank runs and global games (Morris and Shin, 2001; Goldstein and Pauzner, 2005). In the unique equilibrium, a run is the consequence of a coordination failure as bank fundamentals deteriorate. In particular, we build on Rochet and Vives (2004) where unsecured debtholders delegate their rollover decisions to professional fund managers, so the decisions to roll over are global strategic complements.[4] Our contribution is to introduce secured funding and to identify how asset encumbrance affects run risk and the pricing of unsecured debt.

An early contribution on bank funding choices is Greenbaum and Thakor (1987). In a setting with asymmetric information, banks with high-quality assets fund themselves with securitization, while banks with low-quality assets use deposits. In our model, by contrast, a bank uses a mix of unsecured and secured funding. This interaction of funding sources allows us to examine how asset encumbrance affects bank fragility and the role of prudential regulation. These issues are absent in the analysis of Greenbaum and Thakor (1987).

Our paper adds to a nascent literature on the interaction between secured and unsecured debt. In a corporate finance setting, Auh and Sundaresan (2015) study how short-term secured debt interacts with long-term unsecured debt. Ranaldo et al. (2017) studies short-term secured and unsecured debt in money markets, where shocks

[3]Minimum capital requirements sensitive to encumbrance ratios are also an effective tool in our model, though we are unaware of any jurisdiction with such a policy.

[4]Goldstein and Pauzner (2005) study one-sided strategic complementarity due to the sequential service constraint of banks (Diamond and Dybvig, 1983). Matta and Perotti (2017) contrast the sequential service constraint with the mandatory stay of illiquid assets and study its impact on run risk. Eisenbach (2017) shows how rollover risk from demandable debt effectively disciplines banks for idiosyncratic shocks, while a two-sided inefficiency arises for aggregate shocks.

to asset values lead to mutually reinforcing liquidity spirals. Our focus, by contrast, is the interaction between long-term secured and demandable unsecured debt.

Our paper also contributes to the policy debate on the financial stability implications of asset encumbrance. Gai et al. (2013) and Eisenbach et al. (2014) develop partial equilibrium models exploring the interplay between secured and unsecured funding. In a global-game setup, Gai et al. (2013) show that interim liquidity risk and encumbrance intertwine and can generate a 'scramble for collateral' by short-term secured creditors. Eisenbach et al. (2014) study a range of wholesale funding arrangements with exogenous creditor decisions. Their model suggests that asset encumbrance increases insolvency risk when the encumbrance ratio is sufficiently high.

2 Model

Our model builds on Rochet and Vives (2004) and Vives (2014). There are three dates $t = 0, 1, 2$, a single good for consumption and investment, and a large mass of investors. Each investor has a unit endowment at $t = 0$ and can store it until $t = 2$ at a gross return $r > 0$. Although investors are indifferent between consuming at $t = 1$ and $t = 2$, they differ in their risk preferences. A first clientele is risk-neutral, while a second is infinitely risk-averse. The latter group can be thought of large institutional investors mandated to hold high-quality safe assets, e.g. pension funds (IMF, 2012).

A representative risk-neutral bank has access to investments at $t = 0$ that mature at $t = 2$ with return $R > r$. Premature liquidation at $t = 1$ yields a fraction $\psi \in (0, 1)$ of the return. In order to consume at $t = 2$, the bank invests own funds, $E \geq 0$, and obtains funds from the segmented investor base by issuing unsecured demandable debt to risk-neutral investors and secured debt to risk-averse investors.

An exogenous amount of unsecured debt, $U \equiv 1$, is withdrawn at $t = 1$ or rolled over until $t = 2$.[5] Investors delegate the rollover decision to professional fund managers, $i \in [0, 1]$, who are rewarded for making the right decision. If the bank does not fail, a manager's payoff difference between withdrawing and rolling over is a cost $c > 0$. If the bank fails, the differential payoff is a benefit $b > 0$.[6] The conservatism ratio, $\gamma \equiv \frac{c}{b+c} \in (0, 1)$, summarizes these payoffs, with more conservative managers being less likely to roll over.[7] The face value of unsecured debt, D_U, is independent of the withdrawal date.

The banker attracts secured funding by encumbering a proportion $\alpha \in [0, 1]$ of assets and placing them in a bankruptcy-remote entity. Bankruptcy-remoteness ensures an exclusive claim of secured debtholders to encumbered assets. Let $S \geq 0$ be the amount of long-term secured funding and D_S its face value at $t = 2$. Table 1 shows the balance sheet at $t = 0$ after investment $I \equiv E + S + U$ and asset encumbrance.

Assets		Liabilities
(encumbered assets)	αI	S
(unencumbered assets)	$(1 - \alpha)I$	U
		E

Table 1: Balance sheet at $t = 0$ after funding, investment, and asset encumbrance.

The balance sheet is subject to a shock A at $t = 2$. This shock may enhance the value of assets, $A < 0$. But the crystallization of operational, market, credit or legal risks may require writedowns, $A > 0$. The shock has a continuous probability density function $f(A)$ and cumulative distribution function $F(A)$, with decreasing reverse

[5]Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability arises endogenously with liquidity needs (Diamond and Dybvig, 1983), as a commitment device to overcome an agency conflict (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), or to commit to providing a safe claim when investors seek safety (Ahnert and Perotti, 2018).

[6]As an example, suppose the cost of withdrawal is c; the benefit from getting the money back or withdrawing when the bank fails is $b + c$; the payoff for rolling over when the bank fails is zero.

[7]Reviewing debt markets during the financial crisis, Krishnamurthy (2010) argues that investor conservatism was an important determinant of short-term lending behavior. See also Vives (2014).

hazard rate $\frac{d}{dA}\frac{f(A)}{F(A)} < 0$ to ensure a unique equilibrium. Investors and the bank are protected by limited liability. Table 2 shows the bank's balance sheet at $t = 2$ for a small shock and when all unsecured debt is rolled over. Since encumbered assets are ring-fenced, the shock only affects unencumbered assets.[8] The value of bank equity at $t = 2$ is $E_2(A) \equiv \max\{0, RI - A - UD_U - SD_S\}$.

Assets		Liabilities
(encumbered assets)	$R\alpha I$	SD_S
(unencumbered assets)	$R(1-\alpha)I - A$	UD_U
		$E_2(A)$

Table 2: Balance sheet at $t = 2$ after a small shock and rollover of all unsecured debt.

If a proportion $\ell \in [0, 1]$ of unsecured debt is not rolled over at $t = 1$, the bank liquidates an amount $\ell\frac{UD_U}{\psi R}$ to meet withdrawals. A bank fails due to *illiquidity* and is closed early if the liquidation value of unencumbered assets is insufficient,

$$R(1-\alpha)I - A < \frac{\ell UD_U}{\psi}, \tag{1}$$

so the illiquidity threshold of the shock is $A_{IL}(\ell) \equiv R(1-\alpha)I - \frac{\ell UD_U}{\psi}$. Fund managers decisions exhibit strategic complementarity: an individual fund manager's incentive to roll over increases in the proportion of managers who roll over.

In the event of early closure, secured debtholders can recover encumbered assets. But recovery may be partial, reflecting legal difficulties in seizing collateral assets (Duffie and Skeel, 2012; Ayotte and Gaon, 2011), the inability of secured debtholders to properly redeploy these assets (Diamond and Rajan, 2001), or informational losses from the disruption to bank risk management systems (Bolton and Oehmke, 2016). Accordingly, the net return for secured debtholders is $\alpha\lambda R$, where $\lambda \in (0, 1)$ and $1-\lambda$

[8] Our modelling approach is consistent with the notion of collateral replenishment, whereby non-performing encumbered assets are replaced by performing assets from the unencumbered part of the balance sheet. Replenishment concentrates credit and market risks on unsecured creditors.

is the cost of recovering encumbered assets. Unsecured creditors are assumed to face a zero recovery rate upon bank failure.[9]

If the bank is liquid at $t = 1$, then its assets are worth $RI - \ell \frac{UD_U}{\psi} - A$ at $t = 2$. Upon repaying secured debtholders, the bank can use any residual encumbered assets (due to over-collateralization) to repay remaining unsecured debtholders. The bank fails due to *insolvency* at $t = 2$ if

$$RI - A - \frac{\ell UD_U}{\psi} < SD_S + (1 - \ell) UD_U. \tag{2}$$

The insolvency threshold of the shock is thus $A_{IS}(\ell) \equiv RI - SD_S - UD_U \left[1 + \ell \left(\frac{1}{\psi} - 1\right)\right]$.

At $t = 1$, each fund manager bases the rollover decision on a noisy private signal about the shock

$$x_i \equiv A + \epsilon_i, \tag{3}$$

where ϵ_i is idiosyncratic noise drawn from a continuous distribution H with support $[-\epsilon, \epsilon]$ for $\epsilon > 0$. Idiosyncratic noise is independent of the shock and i.i.d. across fund managers. Table 3 summarizes the timeline of events.

$t = 0$	$t = 1$	$t = 2$
1. Issuance of secured and unsecured debt	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
3. Asset encumbrance	3. Unsecured debt withdrawals	3. Debt repayments
	4. Consumption	4. Consumption

Table 3: Timeline of events.

[9] Our results are qualitatively unchanged if positive recovery rates for unsecured debt are assumed.

3 Equilibrium

We focus on the symmetric pure-strategy perfect Bayesian equilibrium and on threshold strategies for the rollover of unsecured debt. Each fund manager rolls over unsecured debt whenever the private signal indicates a healthy balance sheet, $x_i \leq x^*$.[10]

Definition 1. *The symmetric pure-strategy perfect Bayesian equilibrium comprises an encumbrance ratio* (α^*)*, an amount of secured debt* (S^*)*, face values of unsecured and secured debt* (D_U^*, D_S^*)*, and signal and shock thresholds* (x^*, A^*) *such that:*

a. at $t = 1$*, the rollover decisions of all fund managers,* x^**, are optimal and the run threshold* A^* *induces bank failure for any shock* $A > A^*$*, given the encumbrance ratio and secured debt* (α^*, S^*) *and face values of debt* (D_U^*, D_S^*)*;*

b. at $t = 0$*, the banker optimally chooses* (α^*, S^*) *given the face values of debt* (D_U^*, D_S^*)*, the participation of secured debtholders, and the thresholds* (x^*, A^*)*;*

c. at $t = 0$*, secured and unsecured debt are priced by binding participation constraints, given the choices* (α^*, S^*) *and the thresholds* (x^*, A^*)*.*

We construct the equilibrium in four steps. First, we price secured debt. Second, we derive the optimal rollover decision of fund managers. Third, we characterize the optimal asset encumbrance choice of the bank and, in so doing, obtain the endogenous level of secured debt issuance. In a final step, we price unsecured debt.

[10] Since we assume that private information is sufficiently precise relative to public information, the equilibrium is unique (Morris and Shin, 2003). It is also an extremal equilibrium that is in monotone strategies (Vives, 2005). Since the rollover decision is binary, our focus on threshold strategies is without loss of generality.

3.1 Pricing secured debt

Secured debtholders receives either the face value D_S or an equal share of the value of encumbered assets. Since early closure at $t = 1$ occurs for large balance sheet shocks, competitive pricing by infinitely risk-averse investors implies $r = \min\left\{D_S, \lambda\,\frac{R\alpha I}{S}\right\}$.[11]

Lemma 1. Asset encumbrance and secured debt. *Secured debt is cheap, $D_S^* = r$, and the maximum issuance tolerated by risk-averse investors is $S \leq S^*(\alpha) = \alpha\lambda z I^*(\alpha)$, where $z \equiv R/r$ is the relative return and $I^*(\alpha) = \frac{U+E}{1-\alpha\lambda z}$ is investment. Greater encumbrance increases secured debt and investment, $\frac{dS^*}{d\alpha} = \frac{dI^*}{d\alpha} = \frac{\lambda z I^*(\alpha)}{1-\alpha\lambda z} > 0$.*

A binding participation constraint equalizes the face value of secured debt with the outside option of investors. Since infinitely risk-averse investors evaluate secured debt at its worst outcome (the bank is closed early but encumbered assets are legally separated), the maximum level of secured debt increases in the encumbrance ratio.

3.2 Rollover risk of unsecured debt

Asset encumbrance and secured debt issuance have a fundamental impact on run dynamics. Figure 1a shows the illiquidity and insolvency thresholds, $A_{IL}(\ell)$ and $A_{IS}(\ell)$, without encumbrance and secured debt, $\alpha = S = 0$. We recover the dynamics in Rochet and Vives (2004) without liquid reserves. An illiquid bank at $t = 1$ is always insolvent at $t = 2$, so the insolvency threshold is the relevant condition for analysis.

Figure 1b shows the illiquidity and insolvency thresholds with encumbrance and secured debt. Over-collateralization means that the thresholds do not coincide at $\ell = 1$. Additional assets worth $R\alpha I^*(\alpha) - rS^*(\alpha) = R\alpha(1-\lambda)I^*(\alpha) > 0$ are

[11] Incentive compatibility constraints hold if investor types are unobserved. A risk-averse investor strictly prefers safe secured debt, while a risk-neutral investor weakly prefers risky unsecured debt.

available to serve unsecured debt withdrawals at $t = 2$, which are not available at $t = 1$ because of encumbrance. As a result, a bank that is illiquid at $t = 1$ can, nevertheless, be solvent at $t = 2$, that is $R\alpha(1-\lambda)I^*(\alpha) \geq (1-\ell)\,UD_U$. An upper bound on the face value of unsecured debt, $D_U \leq \hat{D}_U \equiv (1-\lambda)R\alpha I^*(\alpha)$, ensures that the illiquidity threshold is the relevant condition for analysis. We suppose that this condition holds and later verify that it does in equilibrium.

We focus on vanishing private noise about the balance sheet shock, $\epsilon \to 0$, so the rollover threshold converges to the run threshold, $x^* \to A^*$.

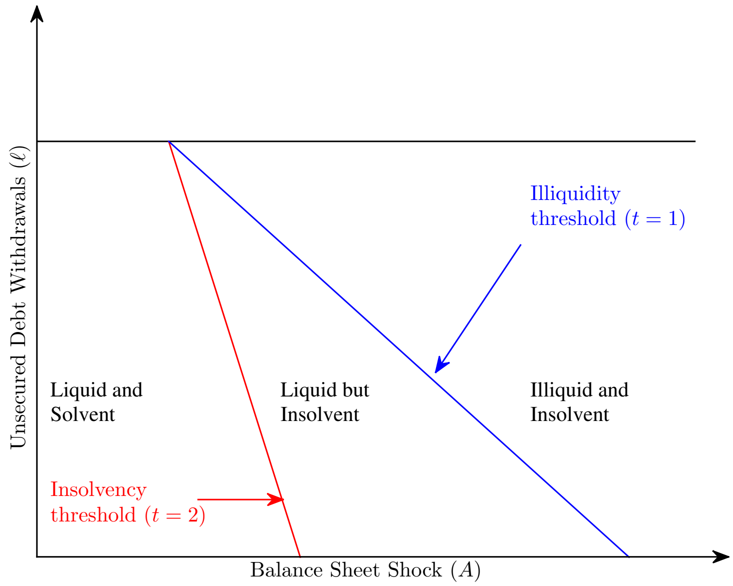
Proposition 1. Run threshold. *There exists a unique run threshold*

$$A^* \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma\,UD_U}{\psi}. \tag{4}$$

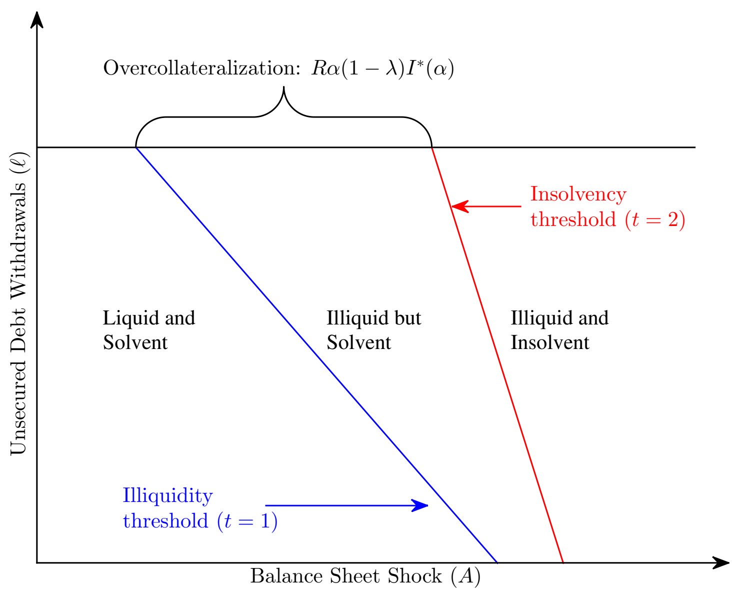
All fund managers withdraw and the bank closes early if and only if $A > A^*$.

Proof. See Appendix A.1. ■

Proposition 1 pins down the unique incidence of an unsecured debt run using global games methods. More conservative fund managers decrease the run threshold, $\frac{\partial A^*}{\partial \gamma} < 0$. A higher return on investment increases the value of unencumbered assets and the amount of secured debt raised for a given encumbrance ratio. Both effects act to reduce run risk, so $\frac{\partial A^*}{\partial R} > 0$. Higher liquidation values decrease the extent of strategic complementarity among fund managers and increases the issuance of secured debt for given encumbrance. Both these effects also lower run risk, $\frac{\partial A^*}{\partial \psi} > 0$. Similarly, a decrease in the cost of recovering encumbered assets after early closure of the bank (higher λ) enables more secured funding for given encumbrance. So the stock of unencumbered assets increases and run risk decreases, $\frac{\partial A^*}{\partial \lambda} > 0$. A higher cost of funding decreases the amount of secured debt for given encumbrance and thereby



(a) Without asset encumbrance ($\alpha = S = 0$)



(b) With asset encumbrance

Figure 1: Run dynamics without and with asset encumbrance.

lowers the value of unencumbered assets, $\frac{\partial A^*}{\partial r} < 0$. Increased bank capital reduces run risk via its effect on increased investment and unencumbered assets, $\frac{\partial A^*}{\partial E} > 0$.

Lemma 2 links bank fragility to secured debt issuance and the recovery of encumbered assets after early closure.

Lemma 2. Encumbrance and fragility. *Asset encumbrance affects bank fragility according to*

$$\frac{dA^*}{d\alpha}\big(\lambda z - 1\big) \geq 0, \tag{5}$$

with strict inequality whenever $\lambda z \neq 1$. More encumbrance increases fragility when the cost of recovering encumbered assets is high, $\lambda z < 1$.

Proof. See Appendix A.1. ■

Greater encumbrance affects the run threshold in two opposing ways. First, for given investment, greater encumbrance reduces the stock of unencumbered assets and increases fragility. Second, greater encumbrance allows the bank to issue more secured debt, which increases investment. As a result, the stock of unencumbered assets increases and run risk is reduced. Whether the second effect dominates depends on the cost of recovering encumbered assets, $1 - \lambda$. If the cost is high, the bank must encumber more than one unit of assets for each unit of secured funding raised. In this case, the stock effect dominates and greater encumbrance increases fragility. Conversely, if recovery is cheap, the banker is able to encumber less than a unit of assets per unit of secured funding – so greater encumbrance reduces fragility.

In what follows, we suppose a high cost of recovering encumbered assets, $\lambda z < 1$. Empirical evidence consistent with this assumption includes Bennett et al. (2005), who show that FHLB advances increase the default probability of U.S. banks, and Garcia et al. (2017), who show that covered bond issuance increases the default risk of European banks. The inability to fully recover encumbered assets also reflects legal reasons – secured senior debtholders often face costly and protracted proceedings in court to recover collateral assets upon bank failure (Duffie and Skeel, 2012). Fleming and Sarkar (2014) highlight how even special-status secured financial instruments (qualifying for so-called 'safe harbor' provisions) are not immune from legal logjams.[12]

3.3 Optimal asset encumbrance and secured debt issuance

The bank encumbers assets to maximize expected equity value, taking as given the face value of unsecured debt D_U, the run threshold $A^*(\alpha)$, and the (maximum) amount of secured debt, $S \leq S^*(\alpha)$. Since more secured debt for a given encumbrance ratio increases expected equity value and lowers fragility, we obtain $S^* = S^*(\alpha)$. Thus:

$$\max_{\alpha} \pi \equiv \int_{-\infty}^{\infty} E_2(A) dF(A) = \int_{-\infty}^{A^*(\alpha)} \left[RI^*(\alpha) - UD_U - S^*(\alpha)\, r - A \right] dF(A). \quad (6)$$

Figure 2 shows the relationship between encumbrance and expected equity value and the unique interior encumbrance ratio for given face value of debt.[13]

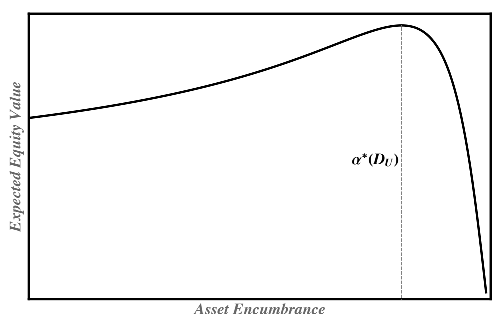


Figure 2: Unique optimal encumbrance ratio.

[12]Fleming and Sarkar document the protracted legal settlements following the failure of Lehman. Although safe harbor provisions allowed retail counterparties to terminate contracts upon bankruptcy, the settlement of their claims on encumbered assets remained incomplete for several years.

[13]Throughout we use the numerical example of $R = 1.5$, $r = 1.1$, $E = 0.5$, $\psi = 0.6$, $\lambda = 0.66$, $\gamma = 0.8$, exogenous $D_U = 3.3$, and the shock is normally distributed with mean -3 and unit variance.

Proposition 2. Asset encumbrance schedule. *There is a unique encumbrance schedule,* $\alpha^*(D_U)$*. If fund managers are sufficiently conservative,* $\gamma > \psi$*, then the schedule decreases in the face value of unsecured debt,* $\frac{d\alpha^*}{dD_U} \leq 0$*, and an interior solution for* $\underline{D}_U < D_U < \overline{D}_U$ *is implicitly given by:*

$$\frac{F(A^*(\alpha^*))}{f(A^*(\alpha^*))} = \frac{(1-\lambda z)}{\lambda(z-1)} \left[RI^*(\alpha^*)\alpha^*(1-\lambda) + UD_U \left(\frac{\gamma}{\psi} - 1 \right) \right]. \tag{7}$$

Proof. See Appendix A.2. ■

The bank balances the marginal benefits and costs of asset encumbrance when choosing the privately optimal encumbrance ratio. The marginal benefit is an increase in the amount of secured funding raised. Since secured debt is cheap and investments are profitable, the equity value of the bank – in the absence of an unsecured debt run – is higher. The marginal cost is greater fragility, that is a higher probability of an unsecured debt run. So a higher face value of unsecured debt exacerbates rollover risk and lowers the run threshold, inducing the bank to encumber fewer assets.

3.4 Pricing of unsecured debt

Repayment of unsecured debt depends on the balance sheet shock. Absent a run, $A < A^*$, unsecured debtholders receive face value D_U, while for larger shocks, $A > A^*$, they receive zero in bankruptcy. The value of an unsecured debt claim is $V \equiv D_U\, F(A^*)$ and competitive pricing equates it with the cost of funding for any encumbrance ratio,

$$r = D_U^*\, F(A^*(\alpha, D_U^*)). \tag{8}$$

Proposition 3. Private optimum. *If bank capital is scarce, $E < \bar{E}$, and managers are conservative, $\gamma \geq \underset{\sim}{\gamma}$, then there exists a unique face value of unsecured debt, $D_U^* > r$. If funding is costly, $r > \underset{\sim}{r}$, asset encumbrance is interior, $\alpha^{**} \equiv \alpha^*(D_U^*) \in (0,1)$.*

Proof. See Appendix A.3. ■

Figure 3 shows the privately optimal allocation and its construction. The condition $\gamma \geq \underset{\sim}{\gamma}$ ensures that the schedule $D_U^*(\alpha)$, derived from the pricing of unsecured debt, is upward-sloping in the vicinity of the encumbrance schedule. Consistently, Garcia et al. (2017) document that spreads on unsecured debt increase in covered bond issuance of European banks. The upward-sloping schedule, in turn, leads to a unique characterization of the joint equilibrium for the encumbrance ratio, α^{**}, and the face value of unsecured debt, D_U^{**}. Next, scarce bank capitalization $E < \bar{E}$ ensures that $D_U^* < \hat{D}_U(\alpha^*)$, so the illiquidity condition is more binding that the insolvency condition, as supposed. Finally, the lower bound on the cost of funding ensures a face value of unsecured debt high enough for an interior encumbrance ratio.

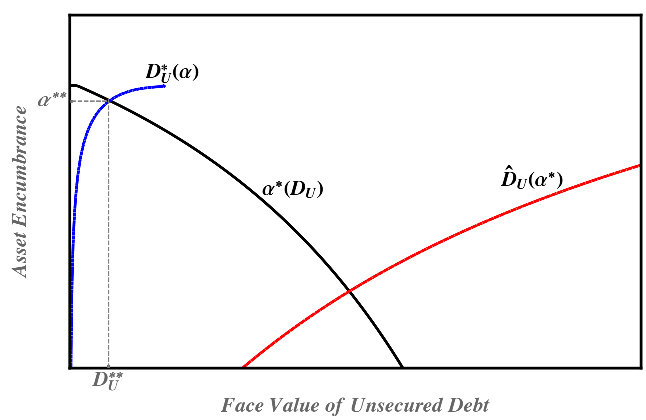


Figure 3: Privately optimal asset encumbrance and face value of unsecured debt.

4 Testable Implications

Our model yields a rich set of comparative static results and testable implications. Parameter changes affect the unique interior equilibrium in two ways. For a given face value of unsecured debt, the bank trades off heightened fragility against more profitable investment funded with cheap secured debt. The face value of unsecured debt required for investor participation also changes with underlying parameter values.

Proposition 4. Comparative Statics. *The privately optimal encumbrance ratio* α^{**} *decreases in funding costs* r*, conservatism of fund managers* γ*, and the costs of recovering encumbered assets,* $1-\lambda$*. Encumbrance increases in investment profitability* R*, the liquidation value* ψ*, and improvements in the shock distribution* F *according to reverse hazard rate dominance. The effect of higher bank capital* E *is ambiguous.*

Proof. See Appendix A.4. ■

Lower funding costs increases the benefits of asset encumbrance. Since the required face value of unsecured debt is also lowered, the two effects combine to increase encumbrance. Empirical evidence consistent with this result includes Meuli et al. (2016) who find that issuance of Swiss covered bonds (Pfandbriefe) between 1932–2014 was lower when interest rates were high.[14]

Krishnamurthy (2010) documents the unwillingness of fund managers to roll over unsecured debt during the global financial crisis. The conservatism parameter γ can thus be interpreted as a measure of market stress. A deterioration in investor sentiment renders the bank more fragile for a given encumbrance ratio. So the bank induces rollovers by fund managers by forgoing secured debt issuance and lowering

[14] Low funding costs may also reflect looser monetary conditions. Juks (2012) and Bank of England (2012) document an increasing trend in the encumbrance ratios of Swedish and UK banks following the extraordinary monetary policy measures in the aftermath of the global financial crisis.

the extent of encumbrance. Our model therefore predicts that increased stress in unsecured markets lowers the share of secured debt (as a proportion of total debt).

A lower cost of recovering encumbered assets implies that, for each unit of secured funding, the bank has to encumber fewer assets. In addition, more unencumbered assets are available for interim withdrawals lowering fragility and, in turn, reducing the face value of unsecured debt. Taken together, encumbrance increases.

Proposition 4 implies that banks with more profitable assets choose to encumber more. Consistent with this, Stojanovic et al. (2008) show that U.S. commercial banks with higher returns on assets were more likely to become FHLB members over the period 1992–2005. Membership proxies for the use of advances on the extensive margin. The evidence on the intensive margin is inconclusive as Ashley et al. (1998) report no significant effect of return on assets on FHLB advances during 1985–1991.

A more favorable distribution of the balance sheet shock reduces fragility and induces greater encumbrance. Evidence consistent with this result includes Banal-Estanol et al. (2017) who find a negative association between CDS spreads, a measure of bank risk, and the asset encumbrance ratios of European banks. Ashley et al. (1998) document how, on the intensive margin, larger ratios of deferred loan losses reduced FHLB advances. Stojanovic et al. (2008) also state that, on the extensive margin, greater interest rate risk and riskier lending reduced the likelihood of U.S. commercial banks becoming a FHLB member.

The effect of higher bank capital on asset encumbrance is ambiguous. More capital allows the bank to withstand larger shocks, lowering fragility. While this 'loss absorption' effect induces greater encumbrance, it also means that the bank risks losing more of its own funds in bankruptcy. The effect of such 'greater skin in the game' is to lower encumbrance. The net result is a non-monotonic relationship between bank capital and asset encumbrance. The relationship between the capital ratio and encumbrance (not plotted) is also non-monotonic. Existing evidence in Ashcraft et al. (2010), Ashley et al. (1998) and Stojanovic et al. (2008) does not suggest a clear relationship between bank capital and FHLB advances.[15]

5 Guarantees and Excessive Encumbrance

We proceed to examine the relationship between asset encumbrance and guarantee schemes. Such schemes, which typically apply to retail depositors, often extend to wholesale unsecured debtholders during times of crisis. While such measures aim to reduce bank fragility ex post, they distort behavior ex ante. In our model, by externalizing the costs of the guarantee upon failure, the bank has incentives to excessively encumber assets that, in turn, creates excessive fragility.

Let $0 < m \leq \widehat{m}$ be the fraction of unsecured debt that is fully and credibly guaranteed.[16] Guaranteed debt is always rolled over.[17] It has face value D_G and ℓ is the withdrawal proportion of non-guaranteed unsecured debt. The bank is illiquid at $t = 1$ if $R(1-\alpha)I - A \leq \frac{\ell(1-m)UD_U}{\psi}$ and insolvent at $t = 2$ if $RI - A - \frac{\ell(1-m)UD_U}{\psi} \leq SD_S + (1-\ell)(1-m)UD_U + mUD_G$. Since guaranteed debt is safe, $D_G^* = r$. From Lemma 1, $D_S^* = r$, $S^* = S^*(\alpha)$, and $I^* = I^*(\alpha)$.[18] And the illiquidity and insolvency

[15] On the extensive margin, Stojanovic et al. (2008) find that higher equity ratios reduce the likelihood of FHLB membership – a negative association between capitalization and encumbrance. On the intensive margin, Ashley et al. (1998) associate lower capital ratios with greater FHLB advances. In contrast, Ashcraft et al. (2010) find that the funding ratio is negatively associated with FHLB advances. Since the capital ratio and the funding ratio should add up to one, their result suggests a positive association between changes in advances and the equity ratio.

[16] The bound $\widehat{m} < 1$ reflects the fiscal capacity of the guarantor (König et al., 2014) or the minimum amount of demandable debt required to control bank moral hazard (Rochet and Vives, 2004).

[17] We follow Allen et al. (2015) and consider guarantees that eliminate both inefficient and efficient runs. Such guarantee schemes better resemble real-world deposit guarantees.

[18] The upper bound on the face value of unsecured debt is less restrictive with guarantees. So the conditions previously imposed continue to suffice for the illiquidity threshold to be more binding.

thresholds in the presence of guarantees are:

$$A_{IL}(\ell) = R(1-\alpha)I^*(\alpha) - \ell\frac{(1-m)UD_U}{\psi}, \tag{9}$$

$$A_{IS}(\ell) = R(1-\alpha\lambda)I^*(\alpha) - UD_U(1-m)\left(1+\ell\left[\frac{1}{\psi}-1\right]\right). \tag{10}$$

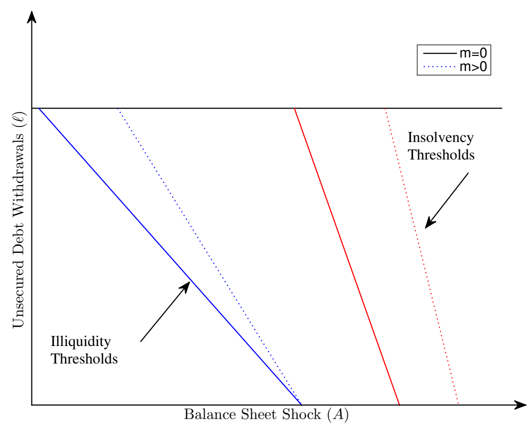


Figure 4: Run dynamics with guarantees: thresholds as functions of the withdrawal proportion without (solid) and with (dashed) guarantees. Guarantees reduces the responsiveness of both thresholds to the withdrawal proportion. The insolvency threshold shifts outwards, while the illiquidity threshold pivots outwards.

Figure 4 depicts run dynamics with guarantees. A run on unsecured non-guaranteed debt occurs whenever $A > A^*_m \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma(1-m)UD_U}{\psi}$. The direct effect of the guarantee is to achieve its intended purpose of increasing the run threshold, $\frac{\partial A^*_m}{\partial m} = \frac{\gamma UD_U}{\psi} > 0$. But the guarantee impacts the bank's incentive to encumber assets as well as the face value of unsecured debt. The bank's problem is now:

$$\alpha^*_m \equiv \max_{\alpha} \pi_m(\alpha) = \int_{-\infty}^{A^*_m(\alpha)} \left[RI^*(\alpha) - S^*(\alpha)\,r - (1-m)UD_U - mUr - A\right] dF(A). \tag{11}$$

Proposition 5. Privately optimal encumbrance with guarantees. *The equilibrium with guarantees is unique and has a higher encumbrance ratio, $\alpha^{**}_m > \alpha^{**}$.*

Proof. See Appendix A.5. ■

Guarantees reduce the stock of non-guaranteed debt that may be withdrawn and, therefore, run risk. This has two effects on encumbrance. First, the bank has greater incentives to encumber assets – an outward shift of the encumbrance schedule $\alpha^*_m(D_U)$. Second, unsecured debt is repaid more often. This reduces the face value of unsecured debt and shifts the participation constraint of risk-neutral investors $D^*_U(\alpha)$ inwards. In sum, guarantees unambiguously increase the encumbrance ratio, α^{**}_m.

Since the bank ignores the social cost of providing the guarantee, its incentives to encumber assets are distorted. In contrast, a social planner accounts for these costs, mUr, incurred upon bank failure with probability $1 - F(A^*_m)$. The planner takes as given the incomplete information structure of the model and chooses an encumbrance schedule for a given face value of non-guaranteed unsecured debt:[19]

$$\alpha^*_P(D_U) \equiv \max_{\alpha} \quad \pi_m(\alpha) - \left[1 - F(A^*_m(\alpha))\right] mUr. \tag{12}$$

Even though the encumbrance schedules of the bank and planner differ, $\alpha^*_P(D_U) \leq \alpha^*_m(D_U)$, the participation constraint of unsecured debtholders is the same, $\tilde{D}^*_U(\alpha)$. Any difference in allocations is, therefore, due to differences in encumbrance schedules. The face value of unsecured debt chosen by the planner D^{**}_P solves the fixed point problem $D^{**}_P \equiv D^*_U(\alpha^*_P(D^*_U))$. Figure 5 shows the private and social optimums.

Proposition 6. Excessive encumbrance. *The privately optimal encumbrance ratio and the face value of unsecured debt are excessive, $\alpha^{**}_m > \alpha^{**}_P$ and $D^{**}_m > D^{**}_P$.*

Proof. See Appendix A.6. ■

[19] The payoffs to fund managers do not enter the planner's objective function. This approach is consistent with taking the limits of $b \to 0$ and $c \to 0$ subject to a constant ratio $\gamma = \frac{c}{b+c}$. Moreover, including the payoffs would add $bF(A^*)$ as each fund manager refuses to roll over unsecured debt exactly when the bank fails (due to vanishing noise). As a result, the gap between the privately and socially optimal encumbrance ratios would actually increase, strengthening our results on regulation.

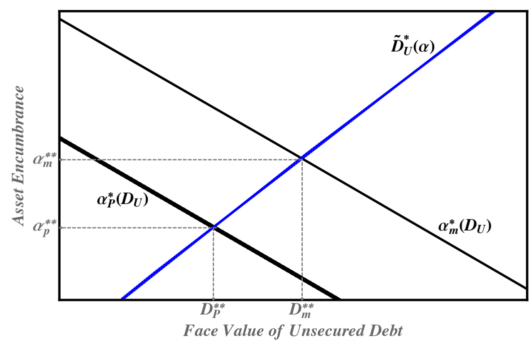


Figure 5: Excessive encumbrance and fragility with guarantees.

Since guarantees are a structural feature of many financial systems, it is worth exploring policy tools that seek to curb excessive encumbrance and fragility within such an environment. Prudential safeguards include explicit limits on encumbrance ($\alpha \leq \bar{\alpha}$) and revenue-neutral linear Pigovian taxes on encumbrance ($\tau \geq 0$). Let $\alpha_R^*(D_U)$ denote the bank's optimal encumbrance schedule subject to regulation.

Proposition 7. Prudential regulation. *For given a guarantee m, a planner achieves the social optimum $(\alpha_P^{**}, D_P^{**})$ by imposing:*

a. a limit on asset encumbrance at α_P^{**};

b. a contingent linear tax on asset encumbrance imposed at $t = 2$, combined with a lump-sum rebate of the generated revenue, $T = \alpha\tau$. The optimal rate is

$$\tau^*(D_U) = \frac{(1 - \lambda z)RI^*(\alpha_P^*)Umrf(A_m^*(\alpha_P^*))}{(1 - \lambda z\alpha_P^*)F(A_m^*(\alpha_P^*))}, \tag{13}$$

which depends on the face value of unsecured non-guaranteed debt.

c. a linear tax on asset encumbrance at $t = 2$ that is not contingent on the face value of debt, combined with a lump-sum rebate of the generated revenue. That is, there exists a unique rate $\tau^* > 0$ such that $\alpha_R^{**}(\tau^*) = \alpha_P^{**}$ and $D_R^{**}(\tau^*) = D_P^{**}$.

Proof. See Appendix A.7. ■

Figure 6 shows the impact of an encumbrance limit. Imposing $\alpha \leq \alpha_P^{**}$, the bank's constrained encumbrance schedule for a given face value of debt is

$$\alpha_R^*(D_U) \equiv \begin{cases} \alpha_m^*(D_U) & \text{if } \alpha_m^*(D_U) < \alpha_P^{**} \\ \alpha_P^{**} & \text{if } \alpha_m^*(D_U) \geq \alpha_P^{**}. \end{cases} \tag{14}$$

The bank chooses the socially optimal encumbrance ratio and, therefore, unsecured debt is also priced at the socially optimal level. When the planner can directly control encumbrance at $t = 0$, an encumbrance limit is effective.

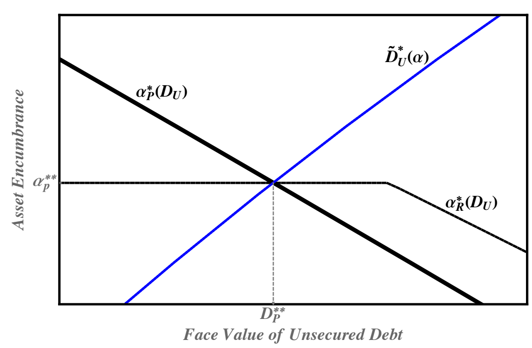


Figure 6: A limit on asset encumbrance avoids excessive encumbrance and fragility.

This encumbrance limit can also be implemented via bank capital regulation. In the model, the bank's capital ratio at $t = 0$, $e \equiv e(\alpha) = \frac{E}{I^*(\alpha)}$, is sensitive to changes in the encumbrance ratio, that is $\frac{de}{d\alpha} < 0$. A minimum capital ratio, $e \geq \underline{e}$, translates to a limit on encumbrance, so the social optimum can be achieved with the capital requirement $e \geq \underline{e}(\alpha_P^{**})$. The argument generalizes to risk-based capital requirements if encumbered and unencumbered assets are assigned different risk-weights.

A limitation of cups on encumbrance is that they require the planner to observe the bank's encumbrance at $t = 0$. As an alternative, we consider a revenue-neutral policy that imposes a linear tax τ on encumbrance at $t = 2$, combined with a lump-sum rebate $T = \tau\alpha$. Higher taxes reduce the privately optimal encumbrance ratio, since the bank's equity value (contingent no run) decreases in encumbrance. When the tax rate can be made contingent on the face value of non-guaranteed debt, the optimal tax rate $\tau^*(D_U)$ ensures that the privately and socially optimal encumbrance schedules are aligned. This policy attains the social optimum (Figure 7).

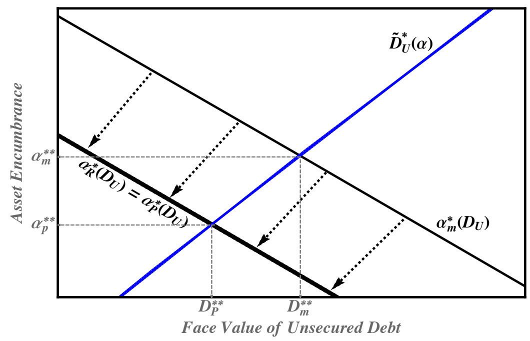


Figure 7: A linear contingent tax on encumbrance with full lump-sum rebate achieves the social optimum by aligning socially and privately optimal encumbrance schedules.

When the tax rate cannot be contingent on the face value of debt, the privately and socially optimal encumbrance schedules do not align (Figure 8). But since a higher tax rate lowers encumbrance, there exists a unique rate at which the social optimum is achieved. Graphically, the private encumbrance schedule shifts inward following the marginal tax on encumbrance so as to intersect with the social optimum.

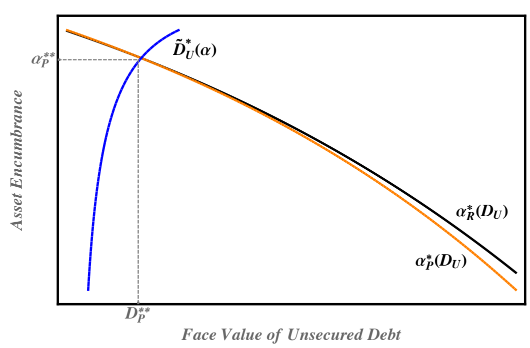


Figure 8: A linear non-contingent lump-sum rebated tax achieves the social optimum.

Our results on prudential regulation are relevant to the policy debate on asset encumbrance. Increasingly, policymakers are expressing concern that the increased collateralization of bank balance sheets may heighten fragility (Haldane, 2012; CGFS, 2013). In many jurisdictions, concerns about excessive encumbrance has resulted in explicit restrictions that apply either (a) through limits on asset that can be pledged when secured debt is issued; or (b) via limits on bond issuance. Table 4 summarizes.

Country	Policies	
	Assets	Liabilities
Australia	8%	
Belgium	8%	
Canada	4%	
Italy	$\begin{cases} 25\% \text{ of assets if } 6\% \leq CET1 < 7\% \\ 60\% \text{ of assets if } 7\% \leq CET1 < 8\% \\ \text{No Limit} \quad \text{if } CET1 \geq 8\% \end{cases}$	
Netherlands	Determined on a case-by-case basis so ratio is 'healthy'	
United States		4%

Table 4: Caps on asset encumbrance across countries

While most countries have opted for limits on encumbered assets, the United States limits the share of secured debt to total liabilities. In our model, these asset and liability-side restrictions are equivalent. In Italy, the encumbrance limit depends on a bank's capital ratio (Core Equity Tier 1), with less capitalized banks facing stricter encumbrance limits. While this approach curbs the incentive to excessively encumber for banks with low capital, it does not reduce the incentives for highly capitalized banks (Proposition 6). Our results suggest that banks of all capital levels should be subject to some limits on encumbrance.

In our model, the optimal level α_P^{**} is sensitive to economic and financial conditions (Proposition 6), so policymakers have to set asset encumbrance limits that vary over time. In the Netherlands, for example, the cap is set on a case-by-case basis for individual banks, taking into account the financial position, solvency risk of the issuing bank, its risk profile, and the riskiness inherent in its assets (DNB, 2015).

A form of Pigovian taxation is also used in Canada to supplement asset encumbrance limits. The deposit insurance premiums levied by the Canadian Deposit Insurance Corporation on systemically important domestic banks reflects the extent to which balance sheets are encumbered. Specifically, 5% of the score used to calculate the premium reflects encumbrance considerations (CDIC, 2017). The surcharge is not revenue neutral and can be viewed as a non-contingent tax on encumbrance.

Next, we consider the case of a regulator who chooses guarantee coverage to minimize fragility, that is to maximize the run threshold A^*.

Proposition 8. Optimal guarantee coverage. *If the recovery value of encumbered assets is sufficiently low, $\lambda \leq \bar{\lambda} \equiv \frac{1}{2z-1}$, the fragility-minimizing coverage is $m^* = \widehat{m}$.*

Proof. See Appendix A.8. ■

There is a positive total effect of greater guarantee coverage on the run threshold. It can be decomposed into a direct effect and two indirect effects. The direct effect is positive and stems from a lower share of non-guaranteed debt. The indirect effects include changes to the encumbrance schedule and the face value of (non-guaranteed) unsecured debt. The former effect is negative but small for low recovery values, while the latter is positive. For a low recovery value, the bank can raise only a small amount of secured debt per unit of encumbered assets, providing few incentives to encumber. A low recovery value also implies greater fragility, further reducing the incentives to encumber. Following a marginal increase in coverage, the bank trades off a low benefit from raising secured debt against a high cost through greater fragility. Overall, encumbrance increases by a small amount and fragility is lower.

6 Additional Implications

In this section, we derive the implications of (i) liquid reserves held by banks; (ii) a risk premium; (iii) the precision of private information about the balance sheet shock.

6.1 Liquid reserves

The model also sheds light on how a bank's liquid reserves shape run dynamics and encumbrance choice. Suppose, as in Rochet and Vives (2004), the bank has exogenous liquid reserves $L \geq 0$ to serve interim withdrawals, thereby lowering the amount of liquidated unencumbered assets. The return on liquid reserves is normalized to one and is below the return on investment, $R > 1$, so it is never optimal to encumber liquid reserves. Table 5 shows the bank's balance sheet at $t = 0$ (now $I + L = S + U + E$).

Assets		Liabilities
(encumbered assets)	αI	S
(unencumbered assets)	$(1-\alpha)I$	U
(liquid reserves)	L	E

Table 5: Balance sheet at $t = 0$ with liquid reserves.

There are three cases. If $\ell U D_U > L + \psi[(1-\alpha)I - A]$, the bank is illiquid and closes early. If $L < \ell U D_U \leq L + \psi[(1-\alpha)I - A]$, the bank is liquid at $t = 1$ and liquidates the amount $\frac{\ell U D_U - L}{\psi}$ to serve withdrawals. The bank is insolvent at $t = 2$ if $RI - A - \frac{\ell U D_U - L}{\psi} < SD_S + (1-\ell)UD_U$. If $L \geq \ell U D_U$, the bank is 'super-liquid' and has enough reserves to serve all interim withdrawals. A super-liquid bank is insolvent at $t = 2$ if[20]

$$RI + L - A < UD_U + SD_S. \tag{15}$$

[20] More liquid reserves reduces investment and increases insolvency risk (König, 2015).

Figure 9 shows the run dynamics with liquid reserves and depicts the thresholds as functions of the balance sheet shock A and the withdrawal proportion ℓ. Unlike Figure 1b, there is now a 'super-liquidity' region. If the bank is super-liquid, the relevant critical mass condition is insolvency at $t = 2$. Otherwise, the relevant condition is illiquidity at $t = 1$ (as in the benchmark model).

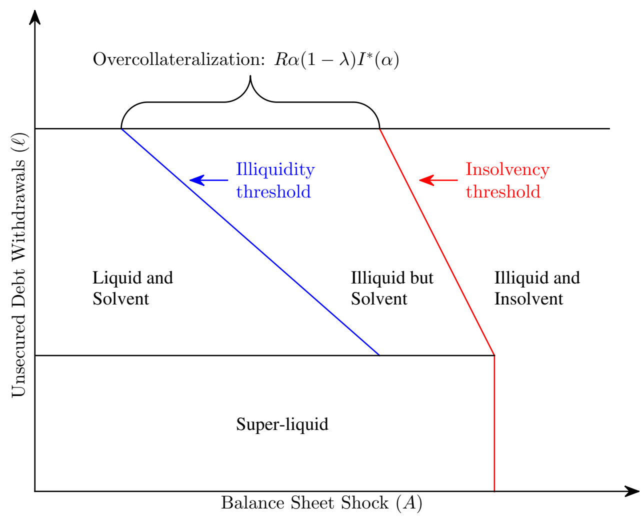


Figure 9: Run dynamics with liquid reserves.

Proposition 9. Runs with liquid reserves. *There exists a unique run threshold:*

$$A^* = \begin{cases} A_1^* \equiv R(1-\alpha)I^*(\alpha) - \frac{\gamma U D_U - L}{\psi} & \gamma > \frac{L}{UD_U} \\ & \text{if} \qquad , \\ A_2^* \equiv RI^*(\alpha) + L - UD_U - S^*(\alpha)r & \gamma \leq \frac{L}{UD_U} \end{cases} \tag{16}$$

where $I^*(\alpha) = \frac{U+E-L}{1-\alpha\lambda z}$ *and* $S^* = \lambda z \alpha I^*$. *More liquid reserves increase fragility of a superliquid bank,* $\frac{dA_2^*}{dL} < 0$. *If* $R < \frac{1}{\psi}$, *they reduce fragility of an illiquid bank,* $\frac{dA_1^*}{dL} > 0$.

Proof. See Appendix A.9. ■

Greater liquid reserves have the opportunity cost of a higher investment return at the final date, R, but can also reduce the required amount of liquidation at the interim date, earning an implicit return $\frac{1}{\psi}$. For a bank with abundant reserves, more reserves always increase fragility since the marginal benefit is zero. By contrast, for a bank with few liquid reserves, more reserves are not idle and reduce fragility when the liquidation cost savings exceed the opportunity cost.

Restricting attention to a bank with scarce liquidity in order to facilitate comparison with the benchmark model, we can show a result on the encumbrance choice:

Proposition 10. Asset encumbrance choice with liquid reserves. *For scarce bank liquidity, $L \leq \gamma U r$, the unique encumbrance schedule is interior and given by:*

$$\frac{F(A_1^*(\alpha^*))}{f(A_1^*(\alpha^*))} = \frac{1-\lambda z}{\lambda(z-1)} \left[RI^*(\alpha^*)\alpha^*(1-\lambda) - L\left(\frac{1}{\psi}-1\right) + UD_U\left(\frac{\gamma}{\psi}-1\right)\right]. \quad (17)$$

The privately optimal level of asset encumbrance increases in liquid reserves, $\frac{d\alpha^*}{dL} > 0$.

Proof. See Appendix A.9 ■

In other words, greater liquid reserves lower fragility and induce greater encumbrance. Ashcraft et al. (2010) document evidence consistent with this implication.

6.2 Risk premium on unsecured debt

We next establish a testable implication about the risk premium on unsecured debt and the encumbrance ratio. Suppose risk-neutral investors have access to a menu of risk-free and risky storage. The return on risk-free storage continues to be r but the expected return on risky storage is $\tilde{r} > r$, implying a risk premium of $p \equiv \tilde{r} - r$. Since the risk premium leaves the pricing of secured debt, the illiquidity condition, and the encumbrance schedule unaffected, $D_S^* = r$, $S^*(\alpha)$, $A^*(\alpha)$, and $\alpha^*(D_U)$ are unchanged. But an increase in the risk premium impacts the pricing of unsecured debt, shifting the schedule $D_U^*(\alpha; p)$ outwards (Figure 10). Proposition 11 summarizes.

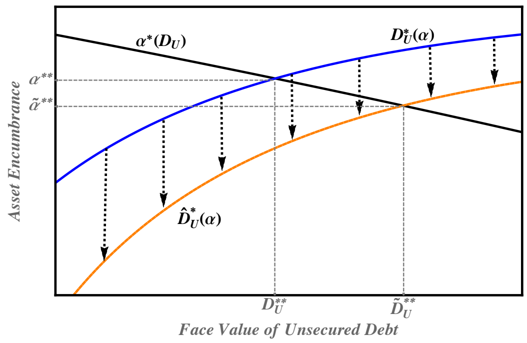


Figure 10: A higher risk premium reduces encumbrance and increases the face value.

Proposition 11. Risk premium. *A higher risk premium p lowers the privately optimal encumbrance ratio,* α^{**}*, and increases the face value of unsecured debt,* D_U^{**}*.*

6.3 Limited precision of information and liquidity support

For infinitely precise private information about the balance sheet shock, $\epsilon \to 0$, the mass of fund managers who withdraw is a step function, $\ell^*(A, x) = \mathbf{1}_{\{\mathbf{A}>\mathbf{A}^*\}}$. If the bank's insolvency and illiquidity lines are sufficiently close, $\ell^*(A, x)$ crosses both curves at the same level A^*, so the illiquidity and insolvency thresholds coincide. For finite precision, by contrast, the mass of fund managers who withdraw is $\ell^*(A, x^*) = H(x^* - A)$ and the illiquidity and insolvency thresholds differ (Figure 11).

For values of the shock between the illiquidity and insolvency thresholds, the bank is illiquid but solvent. This opens up a role for a lender of last-resort, as considered by Rochet and Vives (2004). In particular, if a regulator observes the shock without noise (perhaps due to its supervisory function) it can offer loans at an interest rate $\rho \in [0, \frac{1}{\psi} - 1)$. This policy shifts out the illiquidity threshold for given encumbrance. Since the bank is truly solvent, no taxpayer's money is at risk. Interim

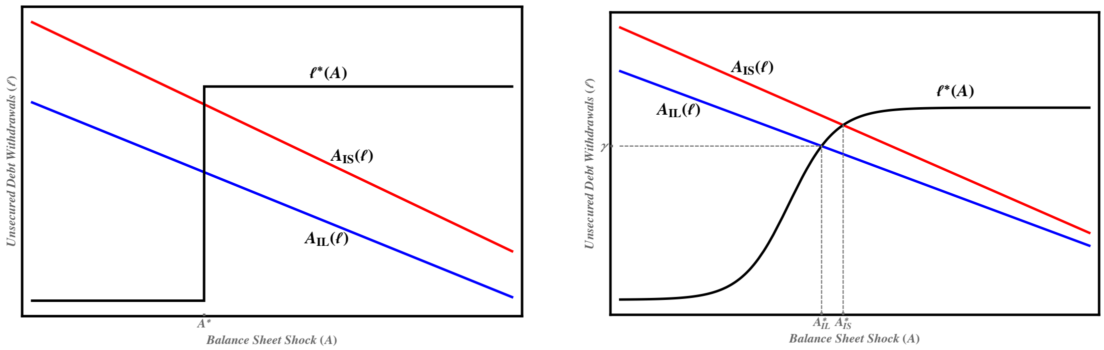


Figure 11: The precision of private information and the range of balance sheet shocks for which a bank is illiquid but solvent. The left panel shows the case of infinitely precise private information and $D_U \geq \frac{\psi}{1-\gamma}\hat{D}_U$, so $A_{IL}^{**} = A_{IS}^{**}$. The right panel shows the case of limited precision of private information and the range $[A_{IL}^{**}, A_{IS}^{**}]$.

liquidity support also affects the initial incentives to encumber assets. A detailed analysis of this issue is left for future research.

7 Conclusion

Our paper has offered a model of asset encumbrance by banks subject to rollover risk to examine its influences on fragility, funding costs, and prudential regulation. The bank's optimal encumbrance ratio trades off profitable investment funded by cheap senior secured debt against greater fragility due to unsecured debt runs. We derive and discuss several testable implications about encumbrance ratios. Our framework is also amenable to normative analysis. The presence of deposit insurance or wholesale funding guarantees induces excessive asset encumbrance and excessive bank fragility. A social planner can eliminate these risk-shifting incentives with prudential safeguards, notably caps on asset encumbrance and revenue-neutral Pigovian taxation.

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A Proofs

A.1 Proof of Proposition 1

The proof is in two steps. First, we show that the dominance regions at the rollover stage, based on the illiquidity threshold, are well defined for any bank funding structure. If the balance sheet shock were common knowledge, the rollover behavior of fund managers would exhibit multiple equilibria (Figure 12). If no unsecured debt is rolled over, $\ell = 1$, the bank is liquid whenever the shock is below $\underline{A} \equiv R(1-\alpha)I - \frac{UD_U}{\psi}$. For $A < \underline{A}$, it is a dominant strategy for fund managers to roll over. If $\ell = 0$, the bank is illiquid whenever the shock is above $\overline{A} \equiv R(1-\alpha)I$. For $A > \overline{A}$, it is a dominant strategy for managers not to roll over.

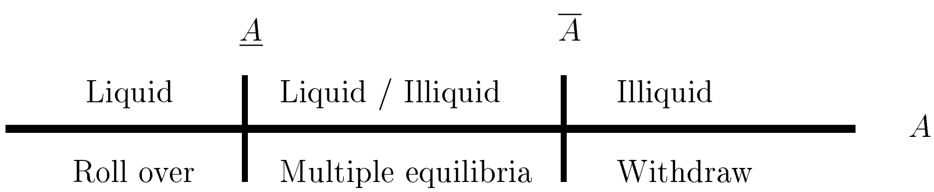

Figure 12: Tripartite classification of the balance sheet shock

Second, we characterize the threshold equilibrium. For a given realization $A \in [\,\underline{A}, \overline{A}\,]$, the proportion of fund managers who do not roll over unsecured debt is:

$$\ell\Big(A, x^*\Big) = \text{Prob}\left(x_i > x^* \middle| A\right) = \text{Prob}\left(\epsilon_i > x^* - A\right) = 1 - H\Big(x^* - A\Big). \tag{18}$$

A **critical mass condition** states that illiquidity occurs when the shock equals A^*, where the proportion of managers not rolling over is evaluated at A^*:

$$R(1-\alpha)I - A^* \equiv \ell\big(A^*, x^*\big)\frac{UD_U}{\psi}. \tag{19}$$

The posterior distribution of the shock conditional on the private signal is derived using Bayes' rule. An **indifference condition** states that the manager who receives the threshold signal $x_i = x^*$ is indifferent between rolling and not rolling over, $\gamma = \Pr\left(A < A^* | x_i = x^*\right)$.

Using the definition $x_j = A + \epsilon_j$, the conditional probability is

$$1-\gamma = \Pr\left(A > A^* | x_i = x^*\right) = \Pr\left(A > A^* | x_i = x^* = A + \epsilon_j\right), \tag{20}$$

$$= \Pr\left(x^* - \epsilon_j > A^*\right) = \Pr\left(\epsilon_j < x^* - A^*\right) = H\Big(x^* - A^*\Big). \tag{21}$$

The indifference condition implies $x^* - A^* = H^{-1}\big(1-\gamma\big)$. Inserting it into $\ell\big(A^*, x^*\big)$, the proportion of managers who withdraw at the threshold shock A^* is $\ell\Big(A^*, x_i = x^*\Big) = 1 - H\Big(x^* - A^*\Big) = 1 - H\Big(H^{-1}\Big(1-\gamma\Big)\Big) = \gamma$. The stated run threshold A^* follows and satisfies:

$$\frac{dA^*}{dD_U} = -\frac{\gamma U}{\psi} < 0, \qquad \frac{dA^*}{d\alpha} = R\left(\lambda z - 1\right)\frac{I^*(\alpha)}{1-\alpha\lambda z}. \tag{22}$$

A.2 Proof of Proposition 2

The banker's problem is given in (6). The total derivative $\frac{d\pi}{d\alpha}$, which takes indirect effects via $A^*(\alpha)$ and $S^*(\alpha)$ into account, yields $\frac{d\pi}{d\alpha} = \frac{RI^*(\alpha)}{1-\alpha\lambda z} f(A^*)G(\alpha)$, where

$$G(\alpha) = \frac{F(A^*)}{f(A^*)}\lambda(z-1) - (1-\lambda z)\left[RI^*(\alpha)\alpha(1-\lambda) + UD_U\left(\frac{\gamma}{\psi} - 1\right)\right]. \tag{23}$$

If an interior solution $0 < \alpha^* < 1$ exists, it is given by $G(\alpha^*) = 0$. It is a local maximum:

$$\frac{dG}{d\alpha} = \underbrace{\frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}}_{+}\underbrace{\frac{dA^*}{d\alpha}}_{-}\lambda(z-1) - (1-\lambda z)\frac{R(1-\lambda)I^*(\alpha)}{1-\alpha\lambda z} < 0, \tag{24}$$

where the signs arise from the decreasing reverse hazard rate of $f(A)$ and $\lambda z < 1$. Using the implicit function theorem (IFT), we obtain $\frac{d\alpha^*}{dD_U} < 0$ for the interior solution since

$$\frac{dG}{dD_U} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dD_U}\lambda(z-1) - (1-\lambda z)U\left(\frac{\gamma}{\psi} - 1\right) < 0, \tag{25}$$

because of $\gamma > \psi$, so $\frac{d\alpha^*}{dD_U} \leq 0$ follows for the entire encumbrance schedule.

An interior solution to $G(\alpha^*) = 0$ requires two conditions, $G(\alpha = 0) > 0$ and $G(\alpha = 1) < 0$. First, evaluating the implicit function when there is no encumbrance, $\alpha = 0$, yields

$$\frac{F(A^*(0))}{f(A^*(0))}\lambda(z-1) - (1-\lambda z)\left(\frac{\gamma}{\psi} - 1\right) U D_U \, , \tag{26}$$

which strictly decreases in D_U. To ensure $\alpha^* > 0$, the face value of unsecured debt must satisfy $D_U < \bar{D}_U$, where $\bar{D}_U$ is uniquely and implicitly defined by

$$\frac{F(R(U+E) - \frac{\gamma}{\psi} U \bar{D}_U)}{f(R(U+E) - \frac{\gamma}{\psi} U \bar{D}_U)}\lambda(z-1) - (1-\lambda z)\left(\frac{\gamma}{\psi} - 1\right) U \bar{D}_U = 0. \tag{27}$$

Second, evaluating the implicit function when all assets are encumbered, $\alpha = 1$, yields

$$\frac{F(A^*(1))}{f(A^*(1))}\lambda(z-1) - (1-\lambda z)\left[\frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left(\frac{\gamma}{\psi} - 1\right) U D_U\right] , \tag{28}$$

which also decreases in D_U. To ensure $\alpha^* < 1$, the expression in equation (28) must be strictly negative. Hence, the face value of unsecured debt must be bounded from below, $D_U > \underline{D}_U$, where $\underline{D}_U < \bar{D}_U$ is uniquely and implicitly defined by

$$\frac{F(-\frac{\gamma}{\psi} U \underline{D}_U)}{f(-\frac{\gamma}{\psi} U \underline{D}_U)}\lambda(z-1) - (1-\lambda z)\left[\frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left(\frac{\gamma}{\psi} - 1\right) U \underline{D}_U\right] = 0\, . \tag{29}$$

A.3 Proof of Proposition 3

The proof is in four steps. First, we ensure that the encumbrance ratio is interior, $\alpha^{**} \in (0,1)$. Since $\alpha = 0$ implies $\hat{D}_U = 0$, there is no equilibrium consistent with the supposition $D_U \leq \hat{D}_U$, so α^{**} must be positive (verified below). If $\alpha = 1$, the run threshold and value of an unsecured debt claim are $A^*(1) = -\frac{\gamma U D_U}{\psi}$ and $V(1, D_U) = D_U F(A^*(1))$, respectively. The value of unsecured debt attains a maximum at $D_U = D_{max}$ uniquely and implicitly defined by $\frac{F\left(-\frac{\gamma}{\psi} U D_{max}\right)}{f\left(-\frac{\gamma}{\psi} U D_{max}\right)} - \frac{\gamma}{\psi} U D_{max} = 0$. $V(\alpha, D_U)$ decreases in α, so any solution for the encumbrance ratio, if it exists, is interior if the outside option satisfies $r > \underset{\sim}{r} \equiv V(1, D_{max})$.

Second, we show that the face value of unsecured debt satisfies $D_U^* > r$. While $\frac{\partial V}{\partial D_U}$ has an ambiguous sign in general, the derivative evaluated at the encumbrance schedule is

$$\left.\frac{\partial V}{\partial D_U}\right|_{\alpha^*(D_U)} = f(A^*)\left[\frac{1-\lambda z}{\lambda(z-1)}R(1-\lambda)\alpha^* I^*(\alpha^*) + \beta_0 U D_U\right], \tag{30}$$

which is non-negative whenever $\beta_0 \equiv \frac{1-\lambda z}{\lambda(z-1)}\left(\frac{\gamma}{\psi}-1\right) - \frac{\gamma}{\psi} \geq 0 \Leftrightarrow \gamma \geq \underset{\sim}{\gamma} \equiv \frac{1-\lambda z}{1-\lambda z-\lambda(z-1)}\psi$. Having established conditions under which V increases in D_U, at least in the vicinity of the encumbrance schedule, it follows that $D_U = r$ always violates the participation constraint, $V(D_U = r) = rF(A^*(D_U = r)) < r$. Thus, $D_U^* > r$.

Third, we establish that the intersection between the encumbrance schedule and participation constraint of unsecured debtholders yields a unique joint equilibrium. Proposition 2 states that $\alpha^*(D_U)$ decreases in D_U. Also, from the second step of this proof, we have sufficient conditions that ensure, in the vicinity of the encumbrance schedule, the market-implied face value of unsecured debt, $D_U^*(\alpha)$, increases in α. Hence, there is at most only one intersection of these two curves, establishing uniqueness. We also show that the equilibrium specified above exists. Define $T(D_U) = r/F(A^*(\alpha^*(D_U), D_U))$ as a mapping from the set $\mathcal{U}$ of face values of unsecured debt into itself. If $\mathcal{U}$ is a closed and compact set then, by Brouwer's fixed-point theorem, there exists at least one fixed-point for the mapping. The lower bound on D_U is r. For the upper bound, note that if the banker could pledge all assets to unsecured investors, then $D_U \leq RI^*(\alpha) - A$. Truncating the shock distribution at some arbitrary $-A_L < 0$ yields a well-defined upper bound on D_U.

Fourth, we verify the supposition $D_U \leq \hat{D}_U$. Denoting the run threshold evaluated at $D_U = \hat{D}_U$ by $\hat{A}^*(\alpha) \equiv A^*(\hat{D}_U(\alpha))$, we have $\hat{A}^*(\alpha) = RI^*(\alpha)\left[1-\alpha\left(1+\frac{\gamma}{\psi}U(1-\lambda)\right)\right]$ with $\frac{d\hat{A}^*}{d\alpha} = -\frac{RI^*(\alpha)}{1-\alpha\lambda z}\left[1-\lambda z+\frac{\gamma}{\psi}U(1-\lambda)\right] < 0$. Next, define $\hat{\alpha}^*$ as the equilibrium encumbrance ratio evaluated at $D_U = \hat{D}_U(\hat{\alpha}^*)$, which is implicitly and uniquely defined by

$$\frac{F(\hat{A}^*(\hat{\alpha}^*))}{f(\hat{A}^*(\hat{\alpha}^*))} = \frac{1-\lambda z}{\lambda(z-1)}\frac{\gamma}{\psi}U(1-\lambda)R\hat{\alpha}^* I^*(\hat{\alpha}^*), \tag{31}$$

because the left-hand side decreases in α, while the right-hand side increases in it. We next translate the condition $D_U \leq \hat{D}_U$ into a condition for unsecured debt pricing:

$$r \leq V(\hat{\alpha}^*, \hat{D}_U(\hat{\alpha}^*)). \tag{32}$$

A stricter sufficient condition is to require $r \leq V(1, \hat{D}_U(1))$. Using the condition for $\hat{\alpha}^*$, we obtain

$$r \leq \frac{\psi\lambda(z-1)}{\gamma U(1-\lambda z)} \frac{F\left(-\frac{R(U+E)}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)^2}{f\left(-\frac{R(U+E)}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)}, \tag{33}$$

where right-hand side decreases in the bank's own funds E. This suggests that imposing an upper bound, $E \leq \bar{E}$, on bank capital ensures that, in equilibrium, $D_U^* \leq \hat{D}_U(\alpha^*)$. The upper bound on capital is implicitly defined as $\frac{\psi\lambda(z-1)}{\gamma U(1-\lambda z)} \frac{F\left(-\frac{R(U+\bar{E})}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)^2}{f\left(-\frac{R(U+\bar{E})}{1-\lambda z}\frac{\gamma}{\psi}U(1-\lambda)\right)} = r$.

A.4 Proof of Proposition 4

The proof is in two steps. First, we show the effect of a parameter on the encumbrance schedule $\alpha^*(D_U)$. Second, we show that this direct effect is reinforced by an indirect effect via the equilibrium cost of unsecured debt D_U^*. For the direct effect via $\alpha^*(D_U)$, we take D_U as given and use the IFT, whereby $\frac{d\alpha^*}{dy} = -\frac{\frac{dG}{dy}}{\frac{dG}{d\alpha}}$ for $y \in \{\gamma, r, R, \psi, E, F(\cdot)\}$:

$$\frac{dG}{d\gamma} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\gamma}\lambda(z-1) - (1-\lambda z)U\frac{D_U}{\psi} < 0, \tag{34}$$

$$\begin{aligned} \frac{dG}{dr} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dr}\lambda(z-1) - \frac{F(A^*)}{f(A^*)}\lambda\frac{z}{r} \\ &- \lambda\frac{z}{r}\left[\frac{R(1-\lambda)\alpha(1-\alpha)I^*(\alpha)}{1-\alpha\lambda z} + D_U\left(\frac{\gamma}{\psi}-1\right)\right] < 0 \end{aligned} \tag{35}$$

$$\begin{aligned} \frac{dG}{dR} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dR}\lambda(z-1) + \frac{F(A^*)}{f(A^*)}\frac{\lambda}{r} \\ &+ \frac{\lambda}{r}D_U\left(\frac{\gamma}{\psi}-1\right) + (1-\lambda)\alpha I^*(\alpha)\left\{\lambda z - \frac{(1-\lambda z)}{1-\alpha\lambda z}\right\} > 0, \end{aligned} \tag{36}$$

where we could sign the expression in equation (36) by evaluating it at α^* and substituting $F(A^*)/f(A^*)$ from the first-order condition in equation (7). Moreover, we have:

$$\frac{dG}{d\psi} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\psi}\lambda(z-1) + (1-\lambda z)UD_U\frac{\gamma}{\psi^2} > 0 \tag{37}$$

$$\begin{aligned} \frac{dG}{d\lambda} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{d\lambda}\lambda(z-1) + \frac{F(A^*)}{f(A^*)}(z-1) + zRI^*(\alpha)\alpha(1-\lambda) \\ &+ zUD_U\left(\frac{\gamma}{\psi}-1\right) + \frac{(1-\lambda z)(1-\alpha z)}{1-\alpha\lambda z}R\alpha I^*(\alpha) > 0, \end{aligned} \tag{38}$$

$$\frac{dG}{dE} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*}\frac{dA^*}{dE}\lambda(z-1) - \frac{1-\lambda z}{1-\alpha\lambda z}R(1-\lambda)\alpha \lesseqgtr 0. \tag{39}$$

Finally, suppose that the balance sheet shock distribution $\widetilde{F}$ first-order stochastically dominates the distribution F according to the reverse hazard rate criterion: $\frac{\widetilde{f}}{\widetilde{F}} \geq \frac{f}{F}$, or, equivalently, $F/f \geq \widetilde{F}/\widetilde{f}$. Let $\widetilde{G}(\widetilde{\alpha}^*) = 0$ denote the implicit function defining the privately optimal encumbrance ratio, $\widetilde{\alpha}^*$, under $\widetilde{F}$. Thus, $\widetilde{G}(\alpha) \leq G(\alpha)$ for all ratios of encumbrance. Since $d\widetilde{G}/d\alpha < 0$ and $dG/d\alpha < 0$, the privately optimal encumbrance ratio satisfy $\widetilde{\alpha}^* \geq \alpha^*$.

The indirect effects arise from the face value of unsecured debt. For any given encumbrance ratio, they are given by the implicit function $J(\alpha, D_U^*) = 0$ where

$$J \equiv -r + D_U F(A^*(\alpha, D_U)). \tag{40}$$

Using the implicit function again, and noting that $\left.\frac{\partial J}{\partial D_U}\right|_{\alpha^*} > 0$, we obtain reinforcing effects:

$$\frac{\partial J}{\partial R} = D_U f(A^*)\frac{dA^*}{dR} > 0, \qquad \frac{\partial J}{\partial \psi} = D_U f(A^*)\frac{dA^*}{d\psi} > 0 \tag{41}$$

$$\frac{\partial J}{\partial \lambda} = D_U f(A^*)\frac{dA^*}{d\lambda} > 0, \qquad \frac{\partial J}{\partial \gamma} = D_U f(A^*)\frac{dA^*}{d\gamma} < 0 \tag{42}$$

$$\frac{\partial J}{\partial E} = D_U f(A^*)\frac{dA^*}{dE} > 0, \qquad \frac{\partial J}{\partial r} = -1 + D_U f(A^*)\frac{dA^*}{dr} < 0. \tag{43}$$

Finally, an improvement in the distribution of the balance sheet shock also increases J.

A.5 Proof of Proposition 5

The derivation for the private optimum with guarantees follows closely that in Appendix A.3. For brevity, we only state the key conditions for the existence of a unique equilibrium.

Taking the derivative of expected equity value with respect to α, the optimal encumbrance ratio, given face value of unsecured debt, is implicitly defined by $G_m(\alpha_m^*(D_U)) = 0$:

$$G_m(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - (1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U\left(\frac{\gamma}{\psi}-1\right) - mUr\right]$$

and $\frac{dG_m}{d\alpha} < 0$, so the solution is a local maximum. As before, for an interior solution, $\alpha_m^*(D_U) \in (0,1)$, we require that $D_U \in (\underline{D}_U(m), \bar{D}_U(m))$. For the rest of the normative analysis, we assume that the encumbrance schedule yields interior solutions that are local maximums. Bounds similar to those in the previous section can be derived.

The face value of the non-guaranteed unsecured debt satisfies $r = V(\alpha, D_U, m) \equiv D_U^* F(A_m^*(\alpha, D_U^*))$. Following the lines of previous reasoning, we obtain that a joint equilibrium exists if (i) $r > \underset{\sim}{r}_m \equiv V(1, D_{max}, m)$, (ii) $\gamma \geq \underset{\sim}{\gamma} \equiv \frac{1-\lambda z}{1-\lambda z-\lambda(z-1)}\psi$, and (iii) $E \leq \bar{E}_m$.

Consider the new comparative static. As before, we derive separately the direct effect on the encumbrance schedule and the indirect effect from the face value of unsecured debt. For the direct effect, the derivative of the implicit function, $G_m(\alpha)$, with respect to coverage is

$$\frac{dG_m}{dm} = \frac{d\frac{F(A_m^*)}{f(A_m^*)}}{dA_m^*}\frac{dA_m^*}{dm}\lambda(z-1) + (1-\lambda z)U\left[D_U\left(\frac{\gamma}{\psi}-1\right) + r\right] > 0\,. \tag{44}$$

Since $dG_m/d\alpha < 0$, the IFT implies $d\alpha_m^*/dm > 0$ for given D_U, so the encumbrance schedule shifts outwards after an increase in coverage. The indirect effect concerns the face value of unsecured debt given by the implicit function $J_m(\alpha, D_U^*(m)) = 0$, where $J_m \equiv -r + D_U F(A_m^*(\alpha, D_U))$. Hence, $\left.\frac{dJ_m}{dD_U}\right|_{\alpha^*} > 0$ and $\frac{dJ_m}{dm} = D_U f(A_m^*)\frac{dA_m^*}{dm} > 0$, so the face value of unsecured debt decreases as coverage increases, reinforcing asset encumbrance.

A.6 Proof of Proposition 6

Taking the derivative of the planner's objective function with respect to the encumbrance ratio, we obtain the first-order condition $\frac{d\pi_m}{d\alpha} + f(A_m^*)\frac{dA_m^*}{d\alpha}mUr = 0$. The planner's encumbrance schedule, $\alpha_P^*(D_U)$, is given by $G_P(\alpha_P^*) = 0$, where

$$G_P(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - (1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U\left(\frac{\gamma}{\psi} - 1\right)\right], \quad (45)$$

which decreases in α. We again focus on the interior solutions. Comparing equation (45) to the implicit function that provides the banker's encumbrance schedule in equation (44), we have that $G_P(\alpha_m^*) < 0$ for all permissible D_U. Hence, $\alpha_m^*(D_U) > \alpha_P^*(D_U)$ for any m.

A.7 Proof of Proposition 7

With a limit on asset encumbrance, the banker's constrained problem is given by

$$\alpha_m^* \equiv \max_{\alpha\in[0,\alpha_P^{**}]} \pi_m(\alpha) = \int^{A_m^*(\alpha)} \left[RI^*(\alpha) - (1-m)UD_U - S^*(\alpha)\,r - mUr - A\right]dF. \quad (46)$$

Since the bank profit is concave around α_m^*, the marginal profit at $\alpha = \alpha_P^{**}$ is positive. The constrained optimum stated in equation (14) follows. For the tax and transfer schemes, the planner imposes a linear tax $\tau > 0$ on encumbrance at $t = 2$ combined with a lump-sum transfer T. Then, the privately optimal encumbrance schedule subject to this regulation is given by the implicit function $G_R(\alpha)$, where $\alpha_R^*(D_U)$ solves $G_R(\alpha_R^*(D_U)) \equiv 0$:

$$\begin{aligned} G_R(\alpha) \quad \equiv \quad & \frac{F(A_m^*)}{f(A_m^*)}\lambda(z-1) - \frac{F(A_m^*)}{f(A_m^*)}\frac{1-\alpha\lambda z}{RI^*(\alpha)}\tau + \cdots \quad (47) \\ & -(1-\lambda z)\left[R\alpha(1-\lambda)I^*(\alpha) - \tau\alpha + T + (1-m)UD_U\left(\frac{\gamma}{\psi}-1\right) - mUr\right]. \end{aligned}$$

Consider the pure transfer scheme, $\tau = 0$. Comparing $G_R(\alpha)|_{\tau=0}$ with $G_P(\alpha)$ in equation (45), the two encumbrance schedules are the same whenever $T = Umr$.

Next, consider a revenue-neutral scheme, $T = \tau\alpha$ for all α. We evaluate $\alpha_R^*(D_U)$ at $T = \tau\alpha$ and solve for the optimal tax. Equalizing the socially optimal and the privately optimal schedule under regulation, $\alpha_R^* = \alpha_P^*$, we obtain τ^* stated in equation (13). This tax rate depends on the face value of unsecured and non-guaranteed debt.

Finally, we consider a linear tax on encumbrance independent of D_U. Using the IFT, we obtain that $\frac{d\alpha_R^*}{d\tau} < 0$ since $\frac{dG_R}{d\alpha} < 0$ by optimality and

$$\left.\frac{dG_R}{d\tau}\right|_{T=\tau\alpha} = -\frac{1-\alpha\lambda z}{RI^*(\alpha)}\frac{F(A_m^*)}{f(A_m^*)} < 0. \tag{48}$$

A.8 Proof of Proposition 8

The equilibrium is given by the triple, α^{**}, D_U^{**} and A^{**} that solve this system of equations:

$$\begin{aligned} g(\alpha^{**}, D_U^{**}, A^{**}) &\equiv \frac{F(A^{**})}{f(A^{**})}\lambda(z-1) & (49)\\ &- (1-\lambda z)\left[R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) + (1-m)UD_U^{**}\left(\frac{\gamma}{\psi}-1\right) - mUr\right] = 0 \\ V(\alpha^{**}, D_U^{**}, A^{**}) &\equiv D_U^{**}F(A^{**}) - r = 0 & (50)\\ T(\alpha^{**}, D_U^{**}, A^{**}) &\equiv A^{**} - R(1-\alpha^{**})I^*(\alpha^{**}) + (1-m)\frac{\gamma}{\psi}UD_U^{**} = 0\,. & (51)\end{aligned}$$

To derive the optimal guarantee, we first need to analyze how the run threshold A^{**} depends on m, both directly and via changes to the level of encumbrance and face value of unsecured debt. By the implicit function theorem and Cramer's rule, we have $\frac{dA^{**}}{dm} = \frac{|J_m^A|}{|J|}$, where $|J|$ is the determinant of the Jacobian:

$$|J_m^A| = \begin{vmatrix} g_{\alpha^{**}} & g_{D_U^{**}} & -g_m \\ V_{\alpha^{**}} & V_{D_U^{**}} & -V_m \\ T_{\alpha^{**}} & T_{D_U^{**}} & -T_m \end{vmatrix}, \quad |J| = \begin{vmatrix} g_{\alpha^{**}} & g_{D_U^{**}} & g_{A^{**}} \\ V_{\alpha^{**}} & V_{D_U^{**}} & V_{A^{**}} \\ T_{\alpha^{**}} & T_{D_U^{**}} & T_{A^{**}} \end{vmatrix}. \tag{52}$$

Since $V_{\alpha^{**}} = V_m = 0$, it follows that

$$\begin{aligned}
|J_m^A| &= V_{D_U^{**}}\left[-g_{\alpha^{**}}T_m + g_m T_{\alpha^{**}}\right] \\
&= -\frac{F(A^{**})RI^*(\alpha^{**})}{1-\alpha^{**}\lambda z}(1-\lambda z)\left[\lambda(z-1)\frac{\gamma}{\psi}UD_U^{**} + (1-\lambda z)U(D_U^{**}-r)\right] < 0, \\
|J| &= g_{\alpha^{**}}\left[V_{D_U^{**}}T_{A^{**}} - V_{A^{**}}T_{D_U^{**}}\right] + g_{D_U^{**}}V_{A^{**}}T_{\alpha^{**}} - g_{A^{**}}V_{D_U^{**}}T_{\alpha^{**}} \\
&= -f(A^{**})\frac{R(1-\lambda z)I^*(\alpha^{**})}{1-\alpha^{**}\lambda z}\left[(1-\lambda)\left\{\frac{F(A^{**})}{f(A^{**})} - (1-m)\frac{\gamma}{\psi}UD_U^{**}\right\}\right. \\
&+ \left.(1-\lambda z)(1-m)\left(\frac{\gamma}{\psi}-1\right)UD_U^{**} + (1-\lambda z)\frac{F(A^{**})}{f(A^{**})}\frac{d\frac{F(A^{**})}{f(A^{**})}}{A^{**}}\right].
\end{aligned}$$

A sufficient condition for $|J| < 0$ is $\frac{F(A^{**})}{f(A^{**})} - (1-m)UD_U^{**}\left[\frac{\gamma}{\psi} - \frac{1-\lambda z}{1-\lambda}\left(\frac{\gamma}{\psi}-1\right)\right] > 0$. Replacing the inverse reverse hazard rate using $g(\alpha^{**}, D_U^{**}, A^{**}) = 0$ yields:

$$\frac{1-\lambda z}{\lambda(z-1)}\left[R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) - mUr\right] > (1-m)UD_U^{**}\left[\frac{\gamma}{\psi} - \left(\frac{\gamma}{\psi}-1\right)\left\{\frac{1-\lambda z}{1-\lambda} + \frac{1-\lambda z}{\lambda(z-1)}\right\}\right].$$

Since $UD_U < U\hat{D}_U$, we have $R\alpha^{**}(1-\lambda)I^*(\alpha^{**}) - mUr > (1-m)UD_U^{**}$ in equilibrium. Thus, the sufficient condition for $|J| < 0$, $\lambda < \bar{\lambda} \equiv \frac{1}{2z-1}$, can be derived from this inequality:

$$\frac{\gamma}{\psi}\left[\frac{\lambda(z-1)}{1-\lambda z} - \frac{1-\lambda}{\lambda(z-1)}\right] + 1 < 0.$$

A.9 Proof of Propositions 9 and 10

These proofs parallel the proofs without liquid reserves.

We start with the dominance bounds. When all fund managers roll over ($\ell = 0$), the bank fails at $t = 2$ if $A > \bar{A} \equiv RI + L - UD_S - SD_S$. For $A > \bar{A}$, withdrawing is a dominant action for fund managers. When all fund managers withdraw ($\ell = 1$), the bank survives if $A < \underline{A} \equiv R(1-\alpha)I - \frac{UD_U - L}{\psi}$. For all $A < \underline{A}$, rolling over is a dominant action.

We turn to the global games solution. Fund managers use threshold strategies and roll over whenever $x_i < x^*$. Since the introduction of liquid reserves does not alter the signals that fund managers receive, the fraction of withdrawing fund managers remains unchanged: $\ell(A, x^*) = Pr(x > x^*|A) = 1 - H(x^* - A)$. Likewise, the indifference condition for fund managers is also unaffected, so $\gamma = 1 - H(x^* - A^*)$. Taken together, $\ell(A^*, x^*) = \gamma$. If $\gamma \leq \frac{L}{UD_U}$, the bank is super-liquid and the critical mass condition is given by the insolvency condition at $t = 2$ in equation (15), yielding the run threshold A_2^*. Otherwise ($\gamma > \frac{L}{UD_U}$), the critical mass condition is given by the usual illiquidity condition at $t = 1$, yielding A_1^*.

Taking into account the pricing of secured debt in the run threshold, we obtain:

$$\frac{dA_1^*}{dL} = \frac{1}{\psi} - \frac{R(1-\alpha)}{1-\alpha\lambda z}, \quad \frac{dA_2^*}{dL} = 1 - \frac{R(1-\alpha\lambda)}{1-\alpha\lambda z} < 0. \tag{53}$$

The cross-derivative of A_1^* with respect to L and α is positive. Also, $\frac{dA_1^*}{dL}\Big|_{\alpha=0} = \frac{1-R\psi}{\psi}$, so $R\psi <$ is sufficient for $\frac{dA_1^*}{dL} > 0$. The illiquidity run threshold changes with encumbrance:

$$\frac{dA_1^*}{d\alpha} = -RI^*(\alpha) + R(1-\alpha)\frac{dI^*(\alpha)}{d\alpha} = \frac{RI^*(\alpha)}{1-\alpha\lambda z}(\lambda z - 1) < 0. \tag{54}$$

The expected bank equity value is $\pi = \int_{-\infty}^{A^*} \left[RI^*(\alpha) + L - UD_U - S^*(\alpha)r - A\right] dF(A)$. For a given D_U, if $\gamma \leq \frac{L}{UD_U}$, we have $A^* = A_2^*$, else $A^* = A_1^*$ with which we proceed. Paralleling the previous steps, the encumbrance schedule is implicitly defined by $G(\alpha^*) = 0$:

$$G(\alpha) \equiv \frac{F(A_1^*(\alpha))}{f(A_1^*(\alpha))}\lambda(z-1) - (1-\lambda z)\left[RI^*(\alpha)\alpha(1-\lambda) - L\left(\frac{1}{\psi} - 1\right) + UD_U\left(\frac{\gamma}{\psi} - 1\right)\right],$$

and $G_\alpha < 0$. To evaluate how encumbrance depends on liquid reserves, we note that

$$G_L = \frac{d\frac{F(A_1^*)}{f(A_1^*)}}{dA_1^*}\frac{dA_1^*}{dL}\lambda(z-1) + (1-\lambda z)\left[\frac{1}{\psi} - 1 + \frac{R\alpha(1-\lambda)}{1-\alpha\lambda z}\right] > 0.$$

Using the implicit function theorem, we have that $\frac{d\alpha^*}{dL} = -\frac{G_L}{G_\alpha} > 0$.