Anticipated Financial Contagion*

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We examine the incidence of financial contagion, bank choices, welfare, and regulation when interconnected banks anticipate an aggregate liquidity shock. Interbank deposits allow banks to co-insure against regional liquidity shocks but can also lead to contagion—the mutual default of banks. We numerically characterize the equilibrium and find that contagion is rare. Moreover, the equilibrium is constrained inefficient. For less likely aggregate liquidity shocks, banks hold inefficiently large interbank positions that over-expose surviving banks to impaired returns from failing banks when resolution occurs at market values. Efficiency can be restored via an alternative bank resolution scheme.

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1 Introduction

Banks can co-insure against liquidity shocks by holding interbank deposits. In a seminal paper, Allen and Gale (2000) show that financial contagion—the joint default of banks—occurs in this environment when one bank faces a large unanticipated aggregate liquidity shock. However, the ex-ante choices of banks are exogenous in Allen and Gale (2000). The natural extension is to allow the aggregate liquidity shock to occur with positive probability, which is analytically intractable. In this paper, we numerically solve the Nash equilibrium of a game between two banks. This approach allows us to characterize how the size and probability of the aggregate liquidity shock affect banks' ex-ante choices, the risk of contagion, welfare, and regulation.

In our environment, detailed in Section 2, consumers live in two regions characterized by negatively correlated regional liquidity shocks and an aggregate liquidity shock. By offering demand deposits, a representative competitive bank in each region insures risk-averse consumers against their idiosyncratic liquidity shocks, whereby consumers may be impatient and consume early (Diamond and Dybvig, 1983). The deposit contract offers a non-contingent return upon early withdrawal, unless the bank defaults. Since consumer types are privately observed, banks are subject to an incentive compatibility constraint, ensuring that patient consumers do not misrepresent their type and withdraw early. Ex ante a bank chooses a portfolio of three assets: a deposit with the bank in the other region, a low-yielding liquid asset, and a higher-yielding but illiquid asset; as well as the return on demand deposits. Consumer and interbank deposits have the same returns and equal seniority upon default.

Describing the decentralized equilibrium analytically is impractical due to numerous non-negativity and incentive compatibility constraints. Thus, we approximate the equilibrium numerically in Section 3. We provide the withdrawal choices of consumers and banks and a numerical algorithm to solve for the equilibrium. The best response of a regional bank is the portfolio and deposit return that maximizes the expected utility of regional consumers, given the choices of the bank in the other region. Regional banks are ex-ante identical, so their best response functions are symmetric. We find the equilibrium by solving for an iteratively stable fixed point of the best response function.

We obtain three main results in Section 4. First, contagion is possible but rare. It occurs in 3% of the total parameter space, where the probability the aggregate shock is low and its size is large. Second, we characterize the individually optimal bank choices of exposure to and withdrawal of interbank deposits and the implications for portfolio choice and demand deposit return. In equilibrium, mutual default obtains for low shock probabilities, no default obtains for high shock probabilities, and single default obtains otherwise. Third, we examine welfare and regulation. The equilibrium is constrained inefficient when the shock is positive but unlikely. We also propose a resolution scheme upon bank default that restores constrained efficiency.

Providing a non-contingent deposit return is costly when there is large aggregate variation in the fraction of impatient consumers. Default is the only tool a bank can use to make the returns for impatient consumers contingent (Allen and Gale, 1998, 2004; Zame, 1993). Contagion occurs when it is individually optimal for a bank to accept a large degree of ex-post inefficiency (mutual default) when the aggregate shock hits either region, allowing the bank to increase ex-ante liquidity insurance to consumers when the aggregate shock does not realize. Ex ante it is always feasible for a bank to insulate itself from the default of the other bank but it is not optimal when the aggregate shock is unlikely. Hence, banks choose to co-insure against regional liquidity shocks only. For intermediate probabilities of the aggregate shock, mutual default is too costly. Banks choose portfolios such that they only default when the shock hits their region (single-default equilibrium). Finally, when the aggregate shock is likely, banks fully co-insure against the shock and no default occurs. Since the non-contingent deposit return is always paid, its level is the lowest in the no-default equilibrium, supported by the largest liquidity choice.

To evaluate the welfare properties of equilibrium, we require an appropriate benchmark. In Section 5 we propose a *global bank with regional subsidiaries* and analytically characterize the resulting allocations. Such a bank faces the same two constraints as individual banks: absent default, a bank must offer (i) a non-contingent deposit return, and (ii) it is incentive compatible for a patient consumer not to withdraw early. The global bank can transfer liquidity across regions or allow a regional subsidiary to fail. In sum, this benchmark maintains the focus on banks and highlights the (social) cost of decentralization via regional banks.

There are three facets of constrained inefficiency of the equilibrium examined in Section 6.1: (i) where the equilibrium is characterized by mutual default, the global bank chooses single default, (ii) there are parameter regions where the equilibrium is characterized by no default, while the global bank chooses single default, and (iii) in the single default equilibrium, bank's choose inefficiently high interbank positions. That is, regional banks are over-exposed to the interbank market, resulting in inefficient choices of liquidity and deposit return. When contagion occurs, regional banks choose higher deposit return and liquid asset holdings than the global bank but effectively offer less overall liquidity insurance to consumers (because the global bank never chooses contagion). Finally, whenever neither the global nor regional banks default, the equilibrium is constrained efficient up to numerical precision.

We wish to elaborate on the source of the inefficiency: the baseline of treating consumer and bank deposits equally means that the interbank deposit is a costly tool to use when only one bank defaults. The defaulting bank liquidates all its assets, including its interbank deposit at the surviving bank, which pays the full deposit return. The market value of this interbank claim is unimpaired. The surviving bank, however, receives only the impaired market value of its interbank claim: its share in the liquidation value of the bank in default.

We consider in Section 6.2 an *alternative resolution scheme*—interbank claims are netted out at book value (rather than at market value as in the baseline resolution scheme) upon default. We show that this scheme changes the incentives so that banks exploit ex ante the full co-insurance value of interbank deposits. There is no net transfer of resources when one bank defaults in equilibrium, because the interbank deposits net out at book value. As a result, the equilibrium among regional banks subject to this alternative resolution regime achieves the constrained efficient allocation (given by the global bank allocation). Intuitively, this resolution scheme creates more contingency of interbank returns by breaking the link with returns on consumer deposits when a bank defaults.

We relate our work to the literature in Section 1.1, while Section 7 concludes. We discuss the numerical accuracy and robustness of our solution in Appendix A, while Appendices B and C contain proofs. Lastly, Appendix D describes the allocation under autarky (absence of banks).

1.1 Literature

Our paper is related to several strands of literature. We build on models of financial intermediation and coordination-based bank-runs in the tradition of Diamond and Dybvig (1983). Ennis and Keister (2006) study the portfolio choice of a single bank subject to a run. We share the focus on ex-ante bank choices but study multiple banks and the interbank market instead.

Our paper is closely related to a literature on financial contagion due to interbank linkages.[1] We have already compared to the work of Allen and Gale (2000), who consider an exchange of interbank deposits. Similarly, Freixas et al. (2000) consider a spatial economy and study how the failure of a bank can spread through the interbank network. Kiyotaki and Moore (2002) study contagion due to the credit chaims among lenders and entrepreneurs. Dasgupta (2004) uses global-game methods (Carlsson and van Damme, 1993; Morris and Shin, 2003) to pin down uniquely the probability of failure in a version of Allen and Gale (2000) without aggregate liquidity shocks but insolvency risk. Financial contagion arises in equilibrium, whereby a run on the creditor bank is more likely after a run on the debtor bank. As a result, full insurance against the regional liquidity shock is shown not to be desirable under some circumstances. In contrast, we follow the setup of Allen and Gale (2000) without solvency shocks, introduce aggregate liquidity risk with positive probability and solve for the ex-ante optimal choices of the bank's portfolio and liquidity provision to consumers as well as derive normative implications.

[1] There are other types of theories of financial contagion. For a common discount factor channel, see Ammer and Mei (1996) and Kodres and Pritsker (2002). Regarding a common investor base, see Goldstein and Pauzner (2004) for wealth effects, Pavlova and Rigobon (2008) for portfolio constraints, Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post exposures, see Basu (1998) for a common risk factor, and Chen (1999), Acharya and Yorulmazer (2008), Manz (2010) and Allen et al. (2012) for asset commonality among banks and information contagion. See Ahnert and Bertsch (2022) for a wake-up call theory of contagion.

Several other aspects of mutual deposit holdings among banks and liquidity co-insurance have been explored in the literature. First, work on bank capital and risk-taking in the context of liquidity co-insurance includes Brusco and Castiglionesi (2007) and Zawadowski (2013). Second, work on state-contingent bank deposit contracts includes Castiglionesi (2007), Zawadowski (2013), and Castiglionesi et al. (2019). Third, Carletti and Leonello (2019) study the role of credit market competition for liquidity crises. Fourth, Leitner (2005) shows that the threat of contagion induces a voluntary bailout among financially linked banks, effectively providing ex-ante commitment. Fifth, work on bilateral and non-exclusive trades among at least three banks includes Castiglionesi and Wagner (2013) and Babus (2016), who models interbank networks as a strategic network formation game and studies the implications for contagion. Sixth, work on contagion in interbank networks with many agents considers both mutual interbank loans (e.g., Acemoglu et al. (2015)) and the exchange of investment projects (e.g., Elliott et al. (2014), Cabrales et al. (2017)) and studies how exogenous shocks propagate in the interbank network. Finally, evidence for interbank contagion induced by depositor withdrawals, consistent with our model, is provided by Iyer and Peydro (2011) in a natural experiment in India.

Our paper is also part of a broader literature of interbank markets. Bhattacharya and Gale (1987) consider such a market in the presence of moral hazard and adverse selection problems (see also Rochet and Tirole (1996)). Freixas and Holthausen (2004) examine the integration and segmentation of money markets in the presence of adverse selection and superior soft information about home country banks. Allen and Carletti (2006) and Allen et al. (2009) consider an interbank market in which the long asset is traded. In Allen et al. (2009), aggregate risk induces excessive price volatility. Acharya and Skeie (2011) study the term interbank market and liquidity hoarding when levered banks are subject to rollover risk. Freixas et al. (2011) examine optimal central bank policy when the short asset is traded in the interbank market.

Our paper furthermore relates to work on bank resolution regimes. The resolution of banks—or rather avoiding bank resolution by providing a government funded bailout—is frequent (Goodhart, 1995) and costly (Claessens et al., 1999; Laeven and Valencia, 2010). Many authors, including Gorton and Huang (2004), point out that such government liquidity provision can lead to excessive risk taking due to moral hazard. Bail-ins are an alternative resolution regime that policy makers have embraced recently, for example in the European Bank Recovery and Resolution Directive 2014/59/EU or the Dodd-Frank Act in the United States. However, bail-ins can have adverse effects as well (Pandolfi, 2022; Colliard and Gromb, 2020) and the question of optimal bank resolution remains pertinent. We propose an alternative bank resolution regime that is neither a bailout nor a bail-in, but can restore constrained efficiency.

2 Environment

There are three dates $t = 0, 1, 2$ and a single divisible good for consumption and investment. Two riskless constant-returns-to scale technologies are universally available. Storage y (*liquidity*) yields a unit return at the subsequent date. Productive investment x (*investment*) at $t = 0$ yields a return of R at $t = 2$ and a liquidation value of r at $t = 1$, where $0 < r < 1 < R$.

There are two regions $k = A, B$ and each is inhabited by a unit mass of consumers. Consumers have a unit endowment at $t = 0$ and are initially identical. At $t = 1$, they privately learn their consumption preference, an idiosyncratic liquidity shock as in Diamond and Dybvig (1983). A fraction of consumers $\nu_{ks} \in (0, 1)$ in region k and state s values consumption at $t = 1$ ('early consumers'), and the remainder values consumption at $t = 2$ ('late consumers'):

$$U(c_{1ks}, c_{2ks}) = \begin{cases} u(c_{1ks}) & \text{w.p. } \nu_{ks} \\ u(c_{2ks}) & \text{w.p. } 1 - \nu_{ks} \end{cases} \tag{1}$$

where c_{tks} denotes consumption at date t in region k and state s. The period utility function $u(c)$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies the Inada conditions, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. Its relative risk-aversion exceeds 1.

Table 1 shows the distribution of regional liquidity demand ν_{ks}, similar to Table 3 in Allen and Gale (2000). The state $s = 1, 2, 3, 4$ is publicly revealed at $t = 1$. In states 1 and 2, a regional liquidity shock $\varepsilon \in (0, \min\{\gamma, 1 - \gamma\})$ is negatively correlated across regions, where $\gamma \in (0, 1)$ is the average liquidity demand. Because of symmetry, the regional liquidity shock generates an insurance motive across regions, for example in the form of interbank deposits. In (symmetric) states 3 and 4, an aggregate liquidity shock hits one region, resulting in possible contagion across regions if such links are formed. The size of the shock is $\alpha \in \left(0, \frac{1-\gamma}{2}\right)$ and its probability is $p \in [0, 1]$. The model of Allen and Gale (2000) with 2 banks arises as a special case for $p = 0$.

Table 1: Distribution of regional liquidity demand ν_{ks} for region k and state s

State s	Probability π_s	Region A	Region B
1	$\frac{1-p}{2}$	$\nu_{A1} = \gamma - \varepsilon$	$\nu_{B1} = \gamma + \varepsilon$
2	$\frac{1-p}{2}$	$\nu_{A2} = \gamma + \varepsilon$	$\nu_{B2} = \gamma - \varepsilon$
3	$\frac{p}{2}$	$\nu_{A3} = \gamma$	$\nu_{B3} = \gamma + 2\alpha$
4	$\frac{p}{2}$	$\nu_{A4} = \gamma + 2\alpha$	$\nu_{B4} = \gamma$

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3 Equilibrium: Decentralized economy with interbank deposits

There is a representative bank in each region indexed by A and B. Because of free entry within a region, each bank maximizes the expected utility of regional consumers subject to non-negative profits (Allen and Gale, 2000, 2007). Hence, all consumers deposit their endowment at their regional bank at $t = 0$, receiving a demand-deposit contract that promises a return d to consumers upon early withdrawal (Diamond and Dybvig, 1983). Banks choose their portfolio of investment x and liquidity y. To smooth regional liquidity shocks, banks can also make interbank deposits z at the other bank, receiving the same demand-deposit contract. The budget constraint at $t = 0$ for bank A thus reads as

$$x_A + y_A + z_A = 1 + z_B, \tag{2}$$

and similarly for bank B. We study symmetric non-cooperative equilibria among regional banks.

We focus on a sufficiently large aggregate liquidity shock throughout:

$$2\alpha > \varepsilon. \tag{3}$$

If the aggregate liquidity shock were small, $2\alpha \le \varepsilon$, an interbank position large enough to insure against the regional liquidity shock would also fully insure against the aggregate liquidity shock. In other words, this parameter constraint ensures that the highest demand for liquidity from consumers arises in states 3 and 4. We also restrict attention to parameters such that it is never individually optimal for a bank to default in states 1 or 2. These states represent "normal times" (i.e. no aggregate shock). Given condition (3), if bank A were to default in state 2 it would also default in state 4 (and bank B will also default also in state 3 if it defaults in state 1).

By offering demand-deposit contracts, regional banks are subject to a non-contingency constraint. In the absence of default, deposit returns do not depend on the aggregate liquidity shock. Thus, a regional bank uses default as a tool to relax this constraint in some states in order to increase the liquidity insurance to consumers in other states (Allen and Gale, 1998, 2004). Thus, a regional bank considers strategies that lead to one of three mutually exclusive default patterns: accepting default in zero, one, or both states in which the aggregate liquidity shock realizes (in states 3 and 4). In a *no-default pattern*, the choices of banks are such that neither bank ever defaults. In a *single-default pattern*, the choices of banks are such only the bank in the region hit by the aggregate liquidity shock defaults. And in a *mutual-default pattern*, the bank choices are such that both banks default if the aggregate liquidity shock hits either bank. These patterns, which conveniently partition the choice space of banks at $t = 0$, are helpful in describing the optimal withdrawal of interbank deposits in each state.

At $t = 1$, state s realizes, consumers learn their type, all early consumers withdraw, and banks serve withdrawals. Banks choose between using liquidity, withdrawing their interbank position (denoted by w_{As} for bank A), and liquidating investment. Late consumers withdraw at $t = 1$ if and only if the implied contract in state s is not incentive compatible, $c_{2As} < d_A$, given our focus on essential runs (Allen and Gale, 1998, 2000, 2007). Withdrawals from late consumers force bank default and full liquidation, where the proceeds are shared equally among the bank's creditors (pro rata), as in Allen and Gale (2000). Otherwise, the remaining investment matures in $t = 2$, the remaining interbank position is withdrawn, and all resources are paid out.

3.1 Interbank withdrawals

We describe the optimal withdrawal of interbank deposits at $t = 1$ given each default pattern. The choice of banks at $t = 0$ need not necessarily be optimal; we only require that they are consistent with the default pattern. (We describe optimality of bank choices in section 3.3.)

No-default pattern. As in Allen and Gale (2000), there is a *pecking order* in serving withdrawals at $t = 1$. In any state in which neither bank defaults, bank A has a preferred sequence in which it uses its assets to serve withdrawals: first it uses liquidity, then withdraws interbank deposits, and then liquidates investment. This pecking order is optimal for bank A as long as the relative returns between period 1 and 2 satisfy

$$1 \leq \frac{c_{2Bs}}{c_{1Bs}} \leq \frac{R}{r}, \tag{4}$$

where c_{tBs} is the return to any depositor at bank B withdrawing in period t in state s and, therefore, $\frac{c_{2Bs}}{c_{1Bs}}$ is the intertemporal trade-off of interbank deposits bank A made in bank B. The first inequality follows directly from incentive compatibility of the contract offered by bank B. Note that $\frac{R}{r}$ is the worst trade-off between late and early consumption levels possible in this environment (occurring, for instance, in autarky that we characterize in Appendix D) Thus, any bank contract that offers some degree of liquidity risk insurance to risk-averse depositors, who prefer a lower spread between early and late consumption, satisfies the second inequality. Going forward, we focus on parameters such that Condition (4) holds.

Lemma 1. ***Interbank withdrawals in the no-default pattern.*** *At $t = 1$, bank A's optimal withdrawals are*

$$\begin{aligned} w_{A1} &= 0 & (5) \\ w_{A2} &= \max\left\{\frac{d_A(\gamma+\varepsilon)-y_A}{d_B}, 0\right\} & (6) \\ w_{A3} &= \max\left\{\frac{d_A(\gamma+z_B)-y_A}{d_B}, 0\right\} & (7) \\ w_{A4} &= z_A. & (8) \end{aligned}$$

The same withdrawals are optimal for bank B (once accounting for the different states).

Proof. See Appendix B.1 ■

In states 3 and 4, banks with the highest regional liquidity demand fully withdraw, so $w_{A4} = z_A$ and $w_{B3} = z_B$. In other states, each bank withdraws what is necessary to serve withdrawals from both consumers and the other bank. Withdrawing more is not optimal because keeping the deposit with the other bank earns a higher return. For bank A in state 2 (symmetric for bank B in state 1), regional consumers require $d_A(\gamma+\varepsilon)$ and demand from bank B is zero, $w_{B2} = 0$, available regional liquidity is y_A, so the shortfall is $d_A(\gamma+\varepsilon)-y_A$ (if positive). The return on the deposit contract at bank B is d_B, so the total withdrawal volume is $\frac{d_A(\gamma+\varepsilon)-y_A}{d_B}$ (if positive). For bank A in state 3 (and similarly for bank B in state 4), the regional consumers require $d_A\gamma$ and bank B fully withdraws, $w_{B3} = z_B$, so the shortfall is $d_A(\gamma+z_B)-y_A$ (if positive) and the total withdrawals is $\frac{d_A(\gamma+z_B)-y_A}{d_B}$ (if positive).

Single-default pattern. Suppose next that banks choose portfolios at $t = 0$ that imply default of only the bank hit by the aggregate liquidity shock.

Lemma 2. ***Interbank withdrawals in the single-default pattern.*** *At $t = 1$, bank A's optimal withdrawals are*

$$\begin{aligned} w_{A1} &= 0 & (9) \\ w_{A2} &= \max\left\{\frac{d_A(\gamma+\varepsilon)-y_A}{d_B}, 0\right\} & (10) \\ w_{A3} &= z_A & (11) \\ w_{A4} &= z_A. & (12) \end{aligned}$$

The same withdrawals are optimal for bank B (once accounting for the different states).

Proof. See Appendix B.2 ■

States 1 and 2 are identical to the no-default pattern. In state 3, bank B defaults in $t = 1$

but bank A does not. Bank B liquidates its entire portfolio—including its interbank deposits. A deviation by bank A means withdrawing less than its full claim on bank B at $t = 1$. This is not optimal because the value of the claim on bank B at $t = 2$ is zero, reducing the resources available to bank A. State 4 is symmetric, with the bank A defaulting and bank B fully withdrawing.

Mutual-default pattern. Suppose finally that banks choose portfolios at $t = 0$ that imply default of both banks when one of them is hit by the aggregate liquidity shock.

Lemma 3. ***Interbank withdrawals in the mutual-default pattern.*** *At $t = 1$, bank A's optimal withdrawals are identical to that in the single-default pattern.*

Proof. See Appendix B.3 ■

While the optimal withdrawal behaviour in the mutual-default pattern is identical to that in the single-default pattern, the level of resources transferred differs. In the single-default pattern, the withdrawal behaviour in states 3 and 4 implies a net transfer of resources to the defaulting bank, as interbank withdrawals are settled *at market value* at $t = 1$: the defaulting bank receives the promised deposit return from non-defaulting bank, whereas the non-defaulting bank receives only its share of the liquidation proceeds from bank in default. In the mutual-default pattern, both banks receive their share of the liquidation value of the other bank, so their interbank positions cancel out if they make the same choices at $t = 0$.

3.2 Consumption Levels

Having established the optimal withdrawal beahviour at $t = 1$, we can state the levels of consumption and expected utility implied by any bank choice at $t = 0$. Formally, let the vector of bank choices be $\theta_k \equiv \{x_k, y_k, z_k, d_k\}$. Each pair of choices $\{\theta_A, \theta_B\}$ then implies a state-dependent consumption level for each region, $\{c_{1As}, c_{2As}, c_{1Bs}, c_{2Bs}\}_{s=1}^{4}$. The resource constraint at $t = 0$ binds in any equilibrium, $x_j^* + y_j^* + z_j^* = 1 + z_k^*$ because $u(c)$ is strictly increasing, so we drop x_k from the vector and state it more compactly as $\theta_k \equiv \{y_k, z_k, d_k\}$. Next, we state these consumption levels in state s at bank A when bank B defaults or not (and similarly for bank B). In other words, we state the consumption levels for each of the three possible default patterns.

No default of bank A requires that the implied consumption allocation is incentive compatible in all states. The total liquidity demand of bank A at $t = 1$ is $d_A(v_{As} + w_{Bs})$, the sum of withdrawing regional consumers and withdrawals of interbank deposits at return d_A. The available resources are regional liquidity (y_A), interbank withdrawals (w_{As}), and partial liquidation of the investment (λ_{As}), which adds to $y_A + d_B w_{As} + r\lambda_{As}$. Bank A may have excess liquidity: $e_{As} = y_A + d_B w_{As} - d_A(v_{As} + w_{Bs}) \geq 0$ paid out to late consumers and bank B at $t = 2$, or may partially liquidate $\lambda_{As} = \frac{d_A(v_{As}+w_{Bs})-(y_A+d_B w_{As})}{r} \geq 0$ of its investment such that $R(x_A - \lambda_{As})$ is paid out to late consumers and bank B. Either excess liquidity or partial liquidation may occur in some states, but never in the same state. In sum, the consumption levels without default are:

$$c_{1As}(\theta_A,\theta_B) = d_A,$$

$$c_{2As}(\theta_A,\theta_B) = \frac{\overbrace{e_{As}}^{\text{excess liquidity}} + c_{2Bs}(\theta_A,\theta_B)\overbrace{(z_A - w_{As})}^{\text{interbank asset}} + R\overbrace{(x_A-\lambda_{As})}^{\text{investment}}}{\underbrace{(z_B - w_{Bs})}_{\text{interbank liability}} + \underbrace{(1-v_{As})}_{\text{regional late consumers}}}.$$

Upon **default**, bank A fully liquidates all assets at $t = 1$. Its interbank deposit yields $c_{1Bs}(\theta_B,\theta_A)z_A$ (which is general to whether bank B also defaults or not). Its investment yields $r x_A$. Thus, the payout per unit of claim to all claimants—corresponding to the *liquidation values* in Allen and Gale (2000)—is $c_{1As}(\theta_A,\theta_B) = c_{2As}(\theta_A,\theta_B) = \frac{y_A + c_{1Bs}(\theta_B,\theta_A)z_A + r x_A}{1+z_B}$.

Taken together, the consumption allocations in state s for any choices $\{\theta_A,\theta_B\}$ are:

$$c_{1As}(\theta_A,\theta_B) = \begin{cases} d_A & \text{if } A \text{ does not default} \\ \frac{y_A + c_{1Bs}(\theta_B,\theta_A)z_A + r x_A}{1+z_B} & \text{if } A \text{ defaults} \end{cases} \tag{13}$$

$$c_{2As}(\theta_A,\theta_B) = \begin{cases} \frac{e_{As} + c_{2Bs}(\theta_B,\theta_A)(z_A - w_{As}) + R(x_A - \lambda_{As})}{(z_B - w_{Bs}) + 1 - v_{As}} & \text{if } A \text{ does not default} \\ \frac{y_A + c_{1Bs}(\theta_B,\theta_A)z_A + r x_A}{1+z_B} & \text{if } A \text{ defaults} \end{cases} \tag{14}$$

These accounting identities are general to all possible outcomes for both bank A and bank B.

3.3 Nash equilibrium

A regional bank cannot observe the types of consumers at date $t = 1$, so it offers incentive-compatible contracts, $c_{2ks}(\theta_A,\theta_B) \geq c_{1ks}(\theta_A,\theta_B)$. This formulation covers two cases: it reads as $c_{2ks}(\theta_A,\theta_B) \geq d_k$ in all states in which the bank wishes to avoid default. A bank may choose default in some states, resulting in an incentive-compatible allocation by construction because all deposits receive an equal value of the bank's liquidation value.

For arbitrary choices θ_A, θ_B, the *constrained expected utility* of a depositor of bank A is

$$\mathbf{u}(\theta_A|\theta_B) \equiv \sum_{s=1}^{4} \pi_s \left[\nu_{As} u(c_{1As}(\theta_A,\theta_B)) + (1-\nu_{As}) u(c_{2As}(\theta_A,\theta_B)) \right]$$

$$\text{s.t.} \quad x_A + y_A + z_A = 1 + z_B, \qquad e_{As}(\theta_A,\theta_B), \lambda_{As}(\theta_A,\theta_B) \geq 0,$$
$$c_{2As}(\theta_A,\theta_B) \geq c_{1As}(\theta_A,\theta_B), \qquad \text{and equations (5–12) and (13–14)} ,$$

where the constraints are the budget constraint at $t=0$, non-negativity constraints on excess liquidity and partial liquidation, the incentive compatibility constraint, and equations (5–12) describe the optimal withdrawal behaviour in the three default patterns and equations (13–14) the consumption identities.

The problem of bank A, (P2), is to choose θ_A to maximize the constrained expected utility of regional consumers, taking θ_B as given:

$$V_A \equiv \max_{\theta_A} \mathbf{u}(\theta_A|\theta_B) \tag{P2}$$

and similarly for bank B as Problem (P2) is symmetric. It defines a simultaneous-move game $\mathcal{G}$ with a pair of symmetric best response functions that maps a compact subspace $F \subset \mathbb{R}^3_+$ (the set of feasible choices) into itself: $\theta^{br}: F \to F$. We study symmetric Nash equilibrium.[2]

Definition 1. *A symmetric Nash equilibrium of $\mathcal{G}$ is a choice, θ^*, that is a best response to itself. Equivalently, θ^* is a fixed point of the symmetric best response function:*

$$\theta^* = \theta^{br}(\theta^*).$$

Three types of equilibrium are possible.[3] In a *no-default equilibrium*, the choices of banks are such that neither bank ever defaults. In a *single-default equilibrium*, the choices of banks are such only the bank in the region hit by the aggregate liquidity shock defaults. And in a *mutual-default equilibrium*, the bank choices are such that both banks default if the aggregate liquidity shock hits either bank. These equilibrium types build on the three patterns of default but also require that the ex-ante choices of banks are optimal (i.e. a best response).

We are now ready to define contagion in our environment.

[2]While asymmetric equilibria may exist in this environment, our focus on symmetric equilibria is motivated by the symmetry of the problem and the symmetry of the global bank allocation, studied in Section 5. Moreover, our numerical results (see section 4) show that the equilibrium is indeed symmetric.

[3]We use the labels ‘regime’ for the global-bank benchmark and ‘equilibrium type’ for the decentralized economy.

Definition 2. *Contagion occurs when the symmetric Nash equilibrium is characterized by the default of both banks whenever the aggregate liquidity shock realizes in either region.*

Contagion occurs when it is a best response for banks to choose portfolios at $t = 0$ that make it infeasible at $t = 1$ for a bank to offer an incentive-compatible consumption profile upon the default of the other bank. It is always *feasible* for a bank to choose a portfolio at $t = 0$ such that it survives when the other bank defaults at $t = 1$. We define contagion as the situation when it is not individually *optimal* to prevent its default upon the other bank's default.

To illustrate this point, consider bank A that can reduce its exposure to bank B in state 3 by reducing its interbank deposit at $t = 0$. However, this involves a trade-off. To avoid default in state 3, bank A lowers interbank co-insurance in all states, resulting in holding more liquidity and lowering the degree of liquidity insurance that can be offered to local consumers in states 1 and 2. These changes allow bank A to provide an incentive-compatible consumption profile in *all* states without default. Contagion occurs when this trade-off favours accepting the risk of default in a state where the aggregate liquidity shock hits the *other* region.

The intuition for this trade-off is clearest when the aggregate liquidity shock is almost zero (that is, an unanticipated shock in the limit of $p \to 0$). Full interbank co-insurance arises almost certainly (i.e. states 1 and 2), while accepting default in states 3 and 4 is individually optimal as $p \to 0$ as states 3 and 4 have vanishing weight in ex-ante expected utility. Consider a deviation by bank A that avoids default in state 3. Bank A can ensure its survival in state 3 by choosing a different portfolio (less exposure to bank B, which requires more local liquidity and a lower local deposit return), but at the cost of lower welfare in states 1 and 2 (by the non-state-contingency constraint), which reduces welfare compared to accepting default in state 3.

3.4 Numerical Implementation

This section describes our numerical strategy and the search algorithm. We first motivate the need for a numerical approach to solve for the equilibrium of the decentralized economy.

3.4.1 Motivating a numerical approach

Solving the game between two strategic banks—one in each region—requires a numerical approach because a fully analytical solution is not practical due to the large number of parameter regions for which the equilibrium would have to be studied separately. To obtain intuition for this observation, consider the impact of the various potentially binding inequality constraints.

Non-negativity constraints. A regional bank may optimally choose to fund late consumption out of excess liquidity in any state without default, or to fund early consumption by partially liquidating the investment. Consider bank A in a no-default allocation: it may choose to hold excess liquidity (e_{As}) in no state, only state 1, states 1 and 3, or states 1, 2 and 3. Similarly, there may be partial liquidation (λ_{As}) in no state, in state 4, states 2 and 4 or states 2, 3 and 4. It is never optimal to have excess liquidity in the state where the largest liquidity demand occurs, nor to fund the lowest liquidity demand with partial liquidation. Thus, there are 6 relevant non-negativity constraints:

$$e_{As} \geq 0 \text{ for } s = 1,2,3 \qquad \text{and} \qquad \lambda_{As} \geq 0 \text{ for } s = 2,3,4.$$

These non-negativity constraints induce 6 different characterizations of an optimal choice of bank A, for a given choice of bank B. The full set of possible outcomes across two banks in a no-default allocation thus has $6^2 = 36$ potentially different characterizations. Adding the single-default and mutual-default allocations adds further constraints.

Incentive compatibility constraints. There are parameter regions for which distinct solution types exist in the decentralized problem: in any state where no default occurs, the optimal allocation may be at a point where the incentive compatibility constraint for that state, $c_{2As} \geq d_A$, does or does not bind. This means the problem has up to four potentially binding incentive compatibility constraints for bank A:

$$c_{2As} \geq d_A \text{ for } s = 1,\ldots,4$$

A numerical approach can account for these cases simply by directly imposing incentive compatibility and non-negativity as a constraint on the allowable search space.

3.4.2 Numerically characterizing the equilibrium

We numerically solve for the symmetric Nash equilibrium of $\mathcal{G}$ by discretizing the choice variables for liquidity y, interbank deposits z, and deposit return d. We make two restrictions on the choice set. First, interbank positions are no greater than the larger of the regional or per capita aggregate shock: $z_{\max} = \max\{\varepsilon, \alpha\}$. This is motivated by fact that this is the largest per capita transfer that the global bank benchmark (Section 5) requires to support an allocation without liquidation. Furthermore, internal risk management and external capital requirements may restrict the amount of interbank loans. Second, the relevant range of the deposit return, $1 \leq d \leq R$, corresponds to the level of liquidity provision when the relative risk aversion exceeds unity (Diamond and Dybvig, 1983). Thus, the search space Θ of the choice variables θ is:

$$\Theta \equiv \{y, z, d \,|\, y \in [0,1], z \in [0, \max\{\alpha, \varepsilon\}] \text{ and } d \in [1, R]\}.$$

Definition 3. *A numerically approximate symmetric pure-strategy Nash Equilibrium of game $\mathscr{G}$ is an iteratively stable fixed point $\theta^* \in \Theta$ of the numerical best response function $\hat{\theta}^{br}$, where the iterations $\theta_i = \hat{\theta}^{br}(\theta_{i-1})$ are computed by direct numerical optimization of the constrained expected utility function:*

$$\hat{\theta}^{br}(\theta_{i-1}) \quad \equiv \quad \arg\max_{\theta \in \Theta} \mathbf{u}(\theta|\theta_{i-1}).$$

3.4.3 Search Algorithm

The algorithm searches over all possible three-dimensional choice vectors[4] to construct the best response function and iterates over those to find a fixed point. In the numerical implementation, we set $\mathbf{u}(\theta_A|\theta_B)$ to a large negative value if any constraint is violated, encoding the constraints directly into the objective function.

For computational stability, we divide the game $\mathscr{G}$ into three subgames $\mathscr{G}_\tau$ where each subgame is constrained to one of the three potential types of symmetric equilibrium default patterns, $\tau \in \mathscr{T} \equiv$ {no default, single default, mutual default}. For each subgame, we compute a candidate equilibrium, then select the candidate equilibrium with the maximal expected utility as the equilibrium of the full game. The algorithm proceeds as follows. Fix a parameter set and then iterate over the following steps until a convergence criterion is met in Step 4:[5]

Step 1: For each subgame $\mathscr{G}_\tau$, perform Steps 2 – 4.

Step 2: Initialize the choice of bank B at a feasible symmetric[6] and incentive compatible choice: $\theta_0 = [y_0, z_0, d_0] \in \Theta$. The initial point is found by fixing the interbank position at the level chosen by the global bank and performing 2-dimensional best response iteration over the choices of liquidity and deposit return.

Step 3: Find the best response of bank A, $\theta_i = [y_i, z_i, d_i]$, to the choice of bank B, θ_{i-1}:

$$\theta_i = \hat{\theta}^{br}(\theta_{i-1}).$$

[4]We impose the optimal withdrawal behaviour derived in 3.1.

[5]Computations are performed on the Stellenbosch University's High Performance Cluster 1 (Rhasatsha): `http://www.sun.ac.za/hpc`, using the global multivariate optimization methods implemented in Matlab (2020).

[6]The algorithm is initialized at a symmetric choice, but no other part of the algorithm imposes symmetry in $t = 0$ choices.

The numerical search for θ_i is done with standard and robust global optimization routines with n_{start} randomly selected starting points. We use a global search method that uses n_{start} stochastically chosen starting points (in every iteration) for the search for the best θ_i. The algorithm then selects the final optimal θ_i as the best among the convergence points of each of the multiple starting points (Ugray et al., 2007).

Step 4: Compute the ℓ_1 distance norm of the difference between θ_i and θ_{i-1}: $\delta_i \equiv \|\theta_i - \theta_{i-1}\|_1$.[7] Replace the choice of bank B, θ_{i-1}, with the found best response, θ_i. Repeat steps 3 and 4 until convergence criterion is met: $\delta_i < \delta_{crit}$.

Step 5: Selection of equilibrium of the full game: steps 1-4 yields a candidate equilibrium with associated value function for each of the subgames $\tau \in \mathcal{T}$. For a given parameter set, we select the equilibrium default pattern τ^* of the full game $\mathcal{G}$ as:

$$\tau^* = \arg\max_{\tau \in \mathcal{T}} V_A(\theta_A^*, \theta_B^* | \tau)$$

Table 2 states the parameters of the optimization routine. For each step within the iteration (i.e., at a given choice for bank B (θ_{i-1})), we use a global search algorithm with n_{start} stochastically selected starting points of the choice of bank A (θ_i) to find the best response. The final best response of bank A (θ_i) is the best among the convergence points of each of the stochastically selected starting points for the algorithm, given the choice of bank B.

Table 2: Optimization control parameters

Parameter Name	Value
Number of Starting Points for Best Response Search (n_{start})	2500
Convergence Tolerance (δ_{crit})	10^{-4}
Maximum Iterations (i_{max})	$256 = 2^8$

4 Results

We present the numerical results about the decentralized economy. As a necessary viability check for our numerical approach, we successfully replicate the analytical results of Allen and Gale (2000) in Appendix A.2, which are a special case for a zero-probability liquidity shock.

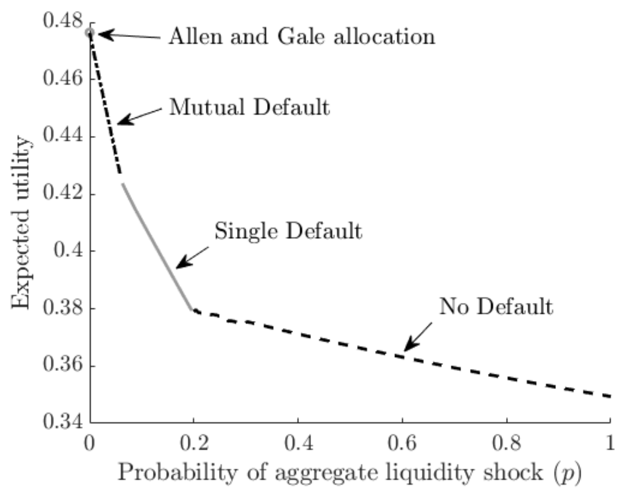
[7] The ℓ_1 norm is the strictest distance metric and corresponds to $\delta_i = |y_i - y_{i-1}| + |z_i - z_{i-1}| + |d_i - d_{i-1}|$.

Numerical Result 1. *For $p = 0$, the algorithm replicates the results of Allen and Gale (2000).*

4.1 Results for the equilibrium default pattern and bank choices

We first characterize the type of equilibrium (no default, single default, or mutual default) and the associated bank choices. For $p = 0$, the equilibrium replicates the expected utility of Allen and Gale (2000), within numerical precision. We summarize the findings for $p > 0$ in the following numerical result. Figure 1 shows the expected utility for each equilibrium default pattern, illustrating the numerical result for a given set of parameters.

Figure 1: Expected utility across probabilities of the aggregate liquidity shock. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, and $\varepsilon = 0.09$.



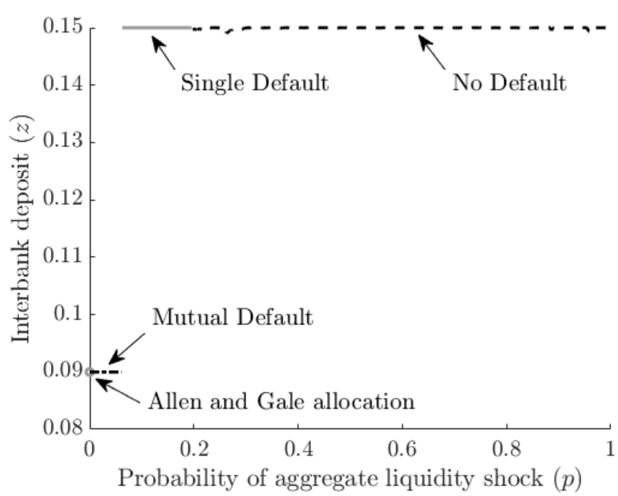
Numerical Result 2. *The type of equilibrium depends on the probability of the aggregate liquidity shock p: the mutual-default equilibrium obtains for low values of p, the single-default equilibrium for intermediate values of p, and the no-default equilibrium for high values of p.*

We refer to the mutual-default equilibrium as *contagion* because the decision of late consumers in one region to run on their bank induces full withdrawals of the interbank position and a bank run by late consumers in the other region (see also Definition 2). Contagion occurs only for a low enough probability of the aggregate liquidity shock. While it is feasible for a bank to choose an ex-ante portfolio that isolates it from the default of the other bank in states 3 and 4, this comes at the cost of lower liquidity risk insurance to local consumers in states 1 and 2, especially since this implies a net transfer to the defaulting bank. For a low enough probability, the trade-off favours accepting a run when the aggregate liquidity shock hits the other region. For a higher probability, in the single-default equilibrium, banks internalize the aggregate liquidity risk to a greater extent by choosing a portfolio such that a run by late consumers of one bank no longer induces a run by the other bank's late consumers. For a high probability, banks fully internalize the ex-post risk of a shock ex ante and the choices are such that default does not occur. We describe these bank choices in greater detail next.

Figure 2 shows interbank deposits (z^*) across the probability of the aggregate liquidity shock (p) for each type of equilibrium.[8] At $p = 0$ the Allen and Gale (2000) allocation is replicated. We summarize the findings for $p > 0$ in the following numerical result.

Numerical Result 3. *Interbank deposits (z^*) are roughly flat within an equilibrium default pattern and discontinuous across at the boundary between mutual and single default. In the mutual default equilibrium we have $z^* = \varepsilon$, and $z^* = \alpha$ in the single and no default equilibria.*

Figure 2: Interbank deposit z^* across the probability of the aggregate liquidity shock. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, and $\varepsilon = 0.09$.



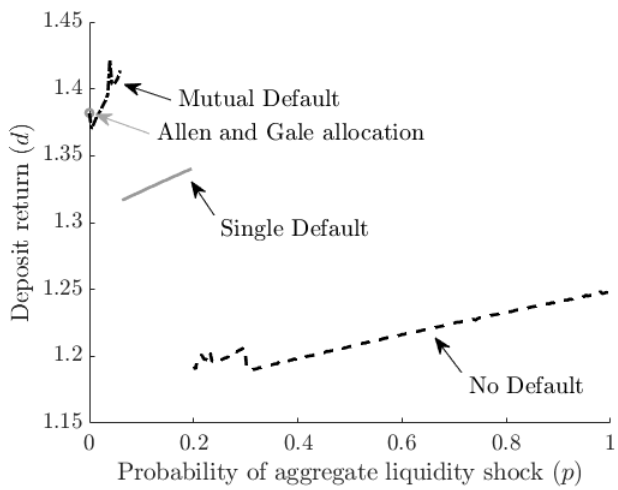
We provide some intuition for these results. When the probability of the aggregate shock is low and the mutual-default equilibrium is superior, banks hold interbank positions to fully co-insure against the regional shock, $z^* = \varepsilon$. Since both banks default when the aggregate shock hits, the size of the aggregate shock is irrelevant as states 3 and 4 are not insured against from an ex-ante perspective. When the probability is higher, such that either the single- or no-default equilibrium obtains, banks hold an interbank position equal to the per-capita size of the aggregate shock, $z^* = \alpha$.

[8]The discontinuities in the optimal choices arise when the equilibrium switches from one type (e.g. mutual default) to another (e.g. single default), mirroring the discontinuities in the benchmark (see Section 5). These discontinuities are features of the solution, rather than anomalies of the numerical approach. First, the equilibrium choices of a bank in each equilibrium have a continuous character (up to numerical precision). Second, the discontinuities between equilibrium default patterns are much larger than any remaining numerical imprecision—see e.g. Figures 2 and 3.

Figure 3 shows the deposit return (d^*) across the probability of the aggregate liquidity shock (p). At $p = 0$ the Allen and Gale (2000) allocation is replicated by the mutual-default equilibrium. The results for $p > 0$ are summarized in the following numerical result.

Numerical Result 4. *The deposit return d^* increases in the probability of the liquidity shock p within each equilibrium default pattern, while it is discontinuous across types: d^* is highest in the mutual-default equilibrium, followed by single-default equilibrium and the no-default equilibrium.*

Figure 3: Deposit return d^* across the probability of the aggregate liquidity shock. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$ and $\varepsilon = 0.09$.



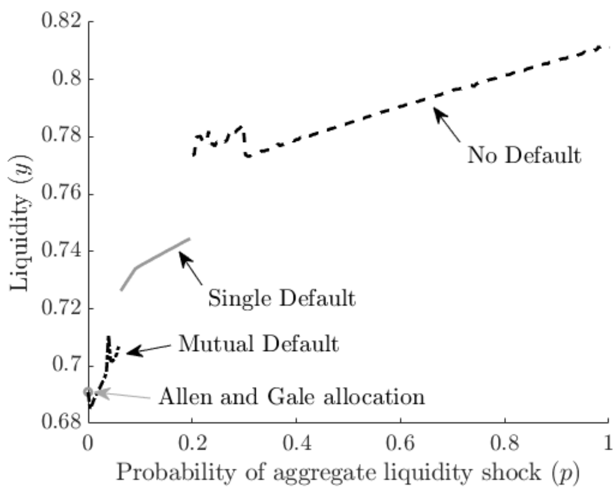
We provide some intuition for these results. The deposit contract provides liquidity to consumers. In the mutual-default equilibrium, banks fully insure the regional shock because the probability of the aggregate shock is low enough to make it optimal to ignore states in which the shock occurs (states 3 and 4). This allows a high degree of ex-ante liquidity provision without the shock (states 1 and 2), at the cost of accepting an ex-post inefficient allocation in states 3 and 4 that have a low ex-ante weight in the objective function. Thus, the deposit return is highest in the mutual-default equilibrium (and the average spread between early and late consumption in states 1 and 2 the lowest). When the mutual-default equilibrium obtains, a deviation to single default is not optimal, as this requires a reduction in liquidity insurance in high probability states (1 and 2) to improve liquidity insurance in a low probability state (3 or 4).

In the single-default equilibrium, each bank must provide the deposit return in three states (states 1-3 for bank A). This allows a lower degree of ex-ante risk sharing, i.e. a lower deposit return. In the no-default equilibrium, banks must provide the deposit return in all states. This allows for the lowest degree of liquidity provision: A lower deposit return and a greater average spread between early and late consumption. Finally, within each equilibrium default pattern, the expected size of early consumers increases in the probability of the aggregate shock, so the deposit return increases in the probability.

Figure 4 shows the equilibrium liquidity (y^*) across the probability of the aggregate liquidity shock (p). At $p = 0$ the Allen and Gale (2000) allocation is replicated by the mutual-default equilibrium. The results for $p > 0$ are summarized in the following numerical result.

Numerical Result 5. *Within each equilibrium default pattern, liquidity y^* increases in the probability of the aggregate liquidity shock p. Liquidity can change discontinuously across equilibrium default patterns.*

Figure 4: Liquidity y^* across probabilities of the aggregate liquidity shock. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$ and $\varepsilon = 0.09$.

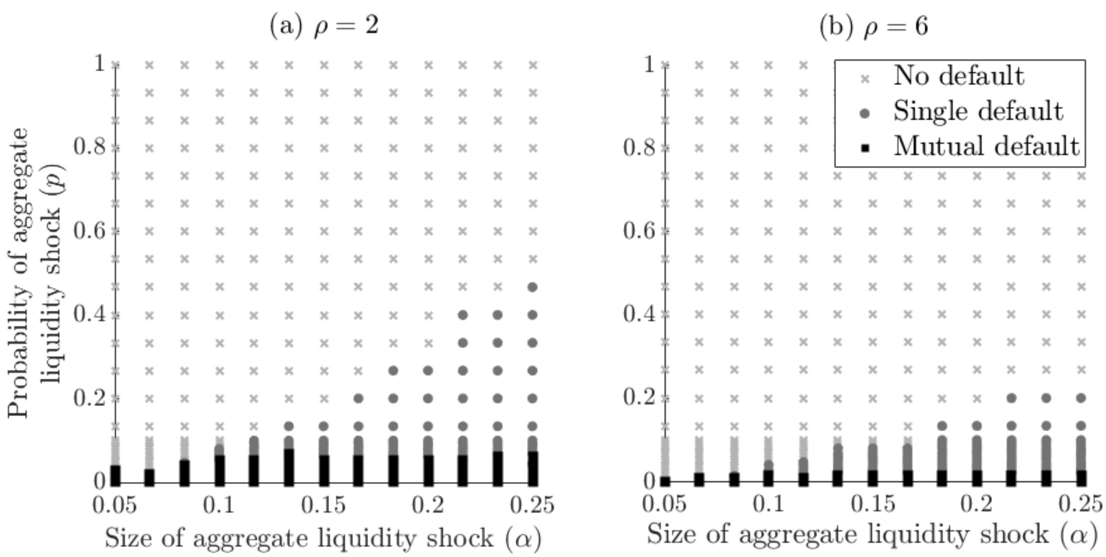


We provide some intuition for these results. Liquidity is held to serve early withdrawals. Thus, within every equilibrium default pattern, liquidity increases in the probability of the aggregate shock (since the deposit return increases). When the equilibrium default pattern switches from mutual- to single-default, liquididty increases discretely (mirroring the discrete decrease in deposit return) as the bank must trade-off pay-out in default and no-default states.

4.2 Prevalence of contagion

In this subsection we examine how prevalent contagion is. We first continue the example from the previous subsection. Figure 5 shows the prevalence of contagion in terms of the probability p and size α of the aggregate liquidity shock. (This figure corresponds to Figure 11 in the benchmark.) We summarize the findings in the following result.

Figure 5: Equilibrium default patterns across the probability p and size α of the aggregate liquidity shock. For illustration, we consider a grid of values of p and α, with a denser grid for low values of p. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$ and $\varepsilon = 0.09$.



Numerical Result 6. *Prevalence of contagion (for parameters used in Section 4.1).*

(i) Contagion occurs when the aggregate liquidity shock is unlikely enough.

(ii) The parameter space for which contagion occurs decreases in the degree of risk aversion (ρ).

(iii) The boundary above which no default occurs increases in α.

Turning to the main analysis, we study the full parameter space via a large number of random draws. Table 3 states the parameter space. For each value of the risk aversion parameter $\rho \in \{2, 4, 6, 8\}$, the other parameters are drawn from independent uniform distributions.

We present the results on the prevalence of contagion via two metrics. The first is a measure of the parameter space in which contagion can occur, denoted $\overline{\mu}_{contagion}$. This metric is the frequency of parameter draws for which the mutual-default equilibrium is superior. For every parameter draw i, the equilibrium default pattern is computed. We construct an indicator variable $i_{contagion}$, where $i_{contagion} = 1$ if the equilibrium default pattern is mutual default. The metric then simply sums the indicator variable standardized by the total number of parameter sets:

$$\overline{\mu}_{contagion} = \frac{1}{N} \sum_{i=1}^{N} i_{contagion}. \tag{15}$$

Table 3: Parameters for analysing the prevalence of contagion (random draws).

Parameter Name	Symbol	Value/Range
Utility function	$u(c)$	$\frac{c^{1-\rho}-1}{1-\rho}$
Risk aversion coefficient	ρ	$\{2,4,6,8\}$
Investment return upon maturity	R	$(1,10)$
Investment return upon liquidation	r	$(0,1)$
Average liquidity demand	γ	$(0,1)$
Aggregate liquidity shock—probability	p	$(0,1)$
Aggregate liquidity shock—size	α	$(0,\frac{1-\gamma}{2})$
Regional liquidity shock—size	ε	$(0,\min\{2\alpha,\gamma,1-\gamma\})$

Table 4: Prevalence of contagion (random draws from the parameter space). N is the number of draws, $\overline{\mu}_{contagion}$ denotes the frequency of parameter draws for which the mutual-default equilibrium is expected utility-superior, and $\underline{\mu}_{contagion}$ is the probability-weighted frequency of parameter draws for which the mutual-default equilibrium is expected utility-superior.

	Risk aversion parameter ρ				
	All	2	4	6	8
N	8 000	2 000	2 000	2 000	2 000
$\overline{\mu}_{contagion}$	0.0301	0.0409	0.0316	0.0305	0.0169
$\underline{\mu}_{contagion}$	0.0025	0.0032	0.0023	0.0025	0.0021

This first metric is an upper bound on the ex-ante likelihood of contagion. Even when the mutual-default equilibrium is superior for a parameter draw i, contagion occurs only if the aggregate liquidity shock is realized (with the probability of the aggregate liquidity shock in that draw, p_i). Thus, the second metric, $\underline{\mu}_{contagion}$, is calculated as the sum of probabilities in the parameter sets where contagion is possible standardized by the total number of parameter sets:

$$\underline{\mu}_{contagion} = \frac{1}{N}\sum_{i=1}^{N} p_i \cdot i_{contagion}. \tag{16}$$

Table 4 presents the results on the prevalence of contagion based on the two metrics described above. We summarize the findings in the following numerical result.

Numerical Result 7. *Contagion is rare. It occurs in* 3% *of the sampled parameter space.*

5 Benchmark: A global bank

In this section, we describe a benchmark for the decentralized interbank market examined in Section 3.[9] We label this benchmark a *global bank* because it is subject to two constraints common in the banking literature (Diamond and Dybvig, 1983; Allen and Gale, 2000). First, a bank is constrained to offer a demand-deposit contract, which is a contract that offers a fixed payment at $t = 1$ unless the bank defaults, $c_{1ks} \equiv c_{1k}$. Second, a bank cannot observe consumer types, so it must offer an incentive-compatible contract, $c_{1ks} \leq c_{2ks}$, in order to avoid default in state s. That is, late consumers can pretend to be early consumers and withdraw from the bank at $t = 1$.

Because of free entry, the global bank maximizes the expected utility of consumers in all regions subject to non-negative profits (Diamond and Dybvig, 1983; Allen and Gale, 2000, 2007). At $t = 0$, the bank chooses the regional portfolios of investment (x) and liquidity (y) and the regional deposit return (d) promised on withdrawals at $t = 1$. The global bank differs from decentralized regional banks (studied in section 3) as there are no explicit interbank deposits at each date. Instead, the global bank can freely transfer resources across regions.

The non-state-contingency of deposits implies that default (in one or both regions) is the only tool for the global bank to shift aggregate liquidity risk from late to early consumers. We allow the global bank to default *selectively* (in only one region). That is, one can interpret this benchmark as a constrained planner that operates independent banks (labelled *subsidiaries* henceforth) in each region and can freely transfer resources between regional subsidiaries.

There are three possible ex-post outcomes labelled as *regimes*: (i) a *no-default (ND) regime* where the neither subsidiary ever defaults; (ii) a *single-default (SD) regime*, where only the subsidiary hit by the aggregate liquidity shock defaults; and (iii) a *mutual-default (MD) regime*, where both subsidiaries default if the aggregate liquidity shock hits either region. We characterize below the model parameters for which each regime is optimal ex ante (at $t = 0$).

The global bank thus has two decisions to make: which regime is optimal; and, conditional on the regime, what choice of portfolios and deposit returns maximizes ex ante utility. The complete problem of the global bank (P1) is therefore characterized by the value function:

$$V = \max\{V_{ND}, V_{SD}, V_{MD}\}. \tag{P1}$$

We start by studying the optimal allocation in each regime.

[9] We also characterize the benchmark of autarky (no banks are available to consumers) in Appendix D.

5.1 No-default regime

Without default, the promised deposit return d is paid in all states. Strictly concave utility combined with ex-ante symmetry implies that (i) the same portfolio y and deposit return is chosen for both subsidiaries at $t = 0$; and (ii) consumers in both regions are treated identically at $t = 1$ and $t = 2$. Hence, the problem reduces to one in which the average liquidity demand is low (γ) in states 1 and 2 and high ($\gamma + \alpha$) in states 3 and 4. The identical treatment is feasibly implemented with transfers between subsidiaries: at $t = 1$ the transfer is $d\,\varepsilon$ in states 1 and 2 and $d\,\alpha$ in states 3 and 4. At $t = 2$, it is $c_{2L}\,\varepsilon$ in states 1 and 2 and $c_{2H}\,\alpha$ in states 3 and 4, where c_{2L} denotes the consumption level of late consumers in states 1 and 2 and c_{2H} in states 3 and 4, respectively.

Optimality requires (i) holding enough liquidity at $t = 0$ to avoid certain liquidation of investment at $t = 1$ as early consumption in states 1 and 2 is best provided with liquidity because $r < 1$; and (ii) not holding more liquidity than needed in states 3 and 4 because $R > 1$. In sum:

$$d\gamma \le y \le d(\gamma + \alpha). \tag{17}$$

There is either *excess liquidity*, $e \equiv y - d\gamma \ge 0$, in states 1 or 2 (when $y \ge d\gamma$), or *partial liquidation*, $\lambda \equiv \frac{d\gamma - y}{r} \ge 0$, in states 3 or 4 (when $y \le d(\gamma + \alpha)$), or both. We state the solution in terms of these choice variables because their optimal values are monotonic in the probability of the aggregate liquidity shock. Using condition (17), the regional transfers, and the definitions for excess liquidity and partial liquidation, the global bank's problem in the no-default regime is

$$V_{ND} \equiv \max_{e\ge 0,\lambda\ge 0} (\gamma + p\alpha)\,u(d) + (1-p)(1-\gamma)\,u(c_{2L}) + p(1-\gamma-\alpha)\,u(c_{2H}) \text{ s.t.} \tag{P1a}$$

$$\begin{aligned}
d &\equiv \frac{e + r\lambda}{\alpha},\\
c_{2L} &\equiv \frac{R}{1-\gamma} - \frac{\frac{\gamma+\alpha}{\alpha}R - 1}{1-\gamma}e - \frac{\frac{r\gamma}{\alpha}R}{1-\gamma}\lambda,\\
c_{2H} &\equiv \frac{R}{1-\gamma-\alpha} - \frac{\frac{\gamma+\alpha}{\alpha}R}{1-\gamma-\alpha}e - \frac{\frac{r\gamma+\alpha}{\alpha}R}{1-\gamma-\alpha}\lambda,\\
c_{2L} &\ge d,\; c_{2H} \ge d,
\end{aligned}$$

where the objective function is normalized to a unit measure of consumers to make it comparable to section 3 and the constraints reflect non-negativity, three accounting identities of consumption levels, and incentive compatibility (ICC).

We characterize the solution to problem (P1a) as follows. Lemma 4 states the solution assuming slack incentive compatibility constraints, $c_{2L} \ge d$ and $c_{2H} \ge d$. Proposition 1 states the parameter regions in which the incentive compatibility constraint is indeed slack. And Proposition 2 states the solution when the incentive compatibility constraint binds. The solution is characterized by various bounds on the probability of the aggregate liquidity shock, summarized in table 7 in Appendix C.

Lemma 4. ***Slack incentive compatibility constraint in no-default regime.*** *The optimal allocation chosen by the global bank is characterized by two unique thresholds* $\underline{p}_{ND} < \overline{p}_{ND}$*:*

(i) For a sufficiently probable aggregate liquidity shock, $p \geq \overline{p}_{ND}$*, no partial liquidation occurs,* $\lambda^*_{ND} = 0$*. There exists a unique level of excess liquidity* $e^*_{ND} > 0$ *that increases in the probability of the aggregate liquidity shock,* $\frac{de^*_{ND}}{dp} > 0$*.*

(ii) For a sufficiently improbable aggregate liquidity shock, $p \leq \underline{p}_{ND}$*, no excess liquidity is held,* $e^*_{ND} = 0$*. There exists a unique level of partial liquidation* $\lambda^*_{ND} > 0$ *that decreases in the probability of the aggregate liquidity shock,* $\frac{d\lambda^*_{ND}}{dp} < 0$*.*

(iii) For $\underline{p}_{ND} < p < \overline{p}_{ND}$*, there exists a unique interior solution in which both excess liquidity is held and partial liquidation occurs,* $\lambda^*_{ND} > 0$ *and* $e^*_{ND} > 0$*, with* $\frac{de^*_{ND}}{dp} > 0$ *and* $\frac{d\lambda^*_{ND}}{dp} < 0$*.*

Proof. See Appendix C.1.1. ∎

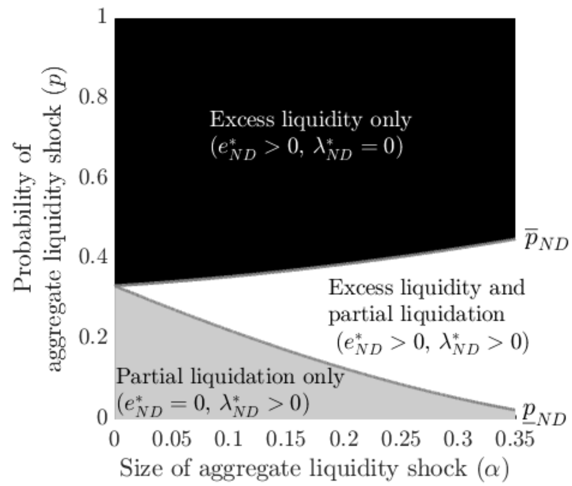
When the no-default regime is characterized by a slack incentive compatibility constraint, the global bank uses two instruments to balance the marginal utility of consumers across states: excess liquidity and partial liquidation (see Figure 6). The relative attractiveness of each depends on parameters, especially the probability and size of the aggregate liquidity shock (p, α).

To obtain intuition for the monotonicity of optimal choices in the probability of the aggregate liquidity shock (shown in Figure 7), we consider two cases. First, when either state 1 or 2 is realized, any excess liquidity is inefficient ex post. More resources could have been held in the productive investment, while whatever level of partial liquidation was chosen for states 3 and 4 does not affect ex-post efficiency. Second, when either states 3 or 4 is realized, any partial liquidation is inefficient ex post, since more liquidity would have been desirable. Taken together, an increase in the probability of states 3 or 4 makes allowing for excess liquidity in states 1 and 2 relatively more desirable than allowing for partial liquidation in states 3 and 4.[10]

We turn next to characterizing the parameter regions for which the incentive compatibility constraint need not be considered for the full problem, i.e. where it is slack.

[10] The same argument does not apply to the choice of liquidity y. When the probability p is low, no excess liquidity is held, there is partial liquidation and $c_{2H} < c_{2L}$ (since there is a low weight on states 3 and 4). As p increases, it is optimal to transfer more resources to states 3 and 4, so c_{2H} increases relative to c_{2L}. Since $e^*_{ND} = 0$ when $p \approx 0$, this happens by *reducing* liquidity (hence increasing investment) and reducing partial liquidation. As p increases further, it eventually is optimal to cease using partial liquidation and to use excess liquidity to finance c_{2L}, which requires *increasing* liquidity along with excess liquidity. Hence, the optimal choice of liquidity is non-monotonic, similar to a result for a single bank in Ennis and Keister (2006) (Proposition 5).

Figure 6: Global bank choices in the no-default regime for slack incentive compatibility constraints. The use of partial liquidation and excess liquidity varies with the size and probability of the aggregate shock (Lemma 4). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 2$, $r = 0.5$, $\gamma = 0.3$.



Proposition 1. *In the no-default regime, the incentive compatibility constraint is always slack in states 1 and 2. In states 3 and 4, it is slack if and only if $p \geq \hat{p}^{IC}$, where $\hat{p}^{IC} \in \left[0, \frac{(R-1)r}{R-r}\right)$ is unique. Therefore, the global bank's choices in the no-default regime are given in Lemma 4 for $p \geq \hat{p}^{IC}$.*

Proof. See Appendix C.1.2. ■

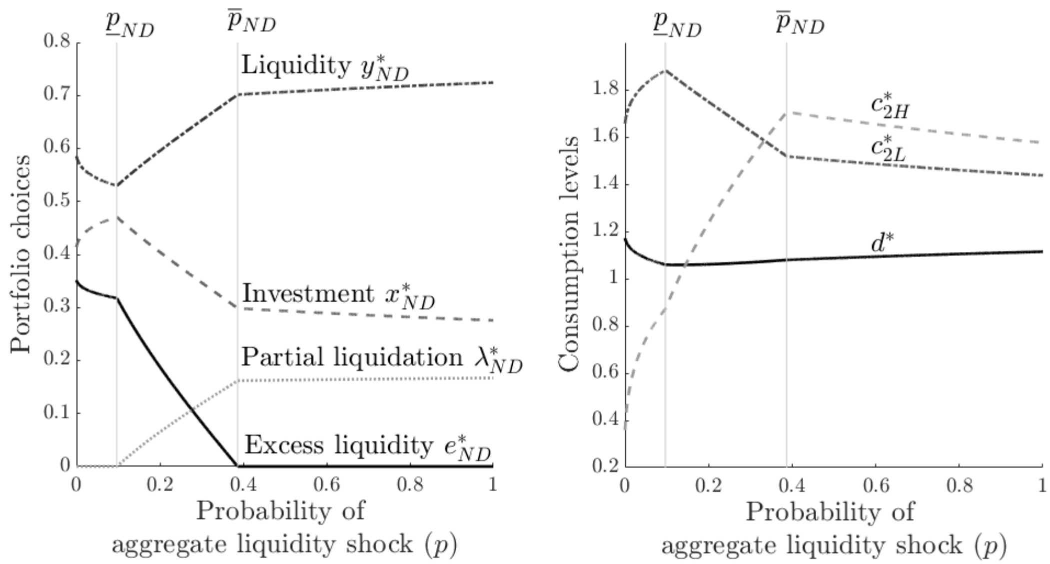
Strict risk aversion implies that $d^* < \max\{c^*_{2L}, c^*_{2H}\}$. Since fewer resources are required to fund d^* when the aggregate liquidity shock does not occur, $c^*_{2L} > d^*$ is feasible whenever $c^*_{2H} \geq d^*$ is. Thus, the incentive compatibility constraint can only bind in states 3 and 4 (see the proof of Proposition 1). When states 1 and 2 (with low aggregate liquidity demand) are likely (low p), optimal liquidity insurance places more weight on these states. Thus, $c^*_{2L} > c^*_{2H}$ and the deposit return d^* is relatively large to minimize spread between early and late consumption. If the aggregate liquidity shock does occur, however, a large amount of costly partial liquidation is required to fund d^*, resulting in a low consumption level for late consumers, c^*_{2H}. It can be so low to violate incentive compatibility if this constraint were excluded (see, for example, Figure 7). Finally, we obtain $\hat{p}^{IC} < \frac{(R-1)r}{R-r} < 1$ because $c^*_{2H} = c^*_{2L}$ at $p = \frac{(R-1)r}{R-r}$.

We next describe the optimal allocation when the incentive compatibility constraints bind.

Proposition 2. ***Binding incentive compatibility constraint in no-default regime.*** *For $p < \hat{p}^{IC}$, the global bank's allocation is characterized by the unique threshold $\underline{p}^{IC}_{ND} < \underline{p}_{ND}$:*

(i) For a sufficiently improbable aggregate liquidity shock, $p \leq \underline{p}^{IC}_{ND}$, only partial liquidation

Figure 7: Global bank choices of portfolio and consumption in the no-default regime when incentive compatibility constraints are ignored. The left panel shows that the choices of excess liquidity (e^*_{ND}) and partial liquidation (λ^*_{ND}) are monotonic in p whereas liquidity (y^*_{ND}) and investment (x^*_{ND}) are not. The right panel shows that, for low enough probability of the aggregate liquidity shock, the solution to the relaxed problem (see Lemma 4) violates incentive compatibility ($c^*_{2H} < d^*$). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 2$, $r = 0.5$, $\gamma = 0.5$, $\alpha = 0.15$.



is used, $e^*_{ND} = 0$, where $\lambda^*_{ND} > 0$ is independent of the utility function and the probability:

$$y^*_{ND} = \frac{\gamma r R}{\gamma r R + \alpha R + r(1-\gamma-\alpha)}, \qquad d^* = c^*_{2H} = \frac{y^*_{ND}}{\gamma}, \qquad c^*_{2L} = \frac{R(1-y^*_{ND})}{1-\gamma}.$$

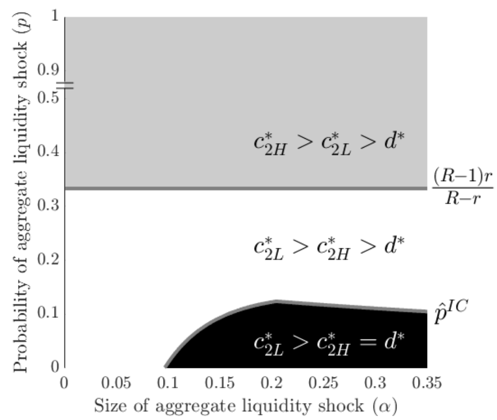
(ii) If $p > \underline{p}^{IC}_{ND}$, a global bank chooses a portfolio with both partial liquidation, $\lambda^*_{ND} > 0$, and excess liquidity, $e^*_{ND} > 0$, with optimal allocation $c^*_{2L} > c^*_{2H} = d^*$. The optimal liquidity holding y^*_{ND} uniquely solves:

$$u'(c_{2H}(y^*_{ND})) = \frac{(1-p)(r(R(1-\alpha-2\gamma)-(1-\alpha-\gamma))+R(R(\alpha+\gamma)-\alpha))}{(1-r)R(\gamma+(1-\gamma)p)} u'(c_{2L}(y^*_{ND})).$$

Proof. See Appendix C.1.3. ■

Lemma 4 characterizes the solution to a relaxed version of problem (P1a) where the incentive compatibility constraint is ignored. Figures 7 and 8 show that there are parameter regions for which incentive compatibility binds. When the probability of the aggregate liquidity shock is low enough and using excess liquidity is not optimal, the optimal allocation is fully pinned down by the constraints ($e^*_{ND} = 0$ and binding incentive compatibility). Otherwise, the binding constraint is $c_{2H} = d$ and only the level of liquidity remains a free choice.

Figure 8: Global bank allocations in the no-default regime and incentive compatibility. The IC constraint binds for a small probability of the aggregate liquidity shock, $p < \hat{p}_{IC}$. If $p < \frac{(R-1)r}{R-r}$, $c^*_{2H} < c^*_{2L}$, otherwise $c^*_{2H} \geq c^*_{2L}$. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 2$, $r = 0.5$, $\gamma = 0.3$.



The additional binding constraint (ICC) also means that the global bank can provide less liquidity insurance. When the incentive compatibility constraint binds, the global bank uses less partial liquidation (because c_{2H} is must be maintained at a higher level and d^* at a lower level) and starts using excess liquidity for lower probabilities of the aggregate liquidity shock than it would without the additional constraint ($\underline{p}^{IC}_{ND} < \underline{p}_{ND}$).

5.2 Single-default regime

In the single-default regime, the global bank chooses the default of the subsidiary in the region hit by the aggregate liquidity shock. Thus, the subsidiary in region A defaults in state 4 and the one in region B defaults in state 3. Upon default both early and late consumers of the defaulting subsidiary receive a pro-rata share of the liquidation value of the subsidiary's assets, $c_D \equiv y + rx$.

By ex-ante symmetry and strict concavity of the utility function, the same liquidity y and deposit return d is chosen for each subsidiary of the global bank. In states 1 and 2, there is no default and early consumers in both regions receive d. Early liquidity demand is $d(\gamma - \varepsilon)$ in region A and $d(\gamma + \varepsilon)$ in region B in state 1, and vice versa in state 2. Equal consumption of all late consumers, c_2, is feasible with a transfer of $d\varepsilon$ across regions at $t = 1$ combined with a reverse transfer of $c_2\varepsilon$ at $t = 2$. In states 3 and 4, it is also feasible to provide c_2 without any transfers in regions not hit by the aggregate liquidity shock because the early liquidity demand $d\gamma$ equals the average early liquidity demand in states 1 and 2. Hence, equal treatment of consumers in regions and states without the aggregate liquidity shock is feasible and optimal by concavity.[11]

[11] The consumption level in the region not hit by the shock generically differs from the level in the shocked region, $c_2 \neq c_D$, as selective default makes consumption levels state-contingent.

In states 3 and 4, there is default and full liquidation of assets of the subsidiary hit by the shock. Partial liquidation without default cannot be optimal ex ante as it would imply some liquidation with certainty. Optimality in the single-default regime thus requires $y \geq d\gamma$. However, some excess liquidity $e \equiv y - d\gamma \geq 0$ may be optimal as it redistributes resources from late consumers in region-state pairs without default to all consumers in region-state pairs with default.

Taken together, the global bank's problem in the single-default regime, (P1b), reduces to:

$$V_{SD} \equiv \max_{y \leq 1, e \geq 0} \frac{2-p}{2} \left[\gamma u(d) + (1-\gamma) u(c_2) \right] + \frac{p}{2} u(c_D) \text{ s.t.} \tag{P1b}$$

$$d \equiv \frac{y-e}{\gamma}, \quad c_2 \equiv \frac{e + R(1-y)}{1-\gamma}, \quad c_D \equiv y + r(1-y), \quad c_2 \geq d,$$

where the first three constraints are accounting identities for consumption (without and with default of the subsidiary) and the final constraint is for incentive compatibility (ICC).

We solve the relaxed problem (without imposing the ICC) and then show that incentive compatibility always holds in the single-default regime. The intuition for this result is as follows. In the no-default regime, the binding ICC arises from early consumption (demand deposits) not being state-contingent such that the cost of the liquidity shock is entirely imposed on late consumers. In the single-default regime, by contrast, default arises in states where the non-state-contingency requirement is most costly, allowing the costs of the liquidity shock to be shared among early and late consumers.

Proposition 3 characterizes the solution to problem (P1b) and Figure 9 visualizes it.

Proposition 3. ***Single-default regime.*** *The optimal allocation chosen by the global bank is characterized by two unique bounds* $\left(\underline{p}_{SD}, \overline{p}_{SD}\right)$ *with* $0 < \underline{p}_{SD} < \overline{p}_{SD}$. *There are three cases:*

(i) For a sufficiently probable aggregate liquidity shock, $p \geq \overline{p}_{SD} \equiv 2\frac{R-1}{R-r}$, *all resources are kept in liquidity,* $y^*_{SD} = 1 = c^*_2 = d^*$, *resulting in excess liquidity* $e^*_{SD} = 1 - \gamma$.

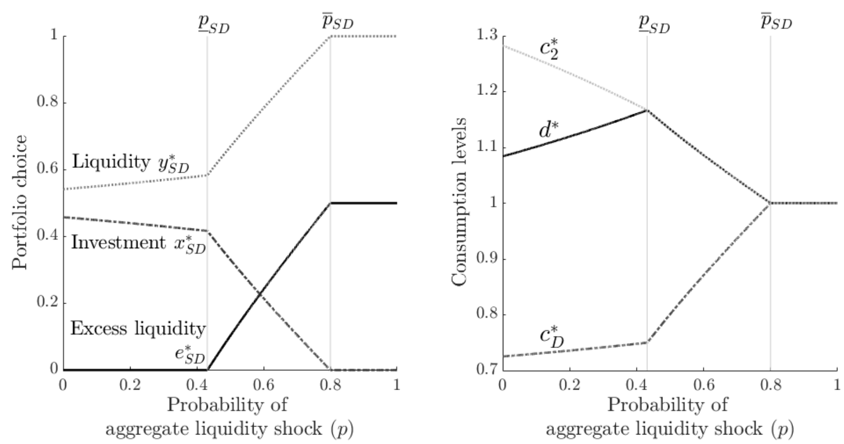
(ii) For a sufficiently improbable aggregate liquidity demand shock, $p \leq \underline{p}_{SD}$, *no excess liquidity is held,* $e^*_{SD} = 0$. *There exists a unique interior optimal level of liquidity* $y^*_{SD} \in (0,1)$ *with* $\frac{dy^*_{SD}}{dp} > 0$ *and associated consumption levels* $c^*_2 > d^* > c^*_D$.

(iii) For an intermediate probability of the aggregate liquidity shock, $\underline{p}_{SD} < p < \overline{p}_{SD}$, *there exists a unique interior solution* $0 < y^*_{SD} < 1$ *with* $\frac{dy^*_{SD}}{dp} > 0$, *and some excess liquidity held:* $e^*_{SD} = y^*_{SD}(1 + (R-1)\gamma) - R\gamma$. *The associated consumption levels are* $c^*_2 = d^* > c^*_D$.

Proof. See Appendix C.2. ■

For a low probability of the aggregate liquidity shock, it is too costly from an ex-post perspective to allow for excess liquidity in states without default. As the probability increases, the expected utility places more weight on states with default, so the consumption in this state, c_D^*, increases, requiring an increase in liquidity, y_{SD}^*. As liquidity increases, d^* increases and c_2^* falls, eventually reaching equality. As p increases further, excess liquidity is held in region-state pairs without default to maintain this equality (because of ICC). While the allocation in this region is incentive compatible, $c_2^* = d^*$, this does not impose an efficiency cost on the allocation. Indeed, it is a feature of the optimal allocation without imposing the incentive compatibility constraint explicitly, which is an optimal insurance outcome due to risk aversion (see also the proof in Appendix C.2).[12] Eventually, the risk of default becomes so probable that it is best not to invest at all in order to avoid costly liquidation of investment.

Figure 9: Global bank portfolio and consumption choices in the single-default regime. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 1.4$, $r = 0.4$, $\gamma = 0.5$, $\alpha = 0.10$.



5.3 Mutual-default regime

The only difference between the single default and mutual-default regime is the weighting of the outcomes with and without default, respectively. Since the analysis and economic intuition follows that of the single-default regime, our exposition in the main text is brief. The difference is that both subsidiaries of the global bank default when the aggregate liquidity shock hits one region. Accordingly, the problem of the global bank in the mutual-default regime, P1c, can be expressed as

$$V_{MD} \equiv \max_{y \le 1, e \ge 0} (1-p)\left[\gamma u\left(d\right) + (1-\gamma) u\left(c_2\right)\right] + p u\left(c_D\right) \text{ s.t.} \tag{P1c}$$

[12]That is, in solving the problem in the single-default regime, $d > c_2 > c_D$ is allowed (the ICC is not imposed) and is feasible (by reducing e) but it is dominated by $d = c_2$ by strict concavity of the utility function (perfect risk sharing). This is different from the no-default regime in which $c_{2L} > d > c_{2H}$ would have been optimal for some parameters but is infeasible because of a violation of the ICC.

$$d \equiv \frac{y-e}{\gamma}, \quad c_2 \equiv \frac{e+R(1-y)}{1-\gamma}, \quad c_D \equiv y+r(1-y), \qquad c_2 \geq d.$$

The incentive compatibility constraint is again slack in the mutual-default regime. Proposition 4 characterizes the solution to problem (P1c).

Proposition 4. ***Mutual-default regime.*** *The global bank's allocation is characterized by two unique thresholds of the probability of the aggregate shock* $\left(\underline{p}_{MD}, \overline{p}_{MD}\right)$ *with* $0 \leq \underline{p}_{MD} < \overline{p}_{MD} < 1$*:*

(i) For $p \geq \overline{p}_{MD} \equiv \frac{R-1}{R-r}$*, all resources are kept in liquidity,* $y^*_{MD} = 1 = c^*_2 = d^*$ *and* $e^*_{MD} = 1-\gamma$*.*

(ii) For $p \leq \underline{p}_{MD}$*, no excess liquidity is held,* $e^*_{MD} = 0$*. There exists a unique interior level of liquidity* $y^*_{MD} \in (0,1)$ *with* $\frac{dy^*_{MD}}{dp} > 0$ *and associated consumption levels* $c^*_2 > d^* > c^*_D$*.*

(iii) For $\underline{p}_{MD} < p < \overline{p}_{MD}$*, there exists a unique interior solution* $y^*_{MD} \in (0,1)$ *with* $\frac{dy^*_{MD}}{dp} > 0$ *and excess liquidity,* $e^*_{MD} = y^*_{MD}(1+(R-1)\gamma)-R\gamma > 0$*. The consumption levels are* $c^*_2 = d^* > c^*_D$*.*

Proof. See Appendix C.3. ∎

5.4 Characterizing the parameter regions where each regime is optimal

We state two analytical results about the ex-ante optimal choice of regime by the global bank.

Proposition 5. ***Comparison of regimes.*** *The global bank never chooses the mutual-default regime as it is (weakly) dominated by the single-default regime. Moreover, there exists a bound on the probability of the aggregate liquidity shock,* $\breve{p}$*, such that single-default regime is preferred over the no-default regime for all* $p < \breve{p}$*.*

Proof. See Appendix C.4. ∎

As intuition for the first result, note that the mutual-default regime involves more default than in the single-default regime, with associated lower consumption levels to investors.

To gain intuition for the second result, consider two figures that show the global bank's optimal choice of regime. Figure 10 shows which regime is optimal for different probabilities of the aggregate liquidity shock (p) when R is relatively high and r is relatively low. It shows that a unique $\breve{p}$ exists at which the regime choice switches from single default to no default.

Figure 11 shows the parameter boundary between the regimes in terms of the probability (p) and size (α) of the aggregate liquidity shock. When α is large enough, there is a positive

Figure 10: Regime choice by the global bank. We plot the expected utility for each regime. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$.

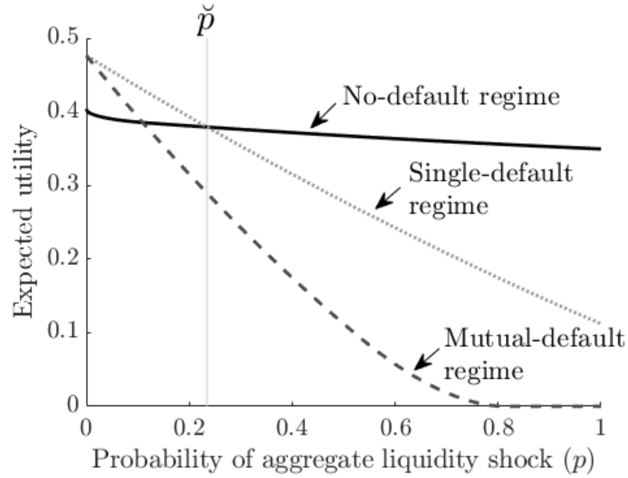
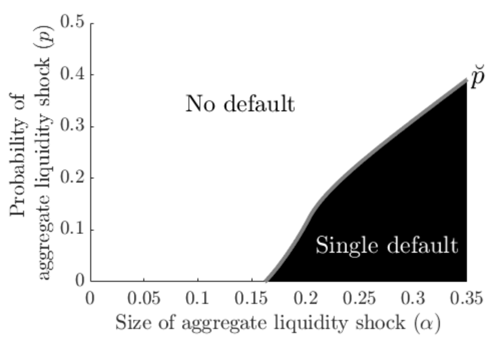


Figure 11: Regime choice by the global bank: single-default is preferred when the aggregate liquidity shock is unlikely ($p < \breve{p}$). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 2$, $r = 0.5$, $\gamma = 0.3$.



probability, $\breve{p}$ below which the single-default regime is optimal and above which the no-default regime is optimal. Moreover, $\breve{p}$ is increasing in α: as the shock becomes larger, it becomes optimal to choose the single-default regime for a larger measure of the probability of the shock.

Intuition for choice of optimal regimes arises from the cost (in expected utility terms) of offering a fixed (that is, non-state-contingent) deposit return, $c_{1ks} = d$ for all k, s. In the no-default regime for an unlikely aggregate liquidity shock, there is more weight on the utility in states without the liquidity shock. After the aggregate liquidity shock occurs, a fixed deposit return can only be achieved by using partial liquidation, so consumption levels at $t = 2$, c^*_{2H}, are low and the allocations are highly inefficient ex post. Since the utility function is concave and the consumption allocations in all states are linked via the fixed deposit return, this ex-post inefficiency is spread across all states. Hence, bank liquidity provision to investors is lower in states where the shock does not occur. (By construction of the no-default regime, default is not used to decouple the consumption levels in states with and without a liquidity shock.)

By contrast, the single-default regime provides the global bank with a tool to mitigate the non-state contingency constraint: default. This tool is equivalent to forcing the global bank to accept a large degree of ex-post inefficiency in the region hit by the aggregate liquidity shock to break the link across states induced by a fixed deposit return. Hence, the allocation without the aggregate liquidity shock is more efficient ex post. Since a low probability of an aggregate liquidity shock implies a high weight on states where such a shock does not occur, the planner chooses the single-default regime for a low enough probability of the aggregate shock, $p < \breve{p}$.

6 Constrained inefficiency and an alternative resolution regime

To study the efficiency of policy interventions, we first have to compare the decentralized equilibrium for non-cooperative banks to the benchmark of a single global bank. We do this numerically and then propose an alternative resolution regime and investigate how it improves efficiency relative to the decentralized equilibrium.

6.1 Comparing the decentralized equilibrium to the global bank benchmark

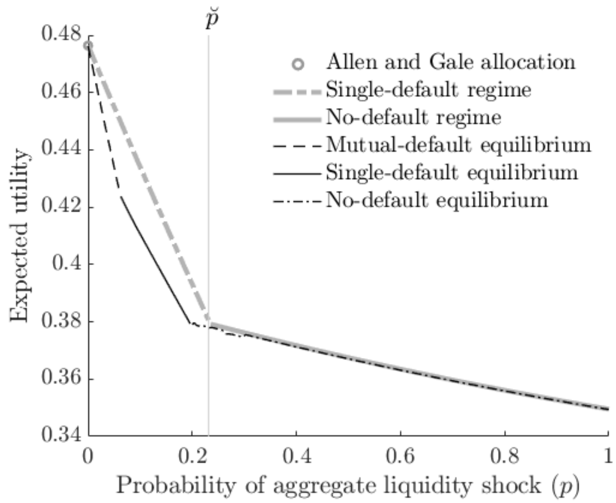
Figure 12 shows the expected utility of consumers in the decentralized equilibrium and in the global bank benchmark. The decentralized equilibrium achieves the benchmark expected utility at $p = 0$ and for high probabilities of the aggregate liquidity shock. But it is inferior for intermediate probabilities of the aggregate liquidity shock.

At $p = 0$ the allocation of Allen and Gale (2000) again obtains. For larger values, $p \in (0, \breve{p})$, the single-default regime obtains in the benchmark (gray dashed line) and is superior to the decentralized equilibrium. In this range, the equilibrium type is mutual default (black dot-dashed line, for small p), single default (dashed black line, for intermediate p), and no default for a small range just below $\breve{p}$ (solid black line). When the no-default regime obtains in the benchmark (solid gray line, $p > \breve{p}$), it achieves the same expected utility as the no-default equilibrium in the decentralized economy. The probability range in which the global bank chooses the no-default regime is smaller than the probability range in which the no default equilibrium obtains. We summarize these findings in the following numerical result.

Numerical Result 8. *The decentralized equilibrium is constrained inefficient for low but positive probability of the aggregate liquidity shock, $p \in (0, \breve{p})$.*

In the single-default regime of the global bank, consumption is identical across state-region pairs without default, $c_{1ks} = d^{GB}$ and $c_{2ks} = c_2^{GB}$ in states 1-3 for region A and in states

Figure 12: Expected utility of consumers in the decentralized equilibrium (black) and the benchmark (gray). The decentralized equilibrium is constrained inefficient for $p \in (0, \breve{p})$. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, $\varepsilon = 0.09$.



1,2 and 4 for region B. All consumers get an equal share of the liquidation return of the global bank's portfolio in states with default: $c_{1ks} = c_{2ks} = y^{GB} + r(1 - y^{GB})$ in state 4 for region A and in state 3 for region B.

To decentralize the global bank allocation, regional banks must hold the same liquidity y^{GB}, offer the same deposit return d^{GB}, and hold interbank deposits equal to the regional shock, $z^{GB} = \epsilon$. The regional bank A is also subject to the following withdrawal scheme (symmetric for bank B):

$$w_{A1}^{GB} = 0, \qquad w_{A2}^{GB} = \varepsilon, \qquad w_{A3}^{GB} = 0, \qquad w_{A4}^{GB} = 0.$$

The global bank allocation implies zero resource transfers between regional banks if the aggregate liquidity shock realises. Consider the decentralized global bank allocation and withdrawal specification from the perspective of bank A in state 3. By definition, bank A is not in default and can pay the promised deposit return d^{GB} to local early consumers. Bank B is defaulting but *not* claiming any of its interbank deposit from bank A. It is thus offering early withdrawal returns of $\frac{y^{GB}+r(1-y^{GB})}{1+w_{A3}}$. The global bank allocation requires bank A to claim none of its interbank position from bank B (i.e. $w_{A3}^{GB} = 0$). By deviating—i.e. unilaterally increasing its interbank withdrawal—bank A can increase local resources for payout to late consumers which is profitable.[13] In other words, these deviations in interim withdrawal behavior improve regional

[13]Similarly, consider state 4: By definition, bank A is in default. In the decentralized benchmark allocation, it liquidates all its assets *except* its interbank position (this is what $w_{A4}^{GB} = 0$ means in this context–abandoning claims on the interbank asset). In the benchmark allocation, bank B is not withdrawing any of its interbank position, so bank A pays out liquidation returns $c_{1A4} = c_{2A4} = y^{GB} + r(1 - y^{GB})$ to each local consumer. Bank B is not in default and pays d_B to early withdrawals. A deviation by bank A would be to increase withdrawals. In the benchmark

outcomes (though at the cost of worsening global outcomes). Thus the decentralized GB allocation with its implied withdrawal behaviour is not an equilibrium of the decentralized game.[14]

Figure 13 shows the interbank deposits in the decentralized economy and in the global bank benchmark. Recall from Section 5 that the equivalent of z for the global bank is the size of the transfer between the regional subsidiaries. Where the equilibrium is characterized by mutual default, the interbank deposit is identical to the benchmark and equal to the regional liquidity shock. Where the equilibrium type is single or no default, the interbank deposit is equal to the aggregate liquidity shock, larger than in the single default allocation of the benchmark. Banks are over-invested in the interbank market and obtain lower welfare than in the benchmark. This result gives further credence to large exposure limits, introduced as a reaction to the 2007/2008 global financial crisis (Basel Committee on Banking Supervision, 2014).[15] Finally, when both the equilibrium and the benchmark feature no default, the interbank deposits are as large as the aggregate liquidity shock.

Figure 13: Excessive co-insurance. Interbank deposit in the decentralized equilibrium (black) and the benchmark (gray). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$ and $\varepsilon = 0.09$.

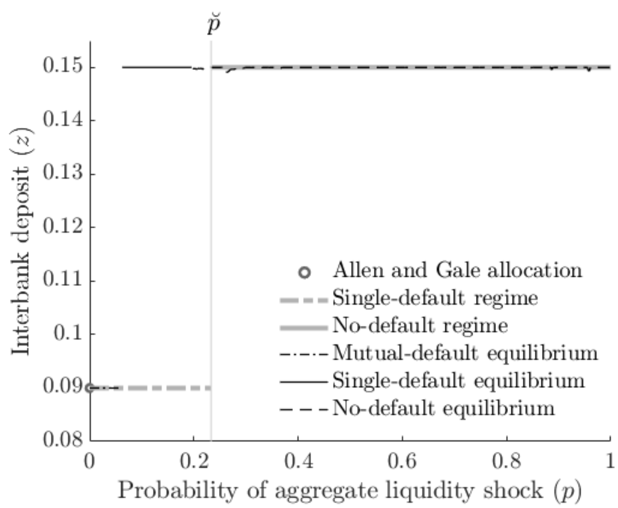


Figure 14 shows the deposit return in the decentralized economy and in the benchmark. When the equilibrium type is mutual default, the deposit return is similar to that of the global

allocation, this would induce default by bank B, but bank A would still receive a positive return on its withdrawals in the form of the liquidation returns from bank B. This deviation necessarily increases the liquidation returns to local consumers in region A, so it is a profitable deviation for bank A.

[14]A similar argument can be made that the global bank allocation in the no-default and mutual-default regimes cannot be supported as a Nash equilibrium. Decentralizing the global bank allocation in either regime also requires $w_{A3}^{GB} = 0$, whereas a unilateral deviation to positive withdrawals is profitable for the same reasons as in in the single-default regime.

[15]Under this framework, a bank's exposures are limited to no more than 25% of the bank's Tier 1 capital. For globally systemically important banks, the limit is even lower at 25% of Tier 1 capital.

bank benchmark. When both the equilibrium and the benchmark are characterized by single default, the deposit return is smaller than in the benchmark. The over-exposure to the interbank market, and hence the default of the other bank in some states allows a smaller degree of liquidity insurance to consumers. When the equilibrium type is no default but the benchmark regime is single default, the deposit return is much smaller than in the benchmark. Finally, when both the equilibrium and the benchmark feature no default, the deposit returns are roughly equal.

Figure 14: Insufficient liquidity provision to consumers. Deposit return in the decentralized equilibrium (black) and the benchmark (gray). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, $\varepsilon = 0.09$.

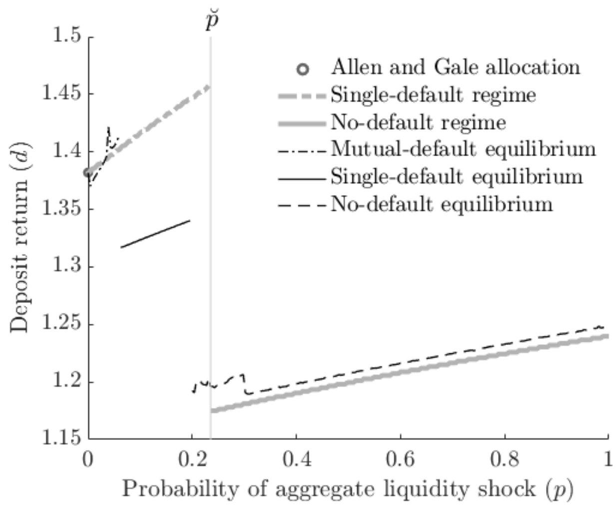
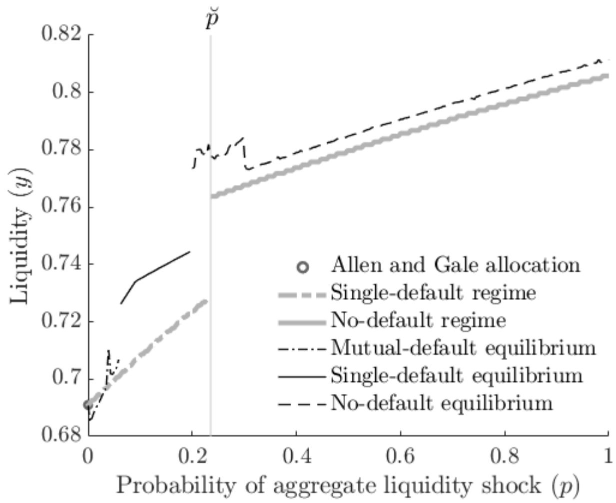


Figure 15 shows liquidity in the decentralized economy and in the benchmark. Except around $p = 0$, liquidity is higher in the decentralized economy than in the benchmark. This difference is most pronounced when the equilibrium type is no default and the benchmark regime is single default. The amount of liquidity directly follows from the equilibrium deposit returns and interbank deposits described above.

In summary, for low probabilities of the aggregate liquidity shock, regional banks promise a high degree of ex-ante liquidity insurance to local consumers in states without default and accept the risk of large degree of ex-post inefficiency due to financial contagion. Thus, banks co-insure against the regional liquidity shock only. For intermediate probabilities, the single-default equilibrium obtains—as in the benchmark regime—but non-cooperative banks choose higher interbank positions than in the benchmark. Combined with the impaired returns on interbank deposits in states where the counterpart bank defaults, this allows for an inefficiently low degree of liquidity insurance to consumers (low deposit returns) that must be supported with an inefficiently high liquidity holding. For a small range of probabilities, the decentralized equilibrium is characterized by no default, while global bank chooses the single-default

Figure 15: Excessive liquidity holdings. Liquidity in the decentralized equilibrium (black) and the benchmark (gray). Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, $\varepsilon = 0.09$.



regime. Finally, for a sufficiently high probability, both the equilibrium and the benchmark are characterized by no default.

6.2 An alternative resolution scheme

We have shown that, absent regulation, the decentralized equilibrium allocation yields lower expected utility than the global bank allocation. In this section, we describe a policy intervention that implements the global bank allocation as a Nash equilibrium. At its core is a *resolution scheme* in the case of bank default that is different from the laissez-faire solution studied above.

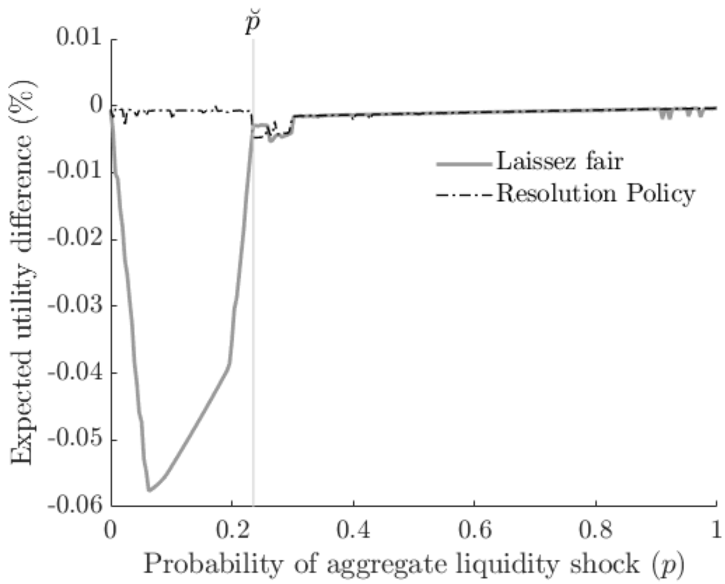
Consider the laissez-faire resolution scheme. For illustration, consider the case in which bank A defaults but bank B does not. Then, bank A is liquidated, so it withdraws the market value of its claim on bank B. Under laissez-faire, this claim is not impaired, $d_B z_A$ with $d_B > 1$. Bank B also receives the market value of its claim on A, but given that A is in default, B only receives the liquidation return $\frac{y_A + d_B z_A + r x_A}{1+z_B}$ that is less than 1 in equilibrium.[16]

Now consider an alternative resolution scheme where interbank claims are netted out at $t = 1$ at book value (i.e. at a gross return of 1) when either bank is in default. We show that this resolution policy at $t = 1$ results in a change to best responses at $t = 0$ such that the Nash equilibrium yields welfare that is numerically indistinguishable from that of the global bank (Figure 16). In this regulated equilibrium, when bank A is in default, it receives only $z_A < d_A z_A$ from its claim on bank B, and bank B receives a higher return than the liquidation returns of

[16]It is easy to construct asymmetric choice pairs where the liquidation return is larger than one, but this never occurs in equilibrium.

bank A.

Figure 16: Welfare comparison of the laissez-faire equilibrium and the resolution policy equilibrium. The figure shows the proportional difference in expected utility between the global bank benchmark and (i) the laissez-faire equilibrium (solid grey line), and (ii) the resolution policy equilibrium (black dash-dot line). The laissez-faire equilibrium yields welfare up to 5% smaller than the benchmark. In contrast the expected utility of the resolution regime is smaller by at most 1%. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, and $\varepsilon = 0.09$.



This resolution scheme affects the seniority of interbank deposits relative to demand deposits from consumers in an asymmetric way: if bank A is in default but bank B is not, bank A receives a lower return than consumer depositors at bank B, since $1 < d_B$. However, bank B receives a higher return than consumer depositors at bank A, who receive only the liquidation return which under the policy is $y_A + z_A + rx_A - z_B < 1$. If both banks are in default, the effect cancels out in symmetric equilibrium, as it does in the symmetric market equilibrium. The intuition behind this is simple: in symmetric equilibrium, interbank positions are identical $z_A = z_B$, so the net transfer of resources in the case of any default is zero, which also characterizes the single default allocation of the global bank.

In the laissez-faire case, the per-unit net transfer to a defaulting bank from a non-defaulting bank makes the interbank deposit a less desirable asset ex ante. This leads to a single-default equilibrium where the interbank position is half the size necessary to insure against regional liquidity risk in the absence of the aggregate shock. The resolution scheme, by contrast, makes the return on an interbank claim state contingent for both defaulting and surviving banks, equal to unity under any default. This scheme removes the net transfer, which makes the interbank deposit more attractive ex ante.

When the probability of the aggregate shock is low enough, the laissez-faire equilibrium features full insurance of regional liquidity demand risk in the absence of the aggregate shock, at the cost of inefficient mutual default if the shock occurs. Under the resolution scheme, the state-contingent return on interbank claims changes the attractiveness of the interbank position in such a way that decentralized banks choose a portfolio that allows them to survive when the other bank fails and the inefficient contagion outcome no longer obtains in equilibrium.

We summarize these findings in our final numerical result:

Numerical Result 9. *Consider a resolution scheme that imposes the netting out of interbank positions at book value given the default of any bank. This scheme leads to an equilibrium (i) without contagion at positive probability of the aggregate liquidity shock, and (ii) achieves the welfare of the aggregate benchmark within numerical precision.*

7 Conclusion

Since the seminal work of Allen and Gale (2000) we know that some networks of interbank deposits are more susceptible to contagion than others. However, these results are obtained under the assumption that banks, when choosing their portfolios ex ante, assign a probability of zero to the aggregate liquidity shock. This raises the question whether contagion is possible if banks are allowed to choose their portfolios while anticipating an ex-post aggregate liquidity shock. The analytical challenges involved in answering this question are formidable, however, making closed-form analytical solutions impractical. In this paper we develop a simple but robust algorithm to approximate pure strategy Nash equilibria in the Allen and Gale (2000) environment in which two banks anticipate a positive probability of an aggregate liquidity shock.

We obtain three main results. First, the type of equilibrium depends on the probability of the aggregate liquidity shock. Mutual default—interbank contagion—is an equilibrium outcome, but only for 5% of the overall parameter space. We furthermore characterize the parameter region in which contagion is an equilibrium outcome. Second, we fully characterize banks' equilibrium portfolio choices, including the amount of co-insurance via interbank deposits. Several authors have studied this question, but a full characterization has been elusive because the problem is not analytically tractable. Since we can study banks' ex-ante portfolio choices, our setup allows us to study welfare and regulation as well. Accordingly, we show, third, that for small shock probabilities the equilibrium outcome is constrained inefficient. We then propose an alternative bank resolution regime in which claims are netted ex-post at their book value when either bank is in default. This regime restores constrained efficiency.

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A Numerical Robustness and Accuracy

A.1 Robustness of Numerical Equilibrium

For each symmetric default pattern independently, the algorithm in section 3.4.3 searches over the three-dimensional choice vector of an individual bank $\theta_k \equiv \{x_k, y_k, z_k\}$ imposing the optimal withdrawal behaviour derived in section 3.1. Here, we show that the equilibrium found in this way is robust to a full seven-dimensional search, which allows arbitrary withdrawal behaviour in period $t = 1$ and arbitrary default patterns.

The generalized search algorithm starts at the equilibrium found with the three-dimensional algorithm, then searches for the best response of the other bank allowing any feasible choice of portfolio and deposit return $\{y_k, z_k, d_k\}$ as well as any pattern of feasible withdrawal in the four states, $\{w_{ks}\}_{s=1}^4 \in [0, z_k]$. In addition, it allows for arbitrary default pattern combinations. As in the main algorithm, a fixed point of the generalized best response function is found by iteration until convergence.

The generalized algorithm is naturally less stable than the restricted 3-dimensional algorithm. Consider, for example, a symmetric choice that is characterized by zero withdrawal by both banks in state s: $w_{As} = w_{Bs} = 0$. Since $d_A = d_B$, any identical positive withdrawal $w_{As} = w_{Bs} > 0$ would cancel out and yield the same expected utility. Allowing for arbitrary default patterns also reduces the numerical stability of the algorithm, as it could get stuck at the local equilibrium for a default pattern that is not the global equilibrium default pattern.

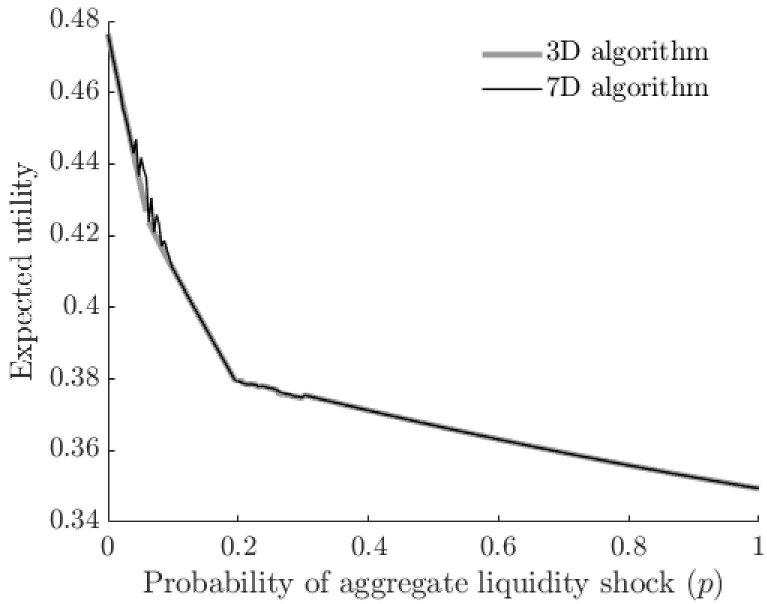
Figure 17 shows the expected utility at the candidate equilibrium for both algorithms. We note that the generalized seven-dimensional algorithm replicates the equilibrium found with the restricted algorithm we use for our main results relatively well. This also serves as numerical confirmation of the analytically derived optimal withdrawal behaviour. Whenever the results of the seven- and three-dimensional algorithms differ significantly, the more general (and less stable) algorithm fails to converge.

A.2 Replicating the Allen and Gale (2000) results

In this section we show that our solution algorithm for the decentralized banking problem replicates the analytical results of Allen and Gale (2000) for $p = 0$.

Allen and Gale (2000) argue that without aggregate risk, the first best allocation is obtained in the decentralized economy. Since regional shocks cancel out when aggregated across regions, the global risk sharing problem is deterministic. Allen and Gale (2000) show that the

Figure 17: Expected utility of the candidate equilibrium found by the 3D algorithm in section 3.4.3 (thick gray line) and the generalized 7D algorithm which allows for arbitrary withdrawal behaviour and default patterns (thin black line). The 7D algorithm most often finds the same equilibrium, but is more unstable in that it sometimes does not converge on the equilibrium. Parameters: $u(c) = \frac{c^{1-\rho}-1}{1-\rho}$, $\rho = 2$, $R = 5$, $r = 0.1$, $\gamma = 0.5$, $\alpha = 0.15$, and $\varepsilon = 0.09$.



first-best arrangement satisfies the FOC, $u'(c_1^{AG}) = Ru'(c_2^{AG})$. This allocation is feasibly decentralized: each bank holds interbank deposit $z_A = z_B = z^{AG} = \varepsilon$ and just enough liquidity to satisfy the average global early liquidity demand $y_A = y_B = y^{AG} = \gamma c_1^{AG}$. If state 1 is realized, bank B needs to withdraw its full interbank holding as it faces regional consumer demand $(\gamma+\varepsilon)c_1^{AG}$ but has liquidity only sufficient to cover γc_1^{AG}. Bank A does not need to withdraw, as the sum of regional demand $(\gamma-\varepsilon)c_1^{AG}$ and demand from bank B εc_1^{AG} is equal to liquidity available locally.

For constant-relative-risk-aversion utility, $u(c) = \frac{c^{\rho-1}-1}{\rho-1}$, the optimal choices are

$$y^{AG} = \frac{\gamma R}{\gamma R + (1-\gamma)R^{\frac{1}{\rho}}}, \qquad z^{AG} = \varepsilon, \qquad d^{AG} = \frac{R}{\gamma R + (1-\gamma)R^{\frac{1}{\rho}}}. \tag{18}$$

For log utility ($\rho = 1$), these choices reduce to $y^{AG} = \gamma$, $z^{AG} = \varepsilon$, and $d^{AG} = 1$.

Our replication approach proceeds as follows. We set $p = 0$ and, for each risk aversion parameter in $\rho_j = \{2,3,4,5,6,7,8\}$, take the following steps:

Step 1: Given ρ_j, take a random draw i of the model parameters $\{R, \gamma, \varepsilon, \alpha\}$, from independent uniform distributions.

Step 2: For each draw i of parameters $\{R_i, \gamma_i, \varepsilon_i, \alpha_i\}$

2.1: Calculate (analytically) the Allen and Gale (2000) allocation and implied expected utility, $\{EU_i^{AG}, y_i^{AG}, z_i^{AG}, d_i^{AG}\}$, using equation (18);

2.2: Compute the numerical equilibrium choices and equilibrium expected utility, $\{EU_i^*, y_i^*, z_i^*, d_i^*\}$, via the search algorithm presented in section 3.4.3; and

2.3: Construct the normalized results: $\{\frac{EU_i^*}{EU_i^{AG}}, \frac{y_i^*}{y_i^{AG}}, \frac{z_i^*}{z_i^{AG}}, \frac{d_i^*}{d_i^{AG}}\}$.

Table 5 shows the descriptive statistics of this replication exercise. While there is naturally some numerical variation, the algorithm succeeds in replicating the results of Allen and Gale (2000). The normalized expected utility results show this most clearly: the median of the numerical results is equal to 1, as required, with the mean being only marginally below 1, and with a very small standard deviation. The skewness of the distribution of the numerical results is negative, showing that the few instances where the goal was missed can be attributed to the algorithm ending before reaching the true equilibrium. The normalized choice variables show somewhat more variability, but are generally centred on unity, except for interbank deposits.

This last result does not imply a failure of the replication exercise, as there is a natural indeterminacy in the interbank deposit in symmetric equilibrium. The interbank deposit in the Allen and Gale (2000) allocation is equal to the size of the regional shock (ε). When the size of the aggregate liquidity demand shock (α) is larger than ε, the search space of the algorithm admits larger than necessary interbank deposits. However, as long as the allocation is symmetric, a larger than necessary interbank deposit still implements the Allen and Gale (2000) allocation. If some symmetric pair of interbank deposits, z^*, feasibly implements a symmetric allocation, any larger symmetric pair of interbank deposits implements the same allocation. Any excess interbank holdings above what is needed to finance early withdrawals (the only purpose of the interbank deposit) pays out at the final date. Since the allocations are symmetric, these final-date payouts cancel out without implications for consumer expected utility.

Additional evidence for the successful replication exercise is presented via scatter plots in Figure 18, where we plot the results of the computed equilibrium against the theoretical values of the Allen and Gale (2000) allocation. The figure shows a near-perfect correspondence of the expected utility in numerical equilibrium with the predicted analytical value (top-left panel). As with the descriptive statistics in Table 5, there is much greater variability in the choice variables than in the expected utility. This illustrates a central feature that complicates the numerical approach: in the region of the equilibrium, the expected utility surface is extremely flat, so that even notable variation in choices have little impact on the implied expected utility of the equilibrium. Therefore, any numerical algorithm tends to struggle to converge.

Table 5: Replicating the Allen and Gale (2000) result for $p = 0$: descriptive statistics of the normalized expected utility and choice variables in numerical equilibrium based on 1856 trials.

Choice	Mean	Median	Std. Dev.	Skewness
Normalized expected utility ($\frac{EU_i^*}{EU_{AG,i}^*}$)	0.997	1	0.011	−10.913
Normalized liquidity ($\frac{y_i^*}{y_{AG,i}}$)	1.021	1	1.368	39.022
Normalized interbank deposit ($\frac{z_i^*}{z_{AG,i}}$)	1.662	1	8.375	26.455
Normalized deposit return ($\frac{d_i^*}{d_{AG,i}}$)	1.008	1	0.0375	7.263

The greatest variability in liquidity (top-right panel of Figure 18) occurs when the analytical allocation has near-zero liquidity. This occurs when the liquidation return on investment is near 1 because then investment and liquidity are near substitutes for funding early investment and the numerical algorithm struggles to precisely pin down the optimal level of liquidity. The interbank deposit (bottom-left panel) is variable in the sense that the numerical equilibrium frequently selects (symmetric) values larger than those predicted by the analytic results. As argued above, this does not affect expected utility, as interbank deposits that are too large cancel out in symmetric equilibrium. Lastly, the deposit return is most variable when it is predicted to be very large, which occurs when both the early liquidation return and the return at maturity are large. Again, even this notable variability has a limited impact on implied expected utility.

A final piece of evidence of a successful replication are the regression results in Table 6. In this analysis, the computed equilibrium values of each measure of interest are regressed on the corresponding analytical values in the Allen and Gale (2000) allocation. We used two methods: the standard ordinary least squares estimator (OLS, in the left panel of the table) and recursively weighted least squares (RWLS, right panel). RWLS is robust to outliers, which in this setting are caused by numerical errors in the computational algorithm. The OLS results show coefficients that are very close to one with very high R^2 values. In the RWLS results, all the coefficients and R^2 values are equal to one (rounded to two decimal places). In the RWLS approach we used a "fair" weighting scheme: the estimation algorithm starts with OLS (i.e. equal weights on all observations) then uses the estimated residuals to down-weight observations with large residuals. This scheme down-weights outliers, but no observation received a zero weight.

We summarize the evidence in this section as the numerical result stated in the main text.

Figure 18: Replicating the Allen and Gale (2000) result for $p = 0$: Scatter plots of the equilibrium expected utility and choice variable values compared with the analytical values in the Allen and Gale (2000) result based on 1856 trials. Each dot is partially transparent to show accumulation around the diagonal line from the origin that correspond to a perfect fit.

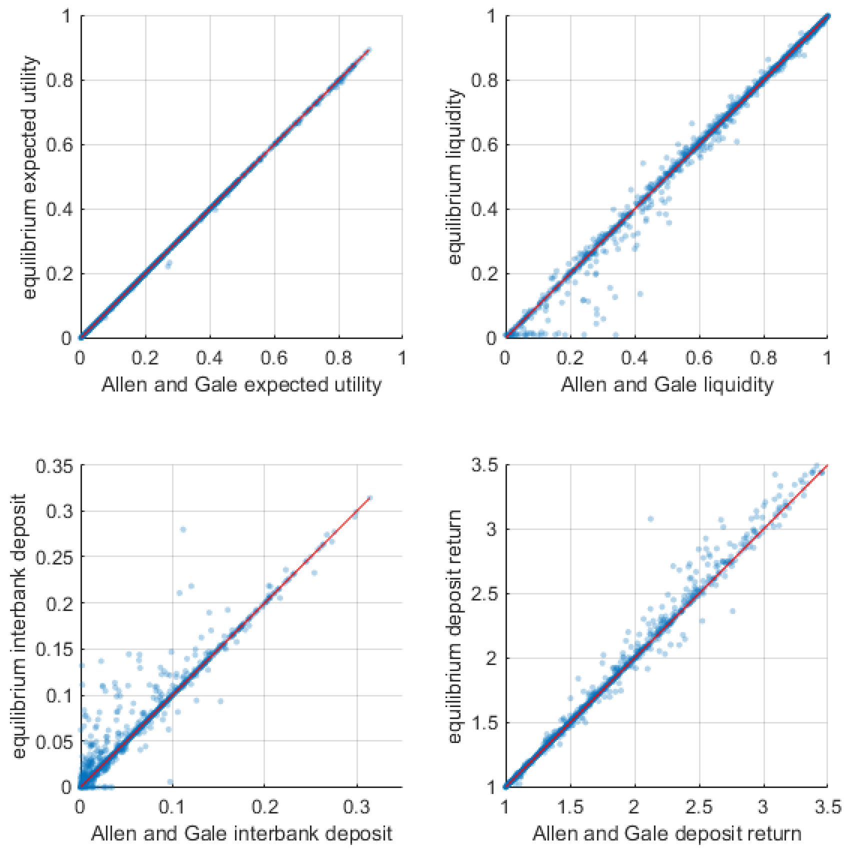


Table 6: Replicating the Allen and Gale (2000) result for $p = 0$: Regression results based on 1856 trials. For each of the values of interest, expected utility and the bank choices, a pair of independent estimations were performed, regressing the computed equilibrium values on the analytical values of the corresponding variable in the Allen and Gale (2000) result. Two methods were used: ordinary least squares (OLS) and recursively weighted least squares (RWLS).

	OLS			RWLS		
	Coefficient	p-value	R^2	Coefficient	p-value	R^2
Expected utility	1.00	0.00	1.00	1.00	0.00	1.00
Liquidity	1.00	0.00	0.99	1.00	0.00	1.00
Interbank deposit	1.02	0.00	0.94	1.00	0.00	1.00
Deposit return	1.02	0.00	0.98	1.00	0.00	1.00

B Optimal withdrawal behaviour

B.1 Proof of lemma 1—optimal withdrawal under no default

The optimal withdrawal behaviour across states yield three cases of withdrawals. Given the pecking order (equation 4), a bank only withdraws the amount of interbank deposits required to serve early withdrawals, because keeping the funds with the other bank until $t = 2$ earns a higher return. Hence, the optimal withdrawal volume in state s is

$$w_{As} = \begin{cases} 0 & \text{if } w_{Bs} \leq y_A - v_{As} \\ \frac{d_A(v_{As}+w_{Bs})-y_A}{d_B} & \text{if } y_A - v_{As} < w_{Bs} < (y_A - v_{As}) + d_B z_A \\ z_A & \text{if } w_{Bs} \geq (y_A - v_{As}) + d_B z_A. \end{cases} \tag{19}$$

Case 1: Suppose $w_{As} = 0$, which occurs when $y_A \geq v_{As} + w_{Bs}$. In words, bank A can serve all early withdrawals (from both its consumers and the other bank) out of liquidity alone. The only feasible deviation by bank A is to increase interbank withdrawals to some $\delta > 0$. Suppose initially that this value is small enough not to cause the default of bank B. Resources of bank A at $t = 1$ increase by $d_B\delta$ and resources in $t = 2$ decrease by $c_{2Bs}\delta$. This is a (weak) net decrease in aggregate resources because the allocation of bank B must be incentive compatible to ensure a no-default pattern, $c_{2Bs} \geq d_B$. Since d_A is fixed and feasible to pay out, the consumption level upon withdrawing at $t = 1$ is unaffected by this deviation, while the consumption level upon waiting until $t = 2$ (weakly) decreases. In sum, a deviation to a small $\delta > 0$ is not optimal. Next, if the deviation is large enough to induce the default of bank B, the deviation returns are even lower because of costly liquidation of investment by bank B.

Case 2: Suppose the withdrawal is positive but not complete, $w_{As} \in (0, z_A)$. This occurs when $y_A - v_{As} < w_{Bs} < (y_A - v_{As}) + d_B z_A$, that is when some but not all interbank withdrawals are required to meet all early withdrawals. In this case, bank A does not have to liquidate any of its investment. As in case 1, withdrawing more than what is required (weakly) reduces the consumption level at $t = 2$. Withdrawing less than what is required to fund early withdrawals implies some costly liquidation of investment. This leaves consumption levels at $t = 1$ unchanged but (strictly) reduces consumption levels at $t = 2$. Hence, these deviations are not optimal.

Case 3: Suppose full withdrawals, which occur when early withdrawals (weakly) exceed liquidity plus the market value of the entire interbank position, $v_{As} + w_{Bs} \geq y_A + d_B z_A$. Thus, some partial liquidation of investment is required. The only feasible deviation is less than full withdrawal of the interbank position, which strictly reduces the consumption at $t = 2$ because of more costly liquidation, as in case 2.

In all 3 cases, there exists no unilateral profitable deviation by a regional bank from the stated symmetric withdrawal scheme in the no-default pattern.

B.2 Proof of lemma 2—optimal withdrawal under no default

States 1 and 2 are identical to the no-default pattern. In state 3, bank B defaults but bank A does not. Bank A must therefore pay the deposit return d_A to all withdrawals in $t = 1$. Bank B defaults in $t = 1$ and liquidates its entire portfolio—including withdrawing its entire interbank deposits, so bank B's pays $\frac{y_B + d_A z_B + r x_B}{1 + w_{A4}} < d_B$ on all deposits. A deviation by bank A means withdrawing less than its full claim on bank B at $t = 1$. This is not optimal because the value of the claim on bank B at $t = 2$ is zero, reducing the resources available to bank A.

The withdrawal behaviour in state 3 implies a net transfer of resources from bank A to bank B. While both banks withdraw fully their interbank claim, these are settled *at market value* at $t = 1$: bank B receives the promised deposit return from bank A, whereas bank A receives only its share of the liquidation proceeds from bank B.

In state 4, bank A defaults and liquidates its entire portfolio (including interbank deposits) to pay out a liquidation returns to all claimants at $t = 1$. That is, full withdrawal of its interbank claim on bank B is implied by the definition of default.

B.3 Proof of lemma 3—optimal withdrawal under no default

States 1 and 2 are again identical to the no-default pattern. In state 3 and 4, both banks default and fully liquidate their portfolios at $t = 1$ that implies the full withdrawal of their respective interbank claims. If we were to allow for a unilateral deviation by bank A, it would reduce withdrawals at $t = 1$ (effectively giving up some of its claims on bank B), which is obviously not optimal for bank A.

C Global bank benchmark

We characterize the solutions to the global bank benchmark (Section 5) (and the case of autarky in Appendix D) by several results in terms of the bounds on the probability of the aggregate liquidity shock (p) summarized in Table 7.

Table 7: Summary of bounds on probability of aggregate liquidity shock p

Notation	Benchmark Setting and Regions Separated
$\underline{p}_{ND}$	If $p \le \underline{p}_{ND}$, the global bank chooses zero excess liquidity in the no-default (ND) regime. Otherwise, excess liquidity is positive.
$\overline{p}_{ND}$	If $p \ge \overline{p}_{ND}$, the global bank chooses zero partial liquidation in the no-default regime. Otherwise, partial liquidation is positive.
$\hat{p}^{IC}$	If $p < \hat{p}^{IC}$, the incentive compatibility constraint binds in the no-default regime. Otherwise, it is slack.
$\underline{p}_{SD}$	If $p \le \underline{p}_{SD}$, the global bank chooses zero excess liquidity in the single-default (SD) regime. Otherwise, excess liquidity is positive.
$\overline{p}_{SD}$	If $p \ge \overline{p}_{SD}$, the global bank chooses zero investment in the single-default regime. Otherwise, investment is positive.
$\underline{p}_{MD}$	If $p \le \underline{p}_{MD}$, the global bank chooses zero excess liquidity in the mutual-default (MD) regime. Otherwise, excess liquidity is positive.
$\overline{p}_{MD}$	If $p \ge \overline{p}_{MD}$, the global bank chooses zero investment in the mutual-default regime. Otherwise, investment is positive.
$\breve{p}$	If $p \le \breve{p}$, the single-default regime is optimal in the global bank problem. Otherwise, the no-default regime is optimal.
$\underline{p}^{Aut}, \overline{p}^{Aut}$	If $p' = \gamma + p\alpha \le \underline{p}^{Aut}$ an investor in autarky (Aut) chooses only investment. If $p' \ge \overline{p}^{Aut}$, an investor in autarky chooses only liquidity. Otherwise, $\underline{p}^{Aut} < p < \overline{p}^{Aut}$ and the allocation is interior.

C.1 No-default regime

Expressing consumption levels in terms of excess liquidity and partial liquidation yields:

$$c_{1As} = c_{1Bs} = \frac{y-e}{\gamma} \text{ for } s = 1,2, \qquad c_{2As} = c_{2Bs} = \frac{e+(1-y)R}{1-\gamma} \text{ for } s = 1,2,$$
$$c_{1As} = c_{1Bs} = \frac{y+r\lambda}{\gamma+\alpha} \text{ for } s = 3,4, \qquad c_{2As} = c_{2Bs} = \frac{(1-y-\lambda)R}{1-(\gamma+\alpha)} \text{ for } s = 3,4$$

Combined with the non-state-contingency constraint $c_{1As} = c_{1Bs} \equiv d \,\forall s$, liquidity as a function of e and λ is $y = \frac{\gamma+\alpha}{\alpha}e + \frac{\gamma r}{\alpha}\lambda$. Rewriting yields consumption levels stated in the main text.

C.1.1 Proof of Lemma 4

Suppose the ICC are slack. The Kuhn-Tucker conditions of this reduced problem (P1a) are

$$\frac{\partial V_{ND}}{\partial e} \leqslant 0 \text{ as } e^* \geqslant 0 : (\gamma + p\alpha)u'(d) \leq (1-p)\big(\gamma R + (R-1)\alpha\big)u'(c_{2L}) + p(\gamma+\alpha)Ru'(c_{2H}) \tag{20}$$
$$\frac{\partial V_{ND}}{\partial \lambda} \leqslant 0 \text{ as } \lambda^* \geqslant 0 : (\gamma + p\alpha)u'(d) \leq (1-p)\gamma Ru'(c_{2L}) + p\left(\gamma + \frac{\alpha}{r}\right)Ru'(c_{2H}) \tag{21}$$

Case 1: partial liquidation only ($e^* = 0$ and $\lambda^* > 0$). We characterize the solution to the problem by first studying the problem at $p = 0$, and then showing that there must exist a neighbourhood $p \in \left[0, \underline{p}_{ND}\right]$ where the solution in the ND regime is characterized by unique, positive level of partial liquidation but no excess liquidity. At $p = 0$, the conditions (20) and (21) are

$$\gamma u'(d) \leq \big(\gamma R + (R-1)\alpha\big)u'(c_{2L}) \tag{22}$$
$$\gamma u'(d) \leq \gamma R u'(c_{2L}). \tag{23}$$

Since $(R-1)\alpha > 0$, condition (22) is slack when (23) binds. Thus $e^* = 0$ and $\frac{\partial e^*}{\partial p}(p=0) = 0$. The optimum is characterized by a binding condition (23) evaluated at $e^* = 0$ and $\lambda^* > 0$:

$$u'\left(\frac{r\lambda^*}{\alpha}\right) = Ru'\left(\frac{R}{1-\gamma}\left[1 - \frac{r\gamma}{\alpha}\lambda^*\right]\right). \tag{24}$$

Because of the Inada conditions and since the left-hand side (LHS) increases in λ, while the right-hand side (RHS) decreases in it, there exists a unique $\lambda^* \in \left(0, \frac{\alpha}{r\gamma}\right)$. Its bounds are consistent with zero consumption in the expression of marginal utilities on both sides of (24).

The envelope theorem implies that e^* is continuous in p, so $\frac{\partial e^*}{\partial p} = 0$ in a neighborhood of $p = 0$. Thus, there exists a $\underline{p}_{ND} \in [0,1]$ such that the optimum is characterized by $e^* = 0$ and

$\lambda^* > 0$ if $p \leq \underline{p}_{ND}$. For $p \in \left(0, \underline{p}_{ND}\right]$, the optimum is given by the first-order condition (FOC):

$$(\gamma + p\alpha)u'\left(\frac{r\lambda^*}{\alpha}\right) = (1-p)R\gamma u'\left(\frac{R}{1-\gamma}\left[1 - \frac{r\gamma}{\alpha}\lambda^*\right]\right) + pR\left(\gamma + \frac{\alpha}{r}\right)u'\left(\frac{R}{1-\gamma-\alpha}\left[1 - \left(1 + \frac{r\gamma}{\alpha}\right)\lambda^*\right]\right), \tag{25}$$

which implies the existence of a unique solution $\lambda^* \in \left(0, \frac{\alpha}{\alpha + r\gamma}\right)$. Note that condition (25) contains an additional term relative to condition (24), so the upper bound on λ is lower.

Case 2: excess liquidity only ($e^* > 0$ and $\lambda^* = 0$). We solve the problem in this case by first studying the problem at $p = 1$ and then showing that there exists a neighbourhood $p \in \left[\overline{p}_{ND}, 1\right]$ in which the solution is characterized by unique, positive level of excess liquidity but no partial liquidation. At $p = 1$, conditions (20) and (21) are

$$(\gamma + \alpha)u'(d) \leq (\gamma + \alpha)Ru'(c_{2H}) \tag{26}$$

$$(\gamma + \alpha)u'(d) \leq \left(\gamma + \frac{\alpha}{r}\right)Ru'(c_{2H}). \tag{27}$$

Since $r < 1$, condition (27) is slack when condition (26) binds. Thus $\lambda^* = 0$ and $\frac{\partial \lambda^*}{\partial p}(p = 1) = 0$. The optimum is characterized by the binding condition (26) evaluated at $\lambda^* = 0$ and e^*:

$$u'\left(\frac{e^*}{\alpha}\right) = Ru'\left(\frac{R}{1-\gamma-\alpha}\left[1 - \left(\frac{\gamma+\alpha}{\alpha}\right)e^*\right]\right). \tag{28}$$

Because of the Inada conditions and since the LHS increases in e, while the RHS decreases in e, there exists a unique solution $e^* \in \left(0, \frac{\alpha}{\gamma + \alpha}\right)$. The bounds are values of e consistent with zero consumption in the marginal utilities on both sides of equation (28).

The envelope theorem implies that λ^* is continuous in p with $\frac{\partial \lambda^*}{\partial p} = 0$ in a neighborhood of $p = 1$. Thus, there exists a $\overline{p}_{ND} \in [0,1]$ such that the optimum is characterized by $\lambda^* = 0$ and $e^* > 0$ if $p \in \left[\overline{p}_{ND}, 1\right)$. This optimum is characterized by the following FOC:

$$(\gamma + p\alpha)u'\left(\frac{e^*}{\alpha}\right) = (1-p)\left[R(\alpha+\gamma) - \alpha\right]u'\left(\frac{1}{1-\gamma}\left[R - \left(R\left(1 + \frac{\gamma}{\alpha}\right) - 1\right)e^*\right]\right) + pR(\gamma+\alpha)u'\left(\frac{R}{1-\gamma-\alpha}\left[1 - \left(1 + \frac{\gamma}{\alpha}\right)e^*\right]\right), \tag{29}$$

which implies the existence of a unique solution $e^* \in \left(0, \frac{\alpha}{\alpha + \gamma}\right)$. Note that condition (29) contains an additional marginal utility term relative to condition (28). As a result, the upper bound on e^* is higher than in condition (28), so the previous bounds on e^* are more restrictive.

Case 3: excess liquidity and partial liquidation ($e^* > 0$ and $\lambda^* > 0$). The above results imply that for $p \in \left(\underline{p}_{ND}, \overline{p}_{ND}\right)$, the optimum is a unique choice of positive levels of both excess liquidity and partial liquidation determined by conditions (20) and (21) holding with equality.

Uniqueness of probability boundaries. Finally, we establish that $\frac{\partial e^*}{\partial p} \geq 0$ and $\frac{\partial \lambda^*}{\partial p} \leq 0$, with strict inequality for positive levels of the choice variables. These monotonicity results imply the uniqueness of $\underline{p}^{ND}$ and $\overline{p}_{ND}$. In case 1 ($e^* = 0$ and $\lambda^* > 0$), total differentiation of condition (21) with respect to p implies that $\frac{\partial \lambda^*}{\partial p} < 0$ whenever $u'(c_{2H}) > \frac{r(\gamma+\alpha)}{r\gamma+\alpha} u'(c_{2L})$, for which $u'\left(c^*_{2H}\right) > u'\left(c^*_{2L}\right)$ or $c^*_{2H} < c^*_{2L}$ suffice. This holds whenever $\frac{R}{1-\gamma-\alpha} - \frac{\left(\frac{r\gamma+\alpha}{\alpha}\right)R}{1-\gamma-\alpha}\lambda^* < \frac{R}{1-\gamma} - \frac{rR\gamma}{\alpha(1-\gamma)}\lambda^* \Leftrightarrow 1-\gamma-\alpha+r\gamma\lambda^* > 0$, which always holds since $\lambda^* \geq 0$ and $1-\gamma-\alpha > 0$. Thus, $\frac{\partial \lambda^*}{\partial p} < 0$ in case 1.

In case 2 ($\lambda^* = 0$ and $e^* > 0$), total differentiation of condition (20) with respect to p implies that $\frac{\partial e^*}{\partial p} > 0$ whenever $u'(c_{2L}) > \frac{R\gamma}{(R\gamma+(R-1)\alpha)} u'(c_{2H})$ for which $c^*_{2L} < c^*_{2H}$ is sufficient:

$$\frac{R}{1-\gamma} - \frac{\left(R\left(\frac{\gamma+\alpha}{\alpha}\right)-1\right)}{1-\gamma}e^* \quad < \quad \frac{R}{1-\gamma-\alpha} - \frac{\left(\frac{\gamma+\alpha}{\alpha}\right)R}{1-\gamma-\alpha}e^* \Leftrightarrow e^* < \tilde{e} \equiv \frac{\alpha R}{(R-1)(\gamma+\alpha)+1}.$$

At $e = \tilde{e}$, we have $c_1(\tilde{e}) = c_{2L}(\tilde{e}) = c_{2H}(\tilde{e}) = \tilde{c}$ and $\frac{dV_{ND}}{de}(e = \tilde{e}, \lambda = 0) = -\frac{(\gamma+\alpha)(R-1)}{\alpha}u'(\tilde{c}) < 0$. Thus $e^* < \tilde{e}$ whenever $\lambda^* = 0$ and $\frac{\partial e^*}{\partial p} > 0$ in case 2.

In case 3 ($e^* > 0$ and $\lambda^* > 0$), total differentiation of the FOCs in (20) and (21) with respect to p yields

$$\frac{d\lambda^*}{dp} = \frac{N^\lambda}{D} < 0, \quad \frac{de^*}{dp} = \frac{N^e}{D} > 0.$$

This result holds since the denominator is negative, $D = -p(1-r)(\gamma+\alpha p)(D_1 + D_2 + D_3) < 0$:

$$\begin{aligned}
D_1 &= u''(c_1)(\gamma+\alpha p)(1-p)r^2(R-1)^2(1-\alpha-\gamma)u''(c_{2L}) > 0 \\
D_2 &= u''(c_1)(\gamma+\alpha p)(1-\gamma)p(1-r)^2R^2u''(c_{2H}) > 0 \\
D_3 &= (1-p)pR^2u''(c_{2H})u''(c_{2L})(\gamma(R-r)+\alpha(R-1))^2 > 0,
\end{aligned}$$

while the numerator of $\frac{\partial e^*}{\partial p}$ is negative, $N^e = -u'(c_{2L})\left(R(\gamma(R-r)+\alpha(R-1))\left(N^e_1 + N^e_2\right) + N^e_3\right) < 0$:

$$\begin{aligned}
N^e_1 &= -(1-\gamma)p^2(1-r)R(\alpha+\gamma)u''(c_{2H})(\alpha+\gamma r) > 0, \quad N^e_2 = -\gamma^2(1-p)^2r^2(R-1)(1-\alpha-\gamma)u''(c_{2L}) > 0 \\
N^e_3 &= -(1-\gamma)(1-r)r^2(R-1)(1-\alpha-\gamma)u''(c_1)(\gamma+\alpha p)^2 > 0
\end{aligned}$$

and the numerator of $\frac{\partial \lambda^*}{\partial p}$ is positive, $N^\lambda = u'(c_{2L})\left((\gamma(R-r)+\alpha(R-1))(N^\lambda_1 + N^\lambda_2) + N^\lambda_3\right) > 0$:

$$\begin{aligned}
N^\lambda_1 &= -(1-\gamma)p^2(1-r)R^2(\alpha+\gamma)^2u''(c_{2H}) > 0 \\
N^\lambda_2 &= -\gamma(1-p)^2r(R-1)(1-\alpha-\gamma)u''(c_{2L})(\alpha(R-1)+\gamma R) > 0 \\
N^\lambda_3 &= -(1-\gamma)(1-r)r(R-1)(1-\alpha-\gamma)(\gamma+\alpha p)^2u''(c_1) > 0.
\end{aligned}$$

As a result, we can define unique bounds on the probability of the aggregate liquidity shock:

$$\underline{p}_{ND} \equiv \max\left\{p | e^* = 0\right\}, \quad \overline{p}_{ND} \equiv \min\left\{p | \lambda^* = 0\right\}.$$

In conclusion, we show that $\overline{p}_{ND} > \underline{p}_{ND}$. Proof by contradiction: suppose that $\underline{p}_{ND} \geq \overline{p}_{ND}$. Then there exists a $p \in \left(\overline{p}_{ND}, \underline{p}_{ND}\right)$ with corresponding optimal choice of $e^* = 0$ and $\lambda^* = 0$. However, this implies $d^* = 0$ which contradicts optimality. Thus, $\overline{p}_{ND} > \underline{p}_{ND}$.

C.1.2 Proof of Proposition 1

We characterize $\hat{p}^{IC}$ below which ICC binds in the ND regime. We characterize each case without imposing the ICC in order to identify parameter regions in which the ICC is violated.

Case 1: $\lambda^* > 0$ **and** $e^* = 0$**.** We have established that $c_{2L}^* > c_{2H}^*$. Combined with $d^* < \max\left\{c_{2L}^*, c_{2H}^*\right\}$, this implies $c_{2L}^* > d^*$. Given $e^* = 0$, the accounting identities imply that $c_{2H}^* < d^*$ if $\lambda^* > \hat{\lambda} \equiv \frac{\alpha R}{r(\gamma R + 1 - \alpha - \gamma) + \alpha R}$. Since $\frac{\partial \lambda^*}{\partial p} < 0$ and $\frac{\partial d}{\partial \lambda} > 0$ and $\frac{\partial c_{2H}}{\partial \lambda} < 0$ by construction, there exists a unique $\hat{p}^{IC(2)} \equiv \left\{p \,\middle|\, \lambda^*(p) = \hat{\lambda}\right\}$ such that the ICC is slack if and only if $p > \hat{p}^{IC(2)}$.

Case 2: $\lambda^* = 0$ **and** $e^* > 0$**.** We have already shown that optimal excess liquidity e^* satisfies $e^* < \tilde{e}$, where $d(\tilde{e}) = c_{2L}(\tilde{e}) = c_{2H}(\tilde{e})$. By construction, $\frac{\partial c_1}{\partial e} > 0$, $\frac{\partial c_{2L}}{\partial e} < 0$, and $\frac{\partial c_{2H}}{\partial e} < 0$, so $e^* < \tilde{e}$ implies $c_{2L}^* > d^*$ and $c_{2H}^* > d^*$ in this case. Hence, $p \geq \overline{p}_{ND}$ is sufficient for a slack ICC.

Case 3: $e^* > 0$ **and** $\lambda^* > 0$**.** Conditions (20) and (21) holding with equality imply:

$$u'\left(d^*\right) = \left(1-p\right)\frac{\left(\alpha(R-1) + \gamma(R-r)\right)}{(1-r)\left(\gamma + p\alpha\right)}u'\left(c_{2L}^*\right), \tag{30}$$

$$u'\left(d^*\right) = p\frac{R}{r}\frac{\left(\alpha(R-1) + \gamma(R-r)\right)}{(R-1)\left(\gamma + p\alpha\right)}u'\left(c_{2H}^*\right). \tag{31}$$

Condition (30) implies $c_{2L}^* \geq d^*$ if $R \geq \frac{1-pr}{1-p}$. Since $\frac{1-pr}{1-p} \leq 1$ and $R \geq 1$, this condition always holds, so $c_{2L}^* \geq d^*$ in this case. Condition (31) implies $c_{2H}^* < d^*$ only if $p < \hat{p}^{IC(1)} \equiv \frac{\gamma r(R-1)}{(R-r)(\alpha(R-1)+\gamma R)}$. Hence, $p \geq \hat{p}^{IC(1)}$ is necessary and sufficient for incentive compatibility in this case.

Last, we establish continuity of the incentive compatibility boundary across different cases: we show $\lim_{p \nearrow \underline{p}_{ND}} \hat{p}^{IC(2)} = \lim_{p \searrow \underline{p}^{ND}} \hat{p}^{IC(1)}$ by contradiction. Suppose that $\lim_{p \nearrow \underline{p}^{ND}} \hat{p}^{IC(2)} > \lim_{p \searrow \underline{p}^{ND}} \hat{p}^{IC(1)}$. Then there must exist a sequence $p_n' < \hat{p}^{IC(2)}$ converging to $\dot{p} = \underline{p}^{ND} < \hat{p}^{IC(2)}$ from below, so that $c_{2H}^*(p_n') < d^*(p_n')$ for all p_n'. There must also be a sequence $p_n'' > \hat{p}^{IC(1)}$ converging to $\dot{p} = \underline{p}^{ND} > \hat{p}^{IC(1)}$ from above, so that $c_{2H}^*(p_n'') > d^*(p_n'')$ for all p_n''. But then there must be a discontinuity in the consumption allocation (and hence the optimal portfolio) at $\dot{p}$, which contradicts the result from the theorem of the maximum that the maximizers of a continuous problem are continuous. A similar argument holds for the opposite inequality. Thus, there is a unique continuous boundary $\hat{p}^{IC}$ where the optimal allocation is characterized by $c_{2H}^*(\hat{p}_{IC}) = d^*(\hat{p}_{IC})$, where

$$\hat{p}_{IC} = \begin{cases} \hat{p}^{IC(1)} & \text{if } p > \underline{p}_{ND} \\ \hat{p}^{IC(2)} & \text{if } p \leq \underline{p}_{ND}. \end{cases}$$

C.1.3 Proof of Proposition 2

We have shown above that the ICC only binds in states 3 or 4. When it does, the optimal allocation is characterized by equal payout to early and late consumers: $c_{2H} = d$. There are two cases: the first characterized by only partial liquidation and the second by partial liquidation and excess liquidity. Throughout this subsection, we maintain the assumption that $p \leq \hat{p}_{IC}$. The general problem of the global bank in the ND regime regime reduces to:

$$V_{ND}(p \leq \hat{p}_{IC}) \equiv \max_{e \geq 0, \lambda \geq 0} \left(p + (1-p)\gamma\right) u(d) + (1-p)(1-\gamma) u(c_{2L})$$

Case 1: partial liquidation only, $\lambda^* > 0$ and $e^* = 0$. When probability of the aggregate liquidity shock is low enough $p \leq \underline{p}^{IC}_{ND}, \hat{p}_{IC}$, it is optimal to avoid using excess liquidity. The proof of the existence and uniqueness of the bound $\underline{p}^{IC}_{ND}$ is similar to the case without the ICC above and is therefore omitted. Without excess liquidity, the only way satisfy the non-state-contingency constraint of early consumption is to set $\lambda = \frac{\alpha}{r\gamma} y$. Adding the additional constraint that $d = \frac{y + r\lambda}{\gamma + \alpha} = \frac{(1-y-\lambda)R}{1-\gamma-\alpha} = c_{2H}$ fully determines the solution independent of the utility function:

$$y^* = \frac{\gamma r R}{\gamma r R + \alpha R + r(1-\gamma-\alpha)}, \qquad d^* = c^*_{2H} = \frac{y^*}{\gamma}, \qquad c^*_{2L} = \frac{R(1-y^*)}{1-\gamma}.$$

Case 2: partial liquidation and excess liquidity, $e^* > 0$ and $\lambda^* > 0$. If the probability is high enough, $\underline{p}^{IC}_{ND} < p < \hat{p}_{IC}$, it is optimal to use excess liquidity without the aggregate shock and partial liquidation with it. Using $d = c_{2H}$, the problem can be stated in terms of y only:

$$\begin{aligned} e &= \frac{y(r(\gamma R + 1 - \alpha - \gamma) + \alpha R) - \gamma r R}{r(1-\alpha-\gamma) + R(\alpha+\gamma)}, \quad \lambda = \frac{R(\alpha+\gamma) - y((R-1)(\alpha+\gamma)+1)}{r(1-\alpha-\gamma) + R(\alpha+\gamma)} \\ d &= \frac{(1-r)Ry + rR}{r(1-\alpha-\gamma) + R(\alpha+\gamma)} \\ c_{2L} &= \frac{rR(1-\alpha-2\gamma) + R^2(\alpha+\gamma)}{(1-\gamma)(r(1-\alpha-\gamma) + R(\alpha+\gamma))} - \frac{rR(1-\alpha-2\gamma) + R^2(\alpha+\gamma) - \alpha R - r(1-\alpha-\gamma)}{(1-\gamma)(r(1-\alpha-\gamma) + R(\alpha+\gamma))} y. \end{aligned}$$

This yields the optimality condition $u'(d) = \frac{(1-p)(r(R(1-\alpha-2\gamma) - (1-\alpha-\gamma)) + R(R(\alpha+\gamma)-\alpha))}{(1-r)R(\gamma+(1-\gamma)p)} u'(c_{2L})$. Since $\frac{\partial d}{\partial y} > 0$ and $\frac{\partial c_{2L}}{\partial y} < 0$, there exists a unique $y^* > 0$ that solves this problem.

C.2 Single-default regime

In the single-default regime, the global bank allows the default of the subsidiary in the region hit by the aggregate liquidity shock. Since deposits have a fixed return d, selective default implies

$$c_{1A1} = c_{1A2} = c_{1A3} = d = c_{1B1} = c_{1B2} = c_{1B4}, \tag{32}$$

because the deposit return is not state-contingent unless the subsidiary defaults.

States 1 and 2 are symmetric across regions. Strict concavity implies that it is optimal to treat late consumers from different regions identically, $c_{2As} = c_{2Bs}$ for $s = 1, 2$. Since regions have the same average liquidity demand, $d\gamma$, and choose the same liquidity y and deposit return d by ex-ante symmetry and concavity, consumption levels for late consumers are

$$c_{2A1} = c_{2A2} = c_{2B1} = c_{2B2} = \frac{y - d\gamma + R(1-y)}{1-\gamma} \equiv c_2,$$

where $e = y - d\gamma$ is again excess liquidity and $x = 1 - y$ from the resource constraint at $t = 0$.

In contrast, the consumption levels for late consumers in states 3 and 4 are asymmetric across regions because of the selective default of the regional subsidiary. In state 3, bank A has liquidity y and faces early demand of $d\gamma$, whereas bank B defaults and pays out liquidation value to all regional consumers. And vice versa in state 4. Thus:

$$c_{1A4} = c_{2A4} = c_{1B3} = c_{2B3} = y + r(1-y) \equiv c_D, \tag{33}$$

$$c_{2A3} = c_{2B4} = c_2. \tag{34}$$

Thus, the global bank with regional subsidiaries in the SD regime solves the general problem:

$$\begin{aligned}
V \equiv \max_{\{c_{1As}, c_{2As}, c_{1Bs}, c_{2Bs}\}_{s=1}^{4}} \sum_{s=1}^{4} \pi_s \Big[\quad & \nu_{As} u\big(c_{1As}\big) + (1-\nu_{As}) u\big(c_{2As}\big) \\
& + \nu_{Bs} u\big(c_{1Bs}\big) + (1-\nu_{Bs}) u\big(c_{2Bs}\big)\Big] \text{ s.t.} \\
x + y = 1, \quad & 0 \le y \le 1, \\
c_{1A1} = c_{1A2} = c_{1A3} \quad & = c_{1B1} = c_{1B2} = c_{1B4} = d, \\
c_{2A1} = c_{2B1} = c_{2A2} \quad & = c_{2B2} = c_{2A3} = c_{2B4} = c_2, \\
c_{1A4} = c_{2A4} = c_{1B3} \quad & = c_{2B3} = c_D.
\end{aligned}$$

Using the liquidity demands in Table 1, the problem can be rewritten in terms of liquidity (y) and excess liquidity (e) and expressed as the reduced problem stated in the main text. Note that $y = 0$ cannot be optimal as it implies zero consumption in several states, while $y = 1$ may be optimal. Hence, the problem can be represented by the Lagrangian:

$$L = \frac{2-p}{2}\left[\gamma u\left(\frac{y-e}{\gamma}\right)+(1-\gamma)u\left(\frac{e+R(1-y)}{1-\gamma}\right)\right]+\frac{p}{2}u\big(y+r(1-y)\big)-\mu(y-1),$$

where μ is the Lagrange multiplier on the constraint $y \leq 1$. The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial y}=0: \qquad u'(d)=Ru'(c_2)-\frac{p(1-r)}{2-p}u'(c_D)+\frac{1}{2-p}\mu \tag{35}$$

$$\frac{\partial L}{\partial e}\lesseqgtr 0 \text{ as } e^* \gtreqless 0: \qquad u'(d)\geq u'(c_2) \tag{36}$$

$$\frac{\partial L}{\partial \mu}\lesseqgtr 0 \text{ as } \mu^* \gtreqless 0: \qquad y\leq 1 \tag{37}$$

Condition (36) implies $d \leq c_2$, so the incentive compatibility constraint is always satisfied in the SD regime. There are three cases: (i) $y^* = 1$ and $e^* > 0$; (ii) $y^* \in (0,1)$ and $e^* = 0$; and (iii) $y^* \in (0,1)$ and $e^* > 0$.

Case 1: $y^* = 1$ with $e^* > 0$. Suppose $y^* = 1$, so that $\mu^* > 0$. Zero excess liquidity, $e = 0$, would imply $c_2 = 0$, which cannot be optimal. Hence, $e^* > 0$ and equation (36) binds. In sum:

$$d^* = c_2^* = c_D^* = 1; \qquad e^* = 1-\gamma. \tag{38}$$

Rewriting equation (35) yields $\mu^* = \left[p(R-r)-2(R-1)\right]u'(1)$, so $\mu^* \geq 0 \Leftrightarrow p \geq \overline{p}_{SD} \equiv 2\frac{R-1}{R-r} > 0$. Moreover, $\overline{p}_{SD} \leq 1$ only if $R + r \leq 2$. That is, for low enough R and r, no investment is made if and only if the probability of an aggregate liquidity shock is high enough.

Case 2: interior liquidity $y^* \in (0,1)$ with no excess liquidity $e^* = 0$. Note that equation (37) is slack at $p = 0$ and $\mu^* = 0$. Thus equation (35) evaluated at at $p = 0$ is:

$$u'(d) = Ru'(c_2) \tag{39}$$

Hence, equation (36) is slack (because $R > 1$). Thus, $e^* = 0$ and $\frac{\partial e^*}{\partial p}(p = 0) = 0$. Since the left-hand-side of equation (39) is decreasing in y while the right-hand-side is increasing, there is a unique $0 < y^* < 1$ that solves the problem. Since e^* is continuous in p, there must exist $\underline{p}_{SD} \geq 0$ such that $e^*(p \leq \underline{p}_{SD}) = 0$ by the theorem of the maximum. Moreover, $p \leq \underline{p}_{SD}$ implies $0 < y < 1$, $e = 0$; and $\gamma < 1$ with $R > 1$ implies $c_2 > d > c_D$.

For $p \in (0, \underline{p}_{SD})$ and $p < \overline{p}_{SD}$ we have $e^* = 0$, $\mu^* = 0$, and there is a unique y^* that solves equation (35) since the left-hand-side is decreasing in y while the right-hand-side is increasing. Lastly, total differentiation of equation (35) where $\mu^* = 0$ yields:

$$\frac{dy^*}{dp} = -\frac{\frac{\gamma(1-\gamma)(1-r)}{2-p}u'\big(y^*+r(1-y^*)\big)}{(2-p)R^2\gamma u''\big(\frac{R(1-y^*)}{1-\gamma}\big)+(1-\gamma)\Big((2-p)u''\big(\frac{y^*}{\gamma}\big)+p(1-r)^2\gamma u''\big(y+r(1-y)\big)\Big)} > 0.$$

If $u(c) = \ln c$ (and the coefficient of relative risk aversion is 1), equation (39) implies $y^* = \gamma$, $c_2 = R$ and $d = 1 > c_D = \gamma + r(1-\gamma)$ at $p = 0$. With higher risk aversion, there is greater risk sharing, so $c_2 > d > 1 > c_D$. We constrain the analysis to cases with risk aversion coefficient exceeds 1. Since d increases in y, $c_2 > d \geq 1 > c_D$ is a general result for $p \geq 0$ in this case.

Case 3: interior liquidity $y^* \in (0,1)$ with excess liquidity $e^* > 0$. If $\underline{p}_{SD} < p < \overline{p}_{SD}$, $e^* > 0$ is optimal, $\mu^* = 0$ and equation (36) holds with equality, which implies:

$$\begin{aligned} e^* &= y^*\big(1+(R-1)\gamma\big) - R\gamma \\ c_2^* = d^* &= y^* + R(1-y^*) > 1 > y^* + r(1-y^*) = c_D^* \end{aligned}$$

This means that early and late consumption are equal in any state-region pair without default, and this consumption level is larger than in any state-region pair with default. Plugging these consumption levels into the equation (35) yields:

$$u'\big(y^*+r(1-y^*)\big) = \frac{(2-p)(R-1)}{p(1-r)}u'\big(y^*+R(1-y^*)\big) \tag{40}$$

The left-hand-side is decreasing in y while the right-hand-side is increasing, so the optimal liquidity y^* is unique. Total differentiation of condition (40) with respect to p yields:

$$\frac{dy^*}{dp} = -\frac{\frac{2(1-r)}{(2-p)}u'(y^*+r(1-y^*))}{p(1-r)^2u''(y^*+r(1-y^*))+(2-p)(R-1)^2u''(y^*+R(1-y^*))} > 0 \quad \Rightarrow \frac{de^*}{dp} > 0,$$

Establishing uniqueness of bounds.

$\frac{dy^*}{dp} > 0$ and $\frac{de^*}{dp} > 0$ implies that the bounds $\underline{p}_{SD}$ and $\overline{p}_{SD}$ are unique. Moreover, $\underline{p}_{SD} \leq \overline{p}_{SD}$ since e^* is continuous in p by the theorem of the maximum, combined with $e^*(p \leq \underline{p}_{SD}) = 0$, $\frac{de^*}{dp}(p > \underline{p}_{SD}) > 0$, and $e^*(p = \overline{p}_{SD}) > 0$.

C.3 Mutual-default regime

In the MD regime, both subsidiaries of the global bank default in states 3 and 4. Consumption in these states are

$$c_{1As} = c_{1Bs} = c_{2As} = c_{2Bs} = y + rx \equiv c_D \text{ for } s = 3,4. \tag{41}$$

The analysis of states 1 and 2 follows the SD regime and is skipped for brevity. The problem can be expressed as stated in the main text.

Its Lagrangian is

$$L = (1-p)\big[\gamma u\big(\frac{y-e}{\gamma}\big) + (1-\gamma)u\big(\frac{e+R(1-y)}{1-\gamma}\big)\big] + pu\big(y+r(1-y)\big) - \mu(y-1)$$

and the associated Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial y} = 0: \qquad u'(d) = Ru'(c_2) - \frac{p(1-r)}{1-p}u'(c_D) + \frac{1}{1-p}\mu \tag{42}$$

$$\frac{\partial L}{\partial e} \leqslant 0 \text{ as } e^* \geqslant 0: \qquad u'(d) \geq u'(c_2) \tag{43}$$

$$\frac{\partial L}{\partial \mu} \leqslant 0 \text{ as } \mu^* \geqslant 0: \qquad y \leq 1 \tag{44}$$

Hence, the ICC constraint is always satisfied, $c_2 \geq d$. There are again three cases.

Case 1: $y^* = 1$ **and** $e^* > 0$**.** If $y^* = 1$, then $\mu^* > 0$. Zero excess liquidity $e = 0$ would imply $c_2 = 0$, which is not optimal. Thus, $e^* > 0$ and equation (43) binds, so $d^* = c_2^*$. Taken together:

$$d^* = c_2^* = c_D^* = 1, \qquad e^* = 1-\gamma.$$

Plugging this into equation (42) yields $\mu^* = [p(R-r)-(R-1)]u'(1)$, so $\mu^* \geq 0$ whenever $p \geq \overline{p}_{MD} \equiv \frac{R-1}{R-r} > 0$. Moreover, $\overline{p}_{MD} < 1$ because $r < 1$. Thus, there always exists a unique probability high enough such that no investment is optimal.

Case 2: $y^* \in (0,1)$ **and** $e^* = 0$**.** Note that $\overline{p}_{MD} > 0$ because $R > 1$. Hence, equation (44) is slack at $p = 0$ and $\mu^* = 0$. Thus equation (42) at $p = 0$ is

$$u'(d) = Ru'(c_2) \tag{45}$$

Hence, equation (43) is slack, so $e^* = 0$ and $\frac{\partial e^*}{\partial p}(p=0) = 0$. Since the left-hand-side of equation (45) is decreasing in y while the right-hand-side is increasing, there is a unique $0 < y^* < 1$. Since e^* is continuous in p, there exists a $\underline{p}_{MD} \geq 0$ such that $e^*(p \leq \underline{p}_{MD}) = 0$ by the theorem of the maximum. Moreover, $p \leq \underline{p}_{MD}$ implies $0 < y < 1, e = 0$. Also, $c_2 > d > c_D$. As in the single default regime, we obtain $c_2 > d >\geq 1 > c_D$ as a general result for the utility functions we consider (with coefficient of relative risk aversion of at least 1).

For $p \in (0, \underline{p}_{MD}) < \overline{p}_{MD}$, $e^* = 0 = \mu^*$ and there is a unique y^* that solves equation (42) since the left-hand-side is decreasing in y while the right-hand-side is increasing. Total differentiation of equation (42) using $\mu^* = 0$ yields:

$$\frac{dy^*}{dp} = -\frac{\frac{\gamma(1-\gamma)(1-r)}{1-p}u'\big(y^* + r(1-y^*)\big)}{(1-p)R^2\gamma u''\big(\frac{R(1-y^*)}{1-\gamma}\big) + (1-\gamma)\Big((1-p)u''\big(\frac{y^*}{\gamma}\big) + p(1-r)^2\gamma u''\big(y + r(1-y)\big)\Big)} > 0.$$

Case 3: $y^* \in (0,1)$ **and** $e^* > 0$**.** If $\underline{p}_{MD} < p < \overline{p}_{MD}$, $e^* > 0$ is optimal, $\mu^* = 0$ and equation (43) holds with equality. Thus:

$$e^* = y^*\big(1 + (R-1)\gamma\big) - R\gamma, \qquad c_2^* = d^* = y^* + R(1-y^*) > 1 > c_D^*.$$

Hence, early and late consumption are equal in any state-region pair without default and this consumption level is larger than in any state-region pair with default. Plugging this into the equation (42) yields:

$$u'\big(y^* + r(1-y^*)\big) = \frac{(1-p)(R-1)}{p(1-r)}u'\big(y^* + R(1-y^*)\big) \tag{46}$$

The left-hand-side of equation (46) is decreasing in y while the right-hand-side is increasing, so the optimal liquidity y^* is unique. Total differentiation of condition (46) with respect to p in case 3, where $e^* > 0$ and $y^* < 1$, yields:

$$\frac{dy^*}{dp} = -\frac{\frac{(1-r)}{(1-p)}u'(y^* + r(1-y^*))}{p(1-r)^2u''(y^* + r(1-y^*)) + (1-p)(R-1)^2u''(y^* + R(1-y^*))} > 0 \Rightarrow \frac{de^*}{dp} > 0.$$

Establishing uniqueness of bounds. Note that $\frac{dy^*}{dp} > 0$ implies that the bounds $\underline{p}_{MD}$ and $\overline{p}_{MD}$ are unique. Moreover, $\underline{p}_{MD} \leq \overline{p}_{MD}$ since e^* is continuous in p by the theorem of the maximum, combined with $e^*(p \leq \underline{p}_{MD}) = 0$, $\frac{de^*}{dp}(p > \underline{p}_{MD}) > 0$, and $e^*(p = \overline{p}_{MD}) > 0$.

C.4 Proof of Proposition 5

We have two results on the ex-ante choice of regime by the global bank. We start with preliminaries. There are three regimes for the global bank (GB). Let d^{ND}, c_{2L}^{ND} and c_{2H}^{ND} be the optimal allocation that solves the no-default (ND) regime. The value function in the ND regime is

$$V_{ND}^{GB} \equiv \max_{\{c_1^{ND}, c_{2L}^{ND}, c_{2H}^{ND}\}} (1-p)\Big[\gamma u(d^{ND}) + (1-\gamma)u(c_{2L}^{ND})\Big] + p\Big[(\gamma+\alpha)u(d^{ND}) + (1-\gamma-\alpha)u(c_{2H}^{ND})\Big].$$

Let c_1^{SD}, c_2^{SD} and c_D^{SD} be the optimal allocation that solves the SD regime. The value function is

$$V_{SD}^{GB} \equiv \max_{\{x^{SD}, y^{SD}, c_1^{SD}, c_2^{SD}, c_D^{SD}\}} \frac{2-p}{2}\left[\gamma u(c_1^{SD}) + (1-\gamma)u(c_2^{SD})\right] + \frac{p}{2}u(c_D^{SD}).$$

Finally, let c_{1L}^{MD}, c_{2L}^{MD} and c_H^{MD} be the optimal allocation that solves the mutual-default (MD) regime. We can state the MD value function as:

$$V_{MD}^{GB} \equiv \max_{\{c_1^{MD}, c_2^{MD}, c_D^{MD}\}} (1-p)\left[\gamma u(c_1^{MD}) + (1-\gamma)u(c_2^{MD})\right] + pu(c_D^{MD}).$$

At $p = 0$, the MD and SD regimes are identical and both are weakly superior to the ND regime. Even though the outcome in states where the aggregate liquidity shock realizes has zero weight even in the ND regime, the definition of the ND regime still requires that early consumption be constant across all states. This imposes a shadow cost on its allocation: $V_{MD}^{GB}(p=0) = V_{SD}^{GB}(p=0) \geq V_{ND}^{GB}(p=0)$. Similarly, the global bank does not choose the MD or SD regimes when $p = 1$ because this implies certain liquidation. Thus, $V_{MD}^{GB}(p=1), V_{SD}^{GB}(p=1) < V_{ND}^{GB}(p=1)$.

Comparing the single and mutual default regimes. Under mutual default, the problem of a global bank reduces to:

$$V_{MD} \equiv \max_{y,e \geq 0}(1-p)\left[\gamma u(\frac{y-e}{\gamma}) + (1-\gamma)u(\frac{e+R(1-y)}{1-\gamma})\right] + pu(y + r(1-y))$$

Under single default, the problem of a global bank reduces to:

$$V_{SD} \equiv \max_{y,e \geq 0} \frac{2-p}{2}[\gamma u\big(\frac{y-e}{\gamma}\big) + (1-\gamma)u\big(\frac{e+R(1-y)}{1-\gamma}\big)] + \frac{p}{2}u\big(y + r(1-y)\big)$$

At $p = 0$, the two problems are exactly identical, hence $V_{MD}^{GB}(p=0) = V_{SD}^{GB}(p=0)$. For $p > 0$, the problems under SD and MD only differ in that the MD regime has a higher weight on the states with default than the SD regime: $p > \frac{p}{2}$. We have shown above that $c_2 \geq d \geq 1 \geq c_D$ obtains as a general result for both the SD and MD regimes, hence SD dominates MD.

Comparing the no default and the single default regimes. We show the existence of $\check{p}$ such that $V_{SD}^{GB}(p) \geq V_{ND}^{GB}(p)$ for $p \leq \check{p}$. At $p = 0$, $V_{SD}^{GB}(p=0) \geq V_{ND}^{GB}(p=0)$. The single default regime is weakly superior as it is less constrained than the no default regime as there are fewer states across which the non-state-contingency constraint on early consumption must be satisfied. In other words, the non-state-contingency constraint imposes a shadow cost on the ND regime which is avoided by allowing for default in the SD regime. At $p = 1$, $V_{SD}^{GB}(p=1) < V_{ND}^{GB}(p=1)$. The no default regime is strictly superior as the single default regime places positive weight on states with certain default when $p = 1$. Thus there exists a $\check{p} \in [0,1)$ such that $V_{SD}^{GB} \geq V_{ND}^{GB} \Leftrightarrow p \leq \check{p}$. Figure 10 shows a numerical example where there is a unique $\check{p} > 0$.

D Autarky

As an additional benchmark, we describe autarky in this section. Without banks, the idiosyncratic liquidity risk cannot be pooled away. The investor splits her endowment between liquidity y and the investment x at $t = 0$, before she learns her type at $t = 1$. Her consumption levels are $c_1 = y + rx$ if early and $c_2 = y + Rx$ if late. By strict monotonicity, all endowment is invested, $y^* = 1 - x^*$, and the consumption levels are a function of investment only, $c_1(x) = 1 - (1 - r)x$ and $c_2(x) = 1 + (R - 1)x$. Since the effective probability at $t = 0$ of being an early investor is $p' \equiv \gamma + p\alpha$, the problem of the investor is to choose her portfolio to maximize her expected utility in autarky:

$$\max_{x \in [0,1]} p' u(c_1(x)) + \left(1 - p'\right) u(c_2(x))$$

Proposition 6. *The optimal portfolio choice in autarky is determined by the effective probability of facing the idiosyncratic liquidity shock, p'. There are three cases:*

(i) For a sufficiently high effective probability, $p' \geq \overline{p}^{Aut} \equiv \frac{R-1}{R-r} \in (0,1)$, no investment occurs, $x^{Aut} = 0$.

(ii) For a sufficiently low effective probability, $p' \leq \underline{p}^{Aut} \equiv \frac{(R-1)u'(R)}{(R-1)u'(R)+(1-r)u'(r)} \in (0,1)$, full investment occurs, $x^{Aut} = 1$.

(iii) For intermediate levels, $\underline{p}^{Aut} < p' < \overline{p}^{Aut}$, there exists a unique interior portfolio choice $0 < x^{Aut} < 1$:

$$p'(1 - r)\, u'\big(c_1(x^{Aut})\big) = (1 - p')(R - 1)\, u'\big(c_2(x^{Aut})\big)$$

Proof. If $p'(1-r)\, u'\big(c_1(x)\big) > (1-p')(R-1)\, u'\big(c_2(x)\big)$ for all x, then it is optimal for the investor not to invest. Thus, $c_1^{Aut} = c_2^{Aut} = 1$ and the optimality condition reduces to a inequality constraint on the effective probability, $p' > \overline{p}^{Aut} \equiv \frac{R-1}{R-r} \in (0,1)$.

If $p'(1 - r)\, u'\big(c_1(x)\big) < (1 - p')(R - 1)\, u'\big(c_2(x)\big)$ for all x, it is optimal for the investor to fully invest. Thus, $c_1^{Aut} = r$ and $c_2^{Aut} = R$ and the optimality condition again reduces to a inequality constraint on the effective probability, $p' < \underline{p}^{Aut} \equiv \frac{(R-1)u'(R)}{(R-1)u'(R)+(1-r)u'(r)} \in (0,1)$.

Otherwise, there must exist an $x \in (0,1)$ where $p'(1 - r)\, u'\big(c_1(x)\big) = (1 - p')(R - 1)\, u'\big(c_2(x)\big)$ which implicitly defines the optimal level of investment.

Lastly, $\underline{p}^{Aut} < \overline{p}^{Aut}$ because $\underline{p}^{Aut} \equiv \frac{(R-1)u'(R)}{(R-1)u'(R)+(1-r)u'(r)} < \frac{(R-1)u'(R)}{(1-r)u'(r)} < \frac{(R-1)u'(1)}{(1-r)u'(1)} \equiv \overline{p}^{Aut}$. ■