Real Interest Rates, Bank Borrowing, and Fragility*

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Abstract

How do real interest rates affect financial fragility? We study this issue in a model in which bank borrowing is subject to rollover risk. A bank's optimal borrowing trades off the benefit from investing additional funds into profitable assets with the cost of greater risk of a run by bank creditors. Changes in the interest rate affect the price and amount of borrowing, both of which influence bank fragility in opposite directions. Thus, the marginal impact of changes to the interest rate on bank fragility depends on the *level* of the interest rate. Finally, we derive testable implications that may guide future empirical work.

Keywords: bank borrowing, rollover risk, fragility, real interest rates, global games, funding liquidity risk channel

JEL classifications: G01, G21, G28.

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1 Introduction

The relationship between interest rates and financial stability has been at the fore of policy and academic debates since the Global Financial Crisis of 2007-08. Recent research has improved our understanding of a risk-taking channel of monetary policy, which posits that lower interest rates increase the riskiness of banks' loan portfolios (e.g., Dell'Ariccia et al., 2014; Martinez-Miera and Repullo, 2017). This literature is, however, silent on the implications of interest rates on fragility, i.e., banks' proclivity for sudden funding dry-ups (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). Fragility is an innate and distinct feature of banks that stems from their reliance on funding subject to rollover risk (e.g., short-term unsecured wholesale debt and interbank loans) to finance investments (Adrian and Shin, 2011).[1] Aside from loan risk, fragility is another key determinant of bank risk and consequently of financial stability. It is, thus, critical to complement our understanding of the risk-taking channel with an assessment for how interest rates affect bank fragility (Figure 1).

This paper constitutes an attempt of such an assessment. In order to focus attention on interest rates and bank fragility, we abstract away from risk-taking incentives. The starting point for our analysis is a standard model of a bank subject to rollover risk (Rochet and Vives, 2004; Vives, 2014). We extend the model by endogenizing both the price of debt and the bank's borrowing volume (Section 2). These features allow us to isolate the funding liquidity risk channel for interest rates and to decompose the impact of increases in the interest rate on fragility into distinct and opposing price and scale effects. We show that bank fragility increases when the

[1] For the United States, Hanson et al. (2015) document that deposits represent over three quarters of funding for commercial banks, of which approximately half of those in large banks are uninsured. Across Europe, Certificates of Deposits (CDs) are a common form of unsecured funding for banks. Perignon et al. (2018) document that the market for CDs is roughly as large as that for repurchase agreements and about ten times as large as the unsecured interbank market.

level of the risk-free interest rate is low, but is decreasing when the level is high. Our analysis, thus, contributes to the debate on the financial stability consequences of monetary policy 'normalization' (Powell, 2019).

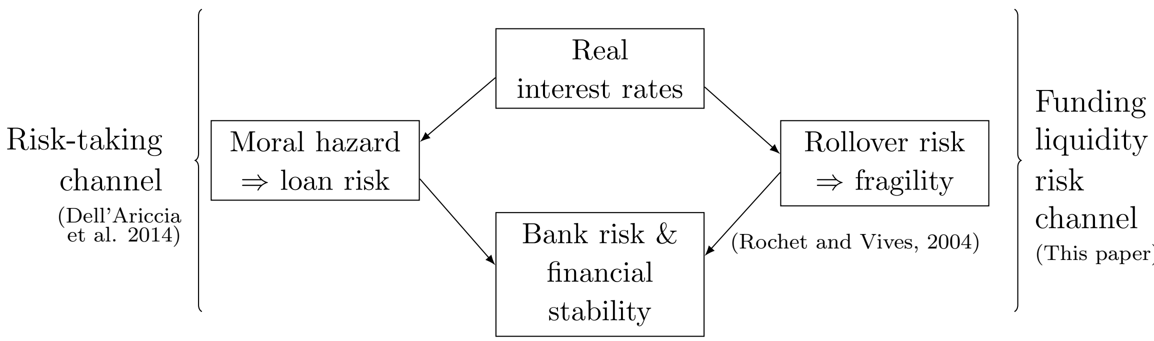

Figure 1: The risk-taking channel, e.g., Dell'Ariccia et al. (2014, 2017); Martinez-Miera and Repullo (2017) has focused on the relationship between interest rates, risk-taking incentives and loan risk. Rochet and Vives (2004), among others, study the relationship between rollover risk, fragility and bank risk. Building on their model, our paper highlights a funding liquidity risk channel, i.e., the relationship between interest rates, rollover risk and fragility.

In our model, a bank chooses how much uninsured demandable debt to issue in order to finance a profitable investment that is costly to liquidate.[2] Following a shock to the bank's balance sheet, concerns over the future viability of the bank can precipitate withdrawals, requiring the bank to liquidate investments. Bank failure is thus driven by both the exogenous balance sheet shock and losses from the costly liquidation of investments to serve endogenous withdrawals. Moreover, individual withdrawal incentives are further sparked by concerns about the withdrawals of other investors, so the run on the bank is also a consequence of a coordination failure.

[2] Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability is a feature of optimal debt contracts in models with uncertain liquidity needs (Diamond and Dybvig, 1983), agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), or expropriation risk (Ahnert and Perotti, 2021). Accordingly, in what follows, we use "uninsured deposits" to refer to short-term or demandable debt including uninsured retail deposits or wholesale funding.

Using global game techniques (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005), we derive the unique equilibrium in Section 3. The bank fails if the shock exceeds a certain threshold. This fragility threshold decreases in the face value of debt, while the impact of greater borrowing is, in general, ambiguous. On the one hand, when the bank borrows more, it becomes more exposed to a higher risk of a run because at the rollover stage a larger amount of debt has to be refinanced. On the other hand, by borrowing more and increasing its profitable investments, the bank raises the resources available to serve its debt. The former effect dominates if the face value of debt is sufficiently high.

In equilibrium, the bank chooses how much debt to borrow (and thus its investment scale) to maximize its expected equity value, taking into account the impact of its choices on the likelihood of failure, i.e., on the fragility threshold. The face value of the bank's debt, in turn, is determined by investors' break-even condition that equates their expected return from lending to the bank with the risk-free real interest rate. Thus, the bank faces the following trade-off: on the one hand, by scaling up its investments using the borrowed funds, it earns the intermediation margin (the difference between the investment return and the face value of its debt). On the other hand, the bank is more likely to fail because financing investments with additional debt increases its susceptibility to a coordination failure and investor run. The risk of such a run, in turn, induces a counteracting effect that leads the bank to scale down its borrowing and investment compared to a situation without coordination failure.

We consider the implications of changes to the real interest rate in Section 4. First, a lower interest rate leads to greater borrowing and investment. This result is in line with empirical evidence of the standard bank lending channel (e.g., Bernanke and Blinder, 1992; Stein and Kashyap, 2000). However, the mechanism in our model differs from other theories (e.g., Peek and Rosengreen, 2013). In particular, changes in the bank's borrowing and investment scale following a change in the interest rate constitute an optimal response to the change in the risk of experiencing a run.

Second, and at the heart of this paper, we consider how changes in the interest rate impact bank fragility. In general, a marginal change to the interest rate exerts three effects on the fragility threshold. First, there is a *price effect* that operates via the break-even condition that pins down the face value of debt. A higher interest rate implies a higher equilibrium face value of bank debt. Thus, when faced with a higher total cost of funding, the bank is more susceptible to runs and fragility is heightened. Second, there is a *scale effect*. When faced with a higher marginal funding cost, the bank has incentives to lower its borrowing. But the less the bank borrows, the smaller is the scope for coordination failures and runs and so fragility is lowered. Finally, there is an *amplification effect*, which captures the feedback between the risk-adjustment in the price of debt and the bank's default risk. This amplification effect alters the magnitude, but not the sign, of the total effect of the interest rate change on fragility.

Our analysis reveals that, in a *high interest rate environment*, increasing the rate reduces fragility. The intuition for our results rests on two key observations. First, fragility depends only on the total cost of funding. And second, when the interest rate is high, so too is the face value of debt, while the level of borrowing is relatively low. Consequently, a marginal decrease in bank borrowing has a more pronounced impact in reducing fragility than a marginal increase in the face value of debt has on increasing fragility. And so, the scale effect dominates the price effect when the level of the interest rate is high. A important corollary of our result is that, in a *low interest rate environment*, the price effect dominates the scale effect. Consequently, increasing the rate can engender greater fragility.

We further consider how loss-absorbing bank equity interacts with the funding liquidity risk channel. Equity exerts a catalytic effect on borrowing and investment: equity mitigates concerns about the bank's future solvency, thus reducing rollover risk and shifting bank borrowing and lending closer to the benchmark of perfect coordination.[3] At the perfect coordination benchmark, changes in interest rates have no effect on the bank's optimal borrowing anymore. Similarly, higher equity reduces the responsiveness of bank borrowing to changes in interest rates, i.e., the transmission of interest rates via the funding liquidity risk channel is weaker for well-capitalized banks.[4]

Finally, our model generates empirical predictions reviewed in Section 6. First, greater rollover risk and higher interest rates reduces bank borrowing. Second, banks with either more equity or lower exposure to rollover risk are less sensitive to interest rate changes. And finally, in a low interest rate environment, higher policy rates or lower market liquidity increase bank fragility. We link these predictions to existing empirical evidence and briefly discuss how they may inform future empirical work.

Literature. Our paper relates to several strands of the literature. A traditional literature on the bank lending channel posits that a monetary policy tightening leads to a shortfall of banks' deposits and reduces lending (e.g., Bernanke and Gertler (1995); Stein and Kashyap (1995); Stein (1998); Boivin et al. (2010); Peek and Rosengreen (2013)). Drechsler et al. (2017) suggest that banks respond to monetary policy shifts by exercising their market power in deposit markets. We complement this literature by arguing that banks' responses are also shaped by rollover risk.

[3] This catalytic effect is similar to that explored by Morris and Shin (2006) in an international finance context. In their model, official sector assistance to a country in the midst of a financial crisis can spur debtors to lend more, thereby alleviating the crisis.

[4] In the main part we focus on an exogenously given level of equity and study comparative statics of changes in this level. In the Online Appendix, we show that this analysis is not particularly restrictive: even if the bank can choose equity, it would either choose zero or as much equity issuance as possible.

We also contribute to the literature on bank runs and global games. In the unique equilibrium, the run is a consequence of a coordination failure following a large shock to the bank's balance sheet. In particular, we build on Rochet and Vives (2004) where investors delegate their rollover decisions to professional fund managers, so the decisions to roll over are global strategic complements.[5] We contribute by showing how, via the endogenous borrowing choice and face value of debt, the effects of changes in the real interest rate on fragility give rise to countervailing effects absent in other studies with exogenous borrowing and investment (e.g., Bebchuk and Goldstein, 2011; Vives, 2014; König, 2015).

In a recent contribution, Leonello et al. (2022) consider how, when depositors do not internalize how their savings choices influence the amount of deposits held by a bank, this can give rise to excessively high bank fragility. Consequently, they argue that, when banks are exposed to rollover risk, banks that issue more deposits are less fragile. In contrast, we consider a setting where investors internalise how their funding choices influence bank fragility. Consequently, the equilibrium level of borrowing by the bank is below the perfect-coordination benchmark.

Our paper also connects to the literature on the risk-taking channel of monetary policy and its implications for financial stability. Dell'Ariccia et al. (2014, 2017) study the risk-shifting incentives of banks and argue that expansionary monetary policy increases bank leverage and risk-taking. Martinez-Miera and Repullo (2017) consider a 'search-for-yield' mechanism: when monetary policy reduces yields on safer assets

[5]Goldstein and Pauzner (2005) study one-sided strategic complementarity due to the sequential service constraint of banks (Diamond and Dybvig, 1983). Liu (2016) studies how runs on banks interact with freezes in the interbank market, giving rise to amplification. Eisenbach (2017) shows how rollover risk from demandable debt effectively disciplines banks for idiosyncratic shocks, while a two-sided inefficiency arises for aggregate shocks. Ahnert et al. (2019) consider how the introduction of senior secured debt influences rollover risk and banks' borrowing and investment choices. Carletti et al. (2020) study the role of liquidity regulation and its interaction with capital requirements. Li and Ma (2021) study the interaction between bank runs and falling asset prices in secondary markets and show the desirability of a committed liquidity support by a regulator.

compared to the interest on their long-term liabilities, financial institutions rebalance their asset portfolios towards riskier short-term assets. Our model complements this literature by focusing on a funding liquidity risk channel: higher failure risk stemming from expanding the bank's balance sheet by increased short-term borrowing.

Finally, Li (2017) also considers the effect of interest rates on bank fragility.[6] Building on Ennis and Keister (2010), the paper studies how the return on bank assets (and the term premium in an extension) affects a measure of fragility, namely the maximum probability of a sunspot variable consistent with partial runs in equilibrium. Li (2017) shows that fragility can be non-monotonic in interest rates, with two counterveiling forces: higher asset returns allow the bank to offer more liquidity provision (reducing ex-ante run incentives) but the spread between good and bad states also widens (increasing ex-ante run incentives). There are several important differences to our paper. While the interest rate refers to bank loan rates or asset profitability in Li (2017), we consider the cost of bank funding on the liability side. In doing so, we endogenize the size of the bank's balance sheet, which is fixed in Li (2017). Second, Li (2017) considers sunspots to address multiple equilibria, while we follow the global-games literature and obtain a unique equilibrium with endogenous bank runs. As a result, our notion of fragility (the ex-ante probability of a run) differs.

2 Model

The model builds on Rochet and Vives (2004), Vives (2014), and Ahnert et al. (2019). There are three dates $t = 0, 1, 2$, a single good for consumption an investment, a continuum $\omega > 0$ of investors and a representative bank owner / manager (henceforth

[6] We thank an anonymous referee for pointing out this reference.

banker for short). Each investor is risk-neutral, endowed with one unit of funds at $t = 0$ and is indifferent between consuming at $t = 1$ and $t = 2$. Investors can store their funds at $t = 0$ to earn a (gross) risk-free return $r > 0$ at $t = 2$. The banker is penniless but has access to illiquid investments that return $R > r$ at $t = 2$ per unit invested at $t = 0$. However, liquidating investments at $t = 1$ yields only a fraction $\psi \in (0, 1)$ of the investment's return. To finance investments, the banker issues $D \geq 0$ of demandable debt to investors at $t = 0$ with face value $F > 0$. The bank's initial balance sheet is $I \equiv D$, where I is the amount invested.

At $t = 1$, investors either redeem their claims against the bank or roll them over until $t = 2$. Each investor delegates the rollover decision to a professional fund manager who is rewarded for making the right decision: if the bank does not fail, a manager's payoff difference between withdrawing and rolling over is a cost $c > 0$; if the bank fails, the differential payoff is a benefit $b > 0$.[7] The conservatism ratio, $\gamma \equiv \frac{b}{b+c} \in (0, 1)$, summarizes these payoffs, with more conservative managers (higher γ) being less inclined to roll over.[8] For simplicity, we assume that the face value of debt, F, is independent of the withdrawal date.

The banker is protected by limited liability and is subject to a shock A at $t = 2$. This shock may improve the bank's balance sheet, $A < 0$, but operational, market, or legal risks may require writedowns, $A > 0$. The shock is drawn at $t = 1$ from a continuous probability distribution with a decreasing reverse hazard rate, $\frac{d}{dA}\frac{g(A)}{G(A)} < 0$, where $G(A)$ denotes the cumulative distribution and $g(A)$ the probability density.[9]

[7] As an example, assume the cost of withdrawal is c; the benefit from getting the money back or withdrawing when the bank fails is $b + c$; the payoff for rolling over when the bank fails is zero.

[8] Reviewing debt markets during the financial crisis, Krishnamurthy (2010) argues that investor conservatism was an important determinant of short-term lending behavior. See also Vives (2014).

[9] This property ensures a unique borrowing choice of the bank. It holds, for instance, for the exponential, (log-)normal, and uniform distributions.

Assets	Liabilities
$RI - A$	FD
	$E_2(A)$

Table 1: Balance sheet at $t = 2$ after a small shock and all debt rolled over.

Table 1 shows the balance sheet at $t = 2$ for a small shock and when all debt is rolled over. In this case, the value of bank equity is given by $E_2(A) \equiv \max\{0, RI - A - FD\}$ and the bank fails whenever $A > \overline{A} \equiv RI - FD$.[10] If, however, a fraction $\ell \in [0, 1]$ of debt is withdrawn at $t = 1$, the banker liquidates a share $\ell \frac{FD}{\psi RI}$ of the investment to repay debt holders. As a result, the banker has insufficient resources to repay the outstanding debt at $t = 2$ and fails whenever

$$RI - A - \ell \frac{FD}{\psi} < (1 - \ell) FD \,, \tag{1}$$

or $A > RI - FD\,[1 + \ell z]$, where $z \equiv \frac{1}{\psi} - 1 > 0$ is the per-unit cost of liquidation.[11] We assume zero recovery upon bank failure at the final date.[12]

At $t = 1$, fund managers base their withdrawal decisions on a private signal (Morris and Shin, 2003):

$$x_i = A + \epsilon_i, \tag{2}$$

where ϵ_i is a mean-zero noise term that is independent of the shock A and identically and independently distributed across fund managers according to a continuous distribution H with support $[-\epsilon, \epsilon]$ for $\epsilon > 0$. Table 2 summarizes the timeline.

[10] Our baseline model assumes an additive or scale-invariant shock to the balance sheet, as in Ahnert et al. (2019), which allows us to better isolate the novel funding liquidity risk channel. In the Online Appendix, we study the consequences of having a multiplicative or scale-dependent shock. While this specification introduces additional effects, our insights are robust.

[11] The bank closes early if it runs out of funds at $t = 1$, $RI - A < \ell \frac{FD}{\psi}$. The relevant failure condition, however, is equation (1), as in Rochet and Vives (2004) or Vives (2014).

[12] Our results are qualitatively unchanged for positive recovery values.

$t = 0$	$t = 1$	$t = 2$
1. Debt issuance	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
	3. Withdrawals	3. Debt repayment
	4. Consumption	4. Consumption

Table 2: Timeline of events.

3 Equilibrium

We solve for the symmetric pure-strategy perfect Bayesian equilibrium in which fund managers at $t = 1$ use threshold strategies, i.e. rolling over if and only if their signal indicates a healthy balance sheet, $x_i \leq x^*$, and the bank fails if and only if $A > A^*$.[13]

Definition 1. *The symmetric pure-strategy perfect Bayesian equilibrium comprises thresholds, x^* and A^*, borrowing volume D^*, and face value of debt F^* such that*

a. at $t = 0$, given the thresholds (x^*, A^*), the banker maximizes his expected equity value by choosing borrowing D and the face value F, subject to a participation constraint of investors that ensures them an expected return of at least r;

b. at $t = 1$, given the banker's choices (D^*, F^*), the threshold strategy, x^*, maximizes fund managers' expected payoffs, given that the banker fails for $A > A^*$.

We solve the model backwards in two steps. First, we pin down x^* and A^* at the rollover stage at $t = 1$. Second, we characterize the banker's borrowing choice at $t = 0$, where the banker takes into account how his choices affect rollover risk.

[13] Since we assume that private information is sufficiently precise, the equilibrium is unique (Morris and Shin, 2003). It is also an extremal equilibrium in monotone strategies (Vives, 2005). Since the rollover decision is binary, our focus on threshold strategies is without loss of generality.

3.1 Rollover risk of debt

For the solution of the rollover subgame at date $t = 1$, we focus on the global game solution of vanishing private noise about the balance sheet shock, $\epsilon \to 0$ (Eisenbach, 2017; Ahnert et al., 2019; Carletti et al., 2020). As a result, the rollover threshold converges to the failure threshold, $x^* \to A^*$. In this case, fund managers face only strategic uncertainty about the behavior of other managers at $t = 1$ because the fundamental uncertainty about the magnitude of the shock vanishes.

Proposition 1. Failure threshold. *There exists a unique failure threshold A^*. All fund managers refuse to roll over and the banker defaults if and only if*

$$A > A^* \equiv RI - FD\,[1 + \gamma z]\,. \tag{3}$$

Proof. See Appendix A1. ■

Proposition 1 pins down the unique incidence of a run on bank debt. The failure threshold can be expressed as $A^* = \overline{A} - \gamma z F D$, so rollover risk causes a deviation of the failure threshold A^* from the perfect-coordination benchmark, $\overline{A}$. In particular, rollover risk is a consequence of the combination of fund manager conservatism, $\gamma > 0$, and asset illiquidity, $\psi < 1$ (or $z > 0$). Without rollover risk, $\gamma z \to 0$, the failure threshold thus converges to its perfect-coordination benchmark, $A^* \to \overline{A}$. We henceforth use A^* as our measure of bank fragility. An increase of A^* reduces fragility as it decreases the range of shocks (A^*, ∞) for which the bank fails.

Lemma 1. Borrowing volume and fragility. *An increase in bank borrowing increases fragility, $\frac{\partial A^*}{\partial D} < 0$, if and only if debt is sufficiently expensive, $F > \frac{R}{1+\gamma z}$.*

Proof. See Appendix A1. ■

Greater borrowing induces two opposing effects on the failure threshold. First, greater borrowing allows the bank to increase the scale of its profitable investments, $\frac{\partial I}{\partial D} = 1$, which reduces fragility because it provides more resources that can be used to repay debt. Second, because the additional borrowing may be withdrawn, it increases the bank's exposure to a debt run, thus heightening fragility. Note that the effective per-unit cost of debt that impacts the bank's fragility at $t = 1$ exceeds the face value F due to the possibility that assets are liquidated at a cost when the debt is withdrawn. This additional per-unit cost of debt is given by $\gamma z F$. As shown in Lemma 1, the second effect dominates the first if the total effective per-unit cost of debt at $t = 1$, $F(1 + \gamma z)$, exceeds the per-unit return from the bank's investment. In this case, more borrowing unambiguously increases fragility. Moreover, a higher liquidation value mitigates the influence of fund manager conservatism and reduces fragility, $\frac{\partial A^*}{\partial \psi} > 0$ (Rochet and Vives, 2004). Finally, with a higher face value of debt, the bank must liquidate a larger share of assets to meet early withdrawals, $\frac{\partial A^*}{\partial F} < 0$. A higher interest rate does not directly affect fragility, $\frac{\partial A^*}{\partial r} = 0$. But, as we subsequently show, it indirectly influences fragility via the volume of borrowing and price of debt.

3.2 Optimal borrowing and investment

The banker chooses the level of borrowing D and offers a face value F to investors in order to maximize his expected equity value at date $t = 2$, subject to the investor participation constraint in funding markets, the balance sheet identity, and the failure threshold $A^*(D, F)$. The banker's problem can be expressed as

$$\max_{D,\,F}\ \pi \quad \equiv \quad \int_{-\infty}^{\infty} E_2(A) dG(A) = \int_{-\infty}^{A^*(D,F)} \left[RI - A - FD\right] dG(A) \tag{4}$$

$$\text{s.t.} \quad F\,G(A^*(F, D)) \geq r, \tag{5}$$

where the participation constraint in (5) states that investors' expected return from funding the bank must be at least the risk-free return earned on the outside option. Since the banker's profit function is strictly decreasing in F for all values of D, profit maximization implies that (5) must hold with equality. Proposition 2 characterizes the interior solution to the constrained optimization problem.

Proposition 2. Bank borrowing and debt pricing. *For any $r \in (\underset{\sim}{r}, \tilde{r})$ there exists a unique interior equilibrium with borrowing volume $D^* \in (0, \omega)$ and face value $F^* \in (\frac{R}{1+\gamma z}, R)$.*

Proof. See Appendix A2. ■

To understand the intuition behind Proposition 2, note that, for a given face value F, a marginal increase in borrowing exerts two effects on the bank's expected equity value, π. First, an increase in D alters the failure threshold, A^*, and the set of shock realizations where the bank fails. As shown in Lemma 1, borrowing lowers the failure threshold if and only if the face value is sufficiently large, $F > R/(1+\gamma z)$. Second, for a given realization of the shock, an increase in D changes the equity value conditional on the bank not failing, $(R - F)D - A$. In particular, greater borrowing increases the bank's equity value if and only if the intermediation margin is positive, i.e., $R - F > 0$. Otherwise, debt is prohibitively expensive and the bank incurs a loss from borrowing any amount.

For an intermediate face value, $R/(1 + \gamma z) < F < R$, the banker's equity value strictly increases in D, while the failure threshold strictly decreases. Hence, the banker's optimal borrowing choice trades off the marginal benefit from increasing the equity value against the marginal cost which stems from the higher expected loss due to the increased likelihood of failure. For given F, the optimal choice of D satisfies

$$R - F = -(1+\gamma z)FD\frac{g(A^*)}{G(A^*)}\frac{dA^*}{dD}. \tag{6}$$

The marginal benefit from additional borrowing on the left-hand side of equation (6) corresponds to the intermediation margin. The right-hand side is the expected loss due to a marginal change in the threshold A^*. The term $\frac{g(A^*)}{G(A^*)}\frac{dA^*}{dD}$ measures the likelihood of default following the total change in the failure threshold conditional on not failing which multiplies the total value of losses in the event of failure. These include the reduction in the banker's equity, $E_2(A^*) = \gamma zFD$, and the losses to creditors, FD, that the banker internalizes through the adjustment in the price of debt via the participation constraint.[14]

The binding participation constraint in Equation (5) and condition in Equation (6) together determine the banker's choice of borrowing and the face value of debt.[15] Figure 2 shows the equilibrium. The solid line is the borrowing schedule $D^*(F)$ derived from (6) and the dashed line $F^*(D)$ is the participation constraint in (5). Both curves intersect at an intermediate r such that $F^* \in (\frac{R}{1+\gamma z}, R)$ and $D^* \in (0, \omega)$.

4 Effects of the real interest rate

In this section, we derive comparative statics for equilibrium borrowing and fragility. Our primary focus is on the risk-free real interest rate, r. Following Dell'Ariccia et al.

[14] If F falls outside of the bounds in Proposition 2, then either debt is sufficiently cheap, $F < R/(1+\gamma z) < R$, the banker borrows as much as possible, $D^* = \omega$ (because borrowing reduces the likelihood that the bank fails and raises the equity value). Or, conversely, debt is very expensive, $F > R$, and the bank completely abstains from any borrowing, $D^* = 0$ (because more borrowing raises the likelihood of failure and reduces the equity value).

[15] The interval in which an interior optimum exists is derived by using the creditors' participation constraint to obtain thresholds such that, at $r = \underset{\sim}{r}$, there is no borrowing, $D^* = 0$, and at $r = \tilde{r}$, the banker borrows $D^* = \omega$.

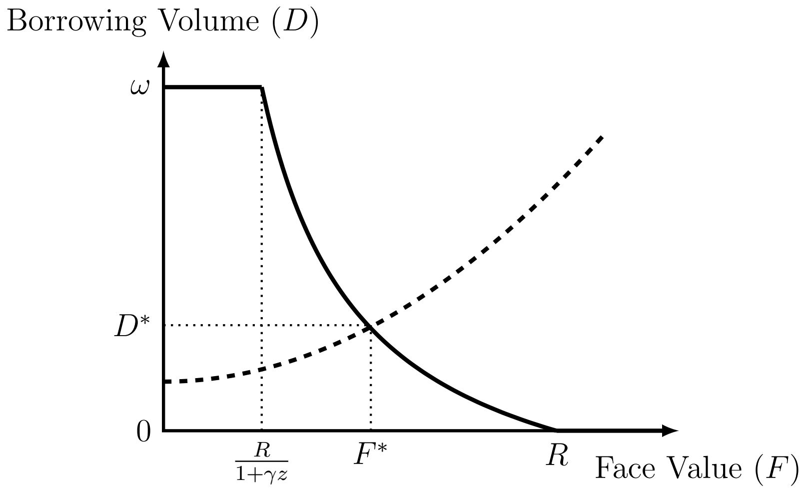

Figure 2: The unique equilibrium borrowing volume and debt face value are determined at the intersection of the creditors' participation constraint (dashed line) and the bank's demand for funds (solid line).

(2014), we assume that the real interest r is determined at $t = 0$ before the bank chooses borrowing and investment.

4.1 Benchmark: no rollover risk

It is instructive to first consider a benchmark without rollover risk. In particular, when $\gamma = 0$, fund managers are no longer subject to strategic complementarities in their rollover decisions and inefficient runs do not occur. Consequently, the bank fails at $t = 2$ whenever $A > \overline{A}$. Moreover, the bank borrows the maximal feasible amount, $D^* = \omega$, as long as $F^* < R$, i.e., whenever the net marginal benefit is positive.

Proposition 3. No rollover risk. *Without rollover risk, $\gamma = 0$, a marginal change to the real interest rate has no effect on the bank's borrowing and investment scale.*

In the absence of rollover risk, changes in the interest rate have no effect on the intensive margin of the banker's borrowing. However, by influencing the face value of debt, a change in the real interest rate can impact the extensive margin of bank borrowing. Whenever F^* increases and exceeds R, the bank stops borrowing.

4.2 Rollover risk and the funding liquidity risk channel

When $\gamma > 0$, the rollover decisions are subject to strategic complementarity. As a result, the bank fails whenever $A > A^*$. Moreover, for any $F \in (R/(1+\gamma z), R)$, the banker optimally responds to an increase in the risk of failure by borrowing less. In other words, the presence of rollover risk gives rise to a 'scale effect' such that $D^* < \omega$. This, in turn, induces the optimal borrowing to respond to changes in interest rate. As such, we refer to this channel as the *funding liquidity risk channel*.

Proposition 4. The funding liquidity risk channel. *An increase in the real interest rate raises the face value of debt,* $\frac{dF^*}{dr} > 0$*, and reduces borrowing,* $\frac{dD^*}{dr} < 0$.

Proof. See Appendix A4. ■

An increase in the interest rate implies that investors require a higher face value in order to supply funding to the bank. A higher face value, in turn, raises the marginal cost of borrowing by increasing the likelihood of the bank failing due to run risk. Moreover, the bank must forego greater equity value in the event of failure. The banker, thus, responds by reducing the amount of borrowing and investment, thereby counteracting the effect of the higher face value on the risk of a run.

Figure 3 shows the scale effect and funding liquidity risk channel. It plots the banker's optimal borrowing against the real interest rate, r. The dashed line depicts the perfect-coordination benchmark in the absence of coordination risk ($\gamma = 0$), while the solid line shows the case with rollover risk ($\gamma > 0$). For values of r that are either sufficiently small ($r < \underset{\sim}{r}$) or sufficiently large ($r > \tilde{r}$), rollover risk has no effect on the bank's borrowing and investment scale. For intermediate values $r \in (\underset{\sim}{r}, \tilde{r})$, however, the scale effect is present and is given by the distance between the dashed line and solid curve. Due to the scale effect, an increase in the interest rate is associated with lower borrowing as the bank trades off the net marginal benefit against the increased likelihood of failure due to a higher borrowing scale.

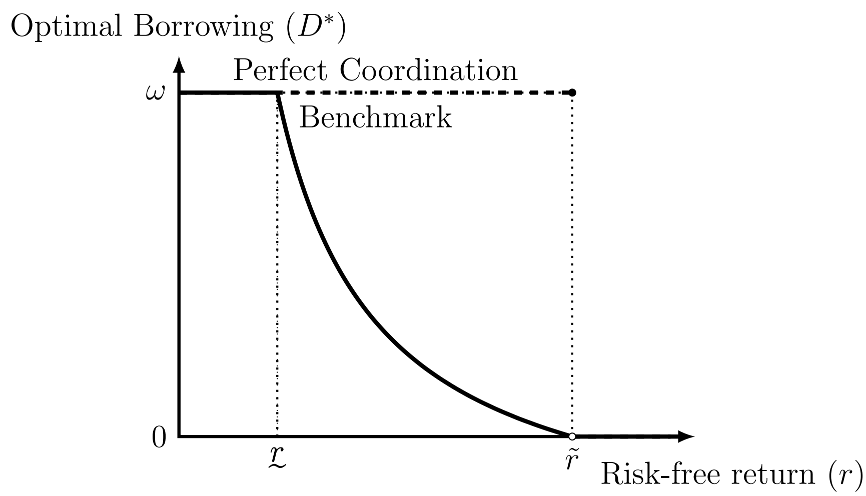

Figure 3: Rollover risk reduces borrowing relative to a perfect-coordination benchmark. Borrowing volume with (solid) and without rollover risk (dashed).

4.3 Funding liquidity risk channel and bank fragility

We next turn to the implications for bank fragility, which we measure by the bank's failure probability. Since the ex-ante failure probability is monotonically decreasing in the failure threshold A^*, to study the effects of the interest rate on bank fragility, it suffices to consider its effects on A^*.

The result in Proposition 4 implies that the total effect of the interest rate on bank fragility is a priori not clear. In particular, it follows from Lemma 1 that for $F \in (R/(1+\gamma z), R)$, the failure threshold decreases in both F and D. But since changes in the risk-free return can have opposing effects on the face value of debt and the bank's borrowing volume, the total effects on the failure threshold are ambiguous. To see this, we decompose the total effect of a marginal increase in the risk-free return on the failure threshold into three separate terms:

$$\frac{dA^*}{dr} = \underbrace{\frac{1}{1+\frac{g(A^*)}{G(A^*)}\frac{\partial A^*}{\partial F}F^*}}_{\text{Amplification effect }(+)} \times \left(\underbrace{\frac{\partial A^*}{\partial D}\frac{dD^*}{dr}}_{\text{Scale effect }(+)} + \overbrace{\frac{1}{G(A^*)}\frac{\partial A^*}{\partial F}}^{\text{Price effect }(-)} \right). \tag{7}$$

The first term in brackets is a scale effect, which affects fragility via the total adjustment in borrowing scale. Since $\frac{dD^*}{dr} < 0$ and $\frac{\partial A^*}{\partial D} < 0$, it follows that the scale effect reduces fragility. The second effect in brackets, which we refer to as the price effect, captures the contribution of the higher face value of debt to the total effect on A^*. Since $\frac{\partial A^*}{\partial F} < 0$, the price effect raises the bank's fragility. Finally, the third effect (that multiplies into the previous two effects) is an amplification that accounts for the fact that an increase in fragility leads to an increase in the face value via the investors participation constraint, which further increases fragility.

As Proposition 5 shows, the effect of changes in the real interest rate on bank fragility (e.g., due to changes in monetary policy) depends on the relative magnitudes of the opposing scale and price effects. These are, in turn, shaped by the *level* of the interest rate.

Proposition 5. The interest rate and bank fragility. *There exists a unique* $\underline{r} \in (\underline{r}, \tilde{r})$*, such that a marginal increase in the risk-free return reduces fragility,* $\frac{dA^*}{dr} > 0$*, if and only if* $r > \underline{r}$*.*

Proof. See Appendix A4. ■

The intuition behind the result of Proposition 5 is as follow. First note that the failure threshold, A^*, depends on the bank's total funding cost, FD. Thus, at any interior equilibrium, the impact of the scale effect in reducing fragility increases with the face value of debt since $\partial A^*/\partial D = R - (1 + \gamma z)F < 0$. At the same time, the negative impact of the price effect increases with the amount of bank borrowing since $\partial A^*/\partial F = -(1+\gamma z)D$. Thus, whenever the risk-free return is sufficiently small, the face value of debt is low, while bank borrowing is relatively high. In such a low interest rate environment, the price effect dominates and financial fragility worsens following an increase in the interest rate.

Conversely, in a high interest rate environment, the face value of debt, and thus the magnitude of the scale effect, are large compared to the borrowing volume and price effect. Consequently, an increase in the interest rate reduces financial fragility. At the threshold, $\underline{r}$, the face value is such that price and scale effects wash out.

The threshold itself crucially depends on both the extent of rollover risk, γ, and market liquidity, ψ. If, for example, market liquidity improves (ψ increases), the bank is better able to serve withdrawals and forestall failure. This, in turn, leads to a lower face value, F. As a consequence, however, the impact of the scale effect in reducing fragility is less pronounced, while the price effect is amplified. In sum, the threshold, $\underline{r}$, at which the price and scale effect perfectly offset each other increases and so the range where a lower interest rate reduces fragility becomes larger.[16]

Corollary 1. *The threshold* $\underline{r}$ *is increasing as market liquidity improves,* $\frac{\partial \underline{r}}{\partial \psi} > 0$.

Figure 4 illustrates the implications for bank fragility emanating from the funding liquidity risk channel. Expansionary monetary policies (a decrease in r) raise bank fragility via the scaling channel whenever $r > \bar{r}$. Thus, for 'high interest envi-

[16] Since ψ and γ only appear together as γz, a similar logic applies when rollover risk γ is reduced.

ronments' the bank fragility consequences of our funding liquidity risk channel are in line with those of the standard risk-taking channel. However, in a 'low interest rate environment', i.e. for $r < \underline{r}$, reductions in the interest rate lower fragility and exert a stabilizing influence, thus contrasting the risk-taking channel.

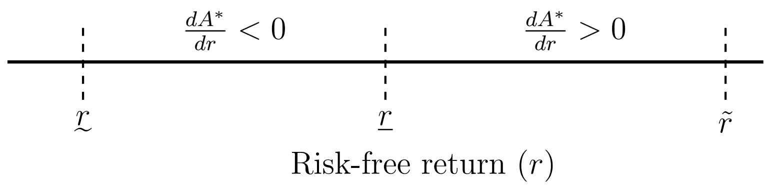

Figure 4: Effect of interest rate on bank fragility for different levels of interest rates.

To further illustrate this point, consider an increase in the interest rate from a low interest environment, e.g., a monetary policy 'exit' from low interest rates. While the standard risk-taking channel predicts that financial stability improves, our funding liquidity risk channel implies the opposite, i.e., a worsening of bank fragility due to the dominance of the price effect.

5 Bank equity and funding liquidity risk channel

In this section, we consider the implications of equity on the bank's borrowing choice. We first focus on the case in which the banker is endowed with an exogenous amount of initial equity, $E > 0$. We subsequently argue that endogenizing equity does not qualitatively alter our results. The endowment of equity has no material impact on the sub-game equilibrium at $t = 1$ except that the balance sheet identity at $t = 0$ now reads $I \equiv D + E$. Substituting this into equation (3), the failure threshold is given by

$$A^* \equiv R(D + E) - FD\,[1 + \gamma z]\,. \tag{8}$$

Since the banker invests all funds into profitable investments, the additional resources $RE > 0$ at $t = 2$ can also be used to serve debt. Hence, equity allows the bank to withstand larger shocks and lowers fragility, $\frac{\partial A^*}{\partial E} = R > 0$. Moreover, the presence of equity requires us to distinguish between the level of borrowing, D^*, and the level of investment, $I^* = D^* + E$. Proposition 6 states the resulting optimal borrowing and investment of the banker, generalizing our previous result.

Proposition 6. Bank equity. *There exist values* $\underaccent{\tilde}{r}_E > \underaccent{\tilde}{r}$ *and* $\tilde{r}_E > \tilde{r}$ *such that the bank's optimal borrowing is given by Equation (6), with* $\underaccent{\tilde}{r}$ *being replaced by* $\underaccent{\tilde}{r}_E$ *and* $\tilde{r}$ *replaced by* $\tilde{r}_E$*. Greater equity increases borrowing,* $dD^*/dE > 0$*, makes debt cheaper,* $dF^*/dE < 0$*, and decreases fragility,* $dA^*/dE > 0$*.*

Proof. See Appendix A5. ■

Bank equity exerts a catalytic effect on bank scale as Figure 5 illustrates. That is, a marginal increase in equity induces the banker to increase borrowing and investment. Along the extensive margin, greater equity shifts the bounds $\underaccent{\tilde}{r}_E$ and $\tilde{r}_E$ outwards. The increase in the lower bound implies that additional equity enables the banker to borrow the maximum $D^* = \omega$ for a larger range of risk-free returns. The increase in the upper bound implies that the range of risk-free returns where borrowing becomes prohibitively costly shrinks. And, on the intensive margin, for any given $r \in (\underaccent{\tilde}{r}_E, \tilde{r}_E)$, an increase in equity increases bank borrowing further, $dD^*/dE > 0$.

How does the presence of bank equity influence the pass-through of changes in the interest rate via the funding liquidity risk channel? Figure 5 conveys the main insights. First, while both bounds, $\underaccent{\tilde}{r}_E$ and $\tilde{r}_E$ increase following a marginal increase in bank capital, the upper bound increases by more. Thus the gap, $\tilde{r}_E - \underaccent{\tilde}{r}_E$, also increases. Second, since $dD^*/dE > 0$ for all $r \in (\underaccent{\tilde}{r}_E, \tilde{r}_E)$, this suggests that, typically,

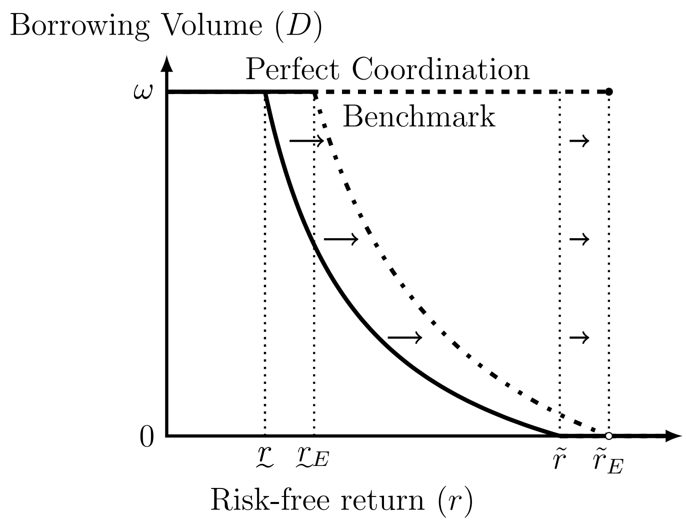

Figure 5: Equity increases the range where over which maximum borrowing takes place and also shifts the borrowed amount outwards (dash-dotted curve). Borrowing volume with (solid curve) and without rollover risk (dashed line).

the borrowing schedule $D^*(r)$ becomes flatter. This implies that marginal changes in the interest rate have a smaller impact on bank borrowing when banks hold more equity. This result follows since more bank equity reduces rollover risk. Hence, the interest rate has a smaller impact on the bank's fragility and therefore dampens the bank's corresponding adjustment of borrowing and investment.

Endogenizing the bank's equity choice. So far we assumed that the bank either finances its investments solely with debt or is endowed with an exogenous amount of equity. In the Online Appendix, we show that these assumptions are not so restrictive. We allow the bank to freely choose equity, $E \in [0, \overline{E}]$, by promising investors an expected marginal return of ρr, where $\rho > 1$ reflects, for example, costs associated with the issuance of equity (Harris et al., 2020) or market segmentation (Carletti et al., 2020). We show that the bank either issues no equity or as much as possible.

To understand the intuition behind this result, note that what matters for whether or not the bank issues equity is just the relative magnitude of the expected marginal benefit from investing into the profitable asset compared to the expected marginal cost of equity. Whenever $G(A^*)R \geq \rho r$, issuing equity increases expected profits which further incentivizes the bank to issue even more equity and thereby ratcheting up the catalytic effect of equity on debt, implying $E^* = \overline{E}$.

Conversely, if $G(A^*)R < \rho r$, while issuing equity continues to reduce fragility, it is now considerably more expensive relative to the gains from additional investments. In this case, the bank could alternatively rely solely on modulating its issuance of debt to influence fragility whilst earning a positive intermediation margin. The marginal impacts of either issuing debt or equity on fragility are rendered equal as these choices are internalized by the bank in setting the face value of debt. Thus, in this case, we obtain $E^* = 0$, which corresponds to the case of our baseline model.

6 Testable implications

In this section, we present several empirical predictions from our model. Wherever possible we provide references to existing empirical work and discuss how our predictions square with these findings. However, our model also generates new predictions which, to our knowledge, have not yet been tested empirically (notably, Predictions 2(ii) and 3). Hence, these predictions may inform future empirical work.

Prediction 1. *Bank borrowing and lending decreases if (i) rollover risk increases, or (ii) the interest rate increases.*

Prediction 1 is consistent with a number of recent empirical findings. With respect to Prediction 1(i), Jasova et al. (2018) document that a reduction in rollover risk due to the ECB's very long-term refinancing operations increases bank lending. Iyer et al. (2013) find that bank lending to firms is reduced if a larger share of its funding comes from the unsecured interbank market in which it is exposed to rollover risk. de Haas and van Lelyfeld (2014) find that lending growth of banks slows more after a shock for banks with higher reliance on wholesale funding. Ivashina and Scharfstein (2010) and Cornett et al. (2011) document a larger reduction in domestic credit following the financial crisis of 2007-08 for U.S. banks with a higher reliance on wholesale funding.

Regarding Prediction 1(ii), Choi and Choi (2021) document how, following a monetary policy tightening, U.S. banks substitute low-interest retail deposits with high-interest wholesale deposits. But, as the substitution is not perfect, there is an overall reduction of bank borrowing, which translates into a reduction of bank lending. Drechsler et al. (2017) document a related result whereby a monetary policy tightening lead to increases in the deposit rate and outflows, thereby leading to a decrease in bank lending. Girotti (2021) also finds a similar result.

Prediction 2. *The borrowing and lending by banks responds less to an increase in the interest rate if (i) the bank has more equity, or (ii) the bank is exposed to higher rollover risk.*

Greater bank equity increases the interval of interest rates over which bank borrowing and lending is strictly positive. Moreover, the size of the interval grows as bank equity increases. As a consequence, the bank's borrowing becomes less responsive to changes in the interest rate. The lower interest-sensitivity of the bank's borrowing and lending stems from the following two observations. First, an increase in rollover risk leads to an increase in the range of the interest rate where bank borrowing is positive and below the perfect-coordination benchmark. Graphically, an increase in rollover risk leads to a decrease in the lower bound, $\underaccent{\tilde}{r}$, while the upper bound, $\tilde{r}$, remains unchanged in Figure 3. And second, since the total supply of funding is fixed, the schedule for the bank's borrowing becomes, generically, flatter. Consequently, marginal changes in the interest rate elicit weaker responses in bank borrowing.

Prediction 2(i) is also in line with several empirical studies. For example, van den Heuvel (2012) finds that output growth of banking sectors in U.S. states that have low bank capital-to-asset ratios are more sensitive to changes in the Federal funds rate. Jiménez et al. (2012) show that monetary policy tightening episodes have larger negative effects on bank lending for banks with lower capital. Using a different identification strategy, Jiménez et al. (2014) show that monetary policy cuts have a more pronounced impact on lowly capitalised banks to lend more. Acharya et al. (2020) demonstrate that 'full allotment' liquidity provision by the ECB resulted in a lowering of deposit spreads and a further extension of credit by highly capitalised banks to their borrowers. Both these results are consistent with the predictions of our model wherein an increase in bank capital leads to a reduction in the face value of debt claims and an increase in lending.

Prediction 3. *In a low interest rate environment, bank fragility increases if (i) the interest rate increases, or (ii) the bank holds more illiquid assets.*

Similar to the effect of interest rates on fragility studied in Proposition 5, the marginal impact of market liquidity on bank fragility can be decomposed into a scale, price and amplification effects (see Appendix A6 for details). In contrast to before, however, bank fragility is increasing in the scale effect and decreasing in the price effect. Thus, in a low interest rate environment – where the scale effect is less pronounced – bank fragility is reduced as market liquidity improves.

7 Conclusion

We consider a parsimonious model for bank borrowing in the presence of rollover risk to study the relationship between real interest rates and fragility. Changing the interest rate creates ambiguous effects on bank fragility which depends on the prevailing level of the interest rate. In particular, increasing the interest rate from a low-interest rate environment can impair bank fragility. Based on this model we derive a number of testable implications that may inform future empirical work on the interaction between monetary policy and financial stability.

To isolate our main channel, we make several simplifying assumptions, e.g., regarding the dead-weight loss from liquidation, or the maturity structure of debt. Nevertheless, these assumptions have no bearing on the model's core trade-off between issuing more debt to raise profitable investment and increasing the bank's fragility. For example, one can relax the assumption on the dead-weight loss from bankruptcy, i.e., assume a positive liquidation value, and allow creditors to obtain a pro-rata share of liquidation value. Furthermore, introducing a reason for different debt maturities (e.g., through alternative investor preferences), implies that the bank raises parts of its funding by issuing long-term debt. Nonetheless, the key trade-off between scaling up profitable investments and increasing fragility which, in turn, gives rise to a downward-sloping demand for short-term debt by the bank and the deviation from the perfect coordination benchmark, would remain unchanged.

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A Proofs

A1 Proof of Proposition 1 and Lemma 1

The proof is in two steps. First, we show that the failure threshold based on condition (1) implies the existence of well-defined dominance regions. In these regions, fund managers have a dominant action to roll over or withdraw irrespective of the actions of other managers. Suppose that all debt is rolled over, $\ell = 0$. The bank still fails if $A > \overline{A} = RI - FD$, so withdrawing is a dominant strategy for fund managers for $x > \overline{A} + \epsilon$. Similarly, suppose that no debt is rolled over, $\ell = 1$. The bank still does not fail if $A < \underline{A} \equiv RI - \frac{FD}{\psi}$. Thus, rolling over is a dominant strategy for $x < \underline{A} - \epsilon$. For $A \in [\underline{A}, \overline{A}]$, whether the bank fails depends on the withdrawal proportion of fund managers. If the realization of A was common knowledge, the model would exhibit multiple self-fulfilling equilibria (Figure 6).

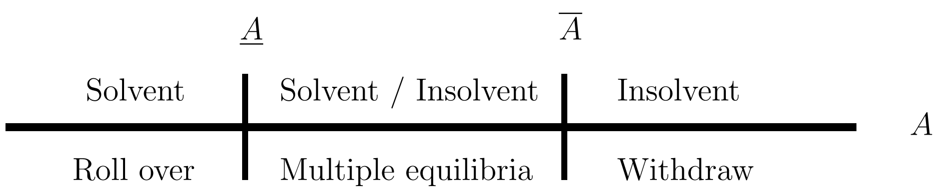

Figure 6: Tripartite classification of the balance sheet shock

Second, we characterize the equilibrium under incomplete information. Suppose fund managers use a symmetric threshold strategy around x^*. For a given realization $A \in [\,\underline{A}, \overline{A}\,]$, the proportion of fund managers who do not roll over debt is $\ell\Big(A, x^*\Big) = \text{Prob}\left(x_i > x^* \middle| A\right) = \text{Prob}\left(\epsilon_i > x^* - A\right) = 1 - H\Big(x^* - A\Big)$. Using condition (1), the failure threshold A^* solves

$$A^* = RI - \big(1 + z\ell\big(A^*, x^*\big)\big)FD. \tag{A1}$$

For given x^*, the left-hand side of (A1) is strictly increasing in A^* and rises from $-\infty$ to ∞. The right-hand side is strictly decreasing in A^* and is bounded above and below by $(1 + z)FD$ and FD, respectively. Hence, there exists a unique failure threshold A^*.

The posterior distribution of the shock conditional on the private signal can be derived using Bayes' rule. The threshold x^* is pinned down by indifference between rolling over and withdrawing of a fund manager who observes $x_i = x^*$: $1-\gamma = \Pr(A > A^* | x_i = x^*)$. For small ϵ, the latter can be written as $\gamma = \Pr(A < A^* | x_i = x^*) = 1 - H\Big(x^* - A^*\Big)$. The indifference condition therefore implies $x^* - A^* = H^{-1}(1-\gamma)$. Inserting it into $\ell(A^*, x^*)$, the withdrawal proportion at the threshold A^* becomes $\ell\Big(A^*, x^*\Big) = 1 - H\Big(x^* - A^*\Big) = 1 - H\Big(H^{-1}\Big(1-\gamma\Big)\Big) = \gamma$. The failure threshold A^* stated in Proposition 1 follows immediately.

The fact that there are no other equilibria in non-threshold strategies follows from standard arguments (Morris and Shin, 2003; Vives, 2005). It exploits the fact that the regions above $\overline{A}$ and below $\underline{A}$ admit equilibria in strictly dominated strategies.

For the proof of Lemma 1, we substitute the balance sheet identity $I \equiv D$ into A^* and differentiate. The derivatives are given by: $\frac{\partial A^*}{dF} = -(1+z\gamma)D < 0$ and $\frac{\partial A^*}{dD} = R - (1+z\gamma)F$. The latter is negative if and only if $R < (1+\gamma z)F$. ■

A2 Proof of Proposition 2

The bank's problem is given by (4). Denote by μ the Lagrange multiplier of the investor participation constraint. The first-order conditions for an interior optimum are:

$$D: \quad \frac{\partial \mathcal{L}}{\partial D} = (R-F)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial D} = 0 \tag{A2}$$

$$F: \quad \frac{\partial \mathcal{L}}{\partial F} = (\mu - D)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial F} = 0 \tag{A3}$$

$$\mu: \quad \frac{\partial \mathcal{L}}{\partial \mu} = -r + FG(A^*) = 0 \tag{A4}$$

Combining equations (A2) and (A3) yields $\mu^* = D + \frac{(R-F)\frac{\partial A^*}{\partial F}}{\frac{\partial A^*}{\partial D}}$. Substituting μ^* into (A2) yields

$$\frac{G(A^*)}{g(A^*)} = \frac{\gamma z F D(-\frac{\partial A^*}{\partial D} + R)}{R-F} = \frac{\gamma z(1+\gamma z)F^2 D}{R-F}. \tag{A5}$$

Equations (A4) and (A5) pin down the interior critical point (F^*, D^*). An interior optimum requires $R/(1+\gamma z) < F < R$. Note that (A3) becomes strictly negative if $F \geq R$. Similarly, $\frac{\partial A^*}{\partial D} > 0$ if $F < R/(1+\gamma z)$, implying that (A2) becomes strictly positive.

The critical point (F^*, D^*) constitutes a maximum as the bordered Hessian, evaluated at (F^*, D^*), is strictly positive. To see this, note that the second derivatives are given by

$$\frac{\partial^2 \mathcal{L}}{\partial D \partial F} = (\mu - D)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial D} - \frac{G(A^*)}{g(A^*)} + (\mu + \gamma z D)F\frac{\partial^2 A^*}{\partial F \partial D} + \gamma z F\frac{\partial A^*}{\partial F} \quad \text{(A6)}$$

$$\frac{\partial^2 \mathcal{L}}{\partial D^2} = (R - F)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial D} + \gamma z F\frac{\partial A^*}{\partial D} \quad \text{(A7)}$$

$$\frac{\partial^2 \mathcal{L}}{\partial F^2} = (\mu - D)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial F} + (\mu + \gamma z D)\frac{\partial A^*}{\partial F} \quad \text{(A8)}$$

Since $\mu^* > D$ and $\frac{R}{1+\gamma z} < F < R$, equations (A6) to (A8) are all negative. Using that the participation constraint is $V(F, D) = FG(A^*) - r$, we further obtain:

$$\frac{\partial V}{\partial F} = G(A^*) + Fg(A^*)\frac{\partial A^*}{\partial F}, \quad \frac{\partial V}{\partial D} = Fg(A^*)\frac{\partial A^*}{\partial D}. \quad \text{(A9)}$$

Thus, $\frac{\partial V}{\partial D}$ is negative for $F > R/(1+\gamma z)$, while $\frac{\partial V}{\partial F}$ is positive when evaluated at (F^*, D^*). To see this, we can express $\frac{\partial V}{\partial F} = g(A^*)\left[\frac{G(A^*)}{g(A^*)} + F\frac{\partial A^*}{\partial F}\right]$ and substitute (A5) for G/g to get:

$$\frac{\partial V}{\partial F} = g(A^*)\left[\frac{FD(1+\gamma z)}{R - F}(F(1+\gamma z) - R)\right] > 0$$

since $F > R/(1+\gamma z)$. Hence, the determinant of the bordered Hessian matrix is

$$H = \begin{vmatrix} 0 & \frac{\partial V}{\partial F} & \frac{\partial V}{\partial D} \\ \frac{\partial V}{\partial F} & \frac{\partial^2 \mathcal{L}}{\partial D^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} \\ \frac{\partial V}{\partial D} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial F^2} \end{vmatrix} > 0.$$

Finally, note that for $F^* \to R$, the left-hand side of (A5) converges to $+\infty$ while the right-hand side remains bounded, implying that $D^* \to 0$. For $F < R/(1+\gamma z)$, the first-order necessary conditions imply that $D^* = \omega$. Using the participation constraint, we can derive bounds on r, given by $\underaccent{\tilde}{r} \equiv RG(0)/(1+\gamma z)$ and $\tilde{r} \equiv RG(0)$, such that for $r \in (\underaccent{\tilde}{r}, \tilde{r})$, it must be that $F \in (R/(1+\gamma z), R)$. Clearly, if $r > \tilde{r}$, then $F > R$ and if $r < \underaccent{\tilde}{r}$, then $F < R/(1+\gamma z)$. This completes the characterization of the bank's optimal choice. ■

A3 Proof of Proposition 3

Let $\gamma \to 0$, then it follows from the proof of Proposition 2 that $\mu^* = 0$. Hence, if $R > F$, then condition (A2) is strictly positive, implying that the bank borrows as much as possible, i.e., $D^* = \omega$. If $F > R$, then $D^* = 0$. Since ω is exogenously given and independent of r, marginal changes in the real interest rate do not affect the bank's borrowing and scale (as long as changes in r do not reverse the relationship between F and R). ■

A4 Proof of Propositions 4 & 5

We can eliminate the Lagrange multiplier μ from the system first-order conditions by combining conditions (A2) and (A3). Hence, we obtain

$$\phi(D,F) \equiv \frac{G(A^*)}{g(A^*)}(R-F) \; - \; \gamma z F D \left(-\frac{\partial A^*}{\partial D} + R\right) \; = \; 0, \tag{A10}$$

with

$$\frac{\partial \phi}{\partial D} = \frac{\partial A^*}{\partial D} \frac{d\,(G/g)}{dA^*}(R-F) + \left(\frac{\partial A^*}{\partial D} - R\right)\gamma z F < 0$$

and

$$\frac{\partial \phi}{\partial F} = \frac{\partial A^*}{\partial F}\left(\frac{d\,(G/g)}{dA^*}(R-F) + 2\gamma z F\right) - \frac{G}{g} < 0.$$

D^* and F^* simultaneously solve $\phi(D^*, F^*) = 0$ and $V(D^*, F^*) = 0$. The Jacobian of this system of two equations is given by

$$\mathbf{J} = \begin{pmatrix} \frac{\partial V(D^*,F^*)}{\partial F} & \frac{\partial V(D^*,F^*)}{\partial D} \\ \frac{\partial \phi(D^*,F^*)}{\partial F} & \frac{\partial \phi(D^*,F^*)}{\partial D} \end{pmatrix}. \tag{A11}$$

At the equilibrium point (D^*, F^*), the determinant of the Jacobian is negative, $|\mathbf{J}| < 0$, since all entries are negative except for $\frac{\partial V(D^*,F)}{\partial F} > 0$. Henceforth, we evaluate all expressions at the equilibrium but suppress the arguments of the derivatives for brevity.

To obtain the effect of r on D^* and F^*, note that $\partial V/\partial r = -1$ and $\frac{\partial \phi}{\partial r} = 0$. Applying the implicit function theorem yields

$$\frac{dD^*}{dr} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & -1 \\ \frac{\partial \phi}{\partial F} & 0 \end{vmatrix}}{|\mathbf{J}|} < 0\,, \qquad \frac{dF^*}{dr} = -\frac{\begin{vmatrix} -1 & \frac{\partial V}{\partial D} \\ 0 & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} > 0\,.$$

The effect of r on fragility can be obtained by totally differentiating A^*:

$$\begin{aligned} \frac{dA^*}{dr} &= \frac{\partial A^*}{\partial D}\frac{dD^*}{dr} + \frac{\partial A^*}{\partial F}\frac{dF^*}{dr} \\ &= \frac{\partial A^*}{\partial D}\frac{dD^*}{dr} + \frac{\partial A^*}{\partial F}\left(\frac{1 - g(A^*)F^*\frac{\partial A^*}{\partial D}\frac{dD^*}{dr}}{g(A^*)F^*\frac{\partial A^*}{\partial F} + G(A^*)}\right) \\ &= \frac{1}{1 + F^*\frac{g(A^*)}{G(A^*)}\frac{\partial A^*}{\partial F}}\left(\frac{\partial A^*}{\partial D}\frac{dD^*}{dr} + \frac{1}{G(A^*)}\frac{\partial A^*}{\partial F^*}\right), \end{aligned}$$

where the second line follows by totally differentiating the participation constraint and substituting dF^*/dr. The third line corresponds to the decomposition in equation (7) and follows after re-arranging.

Substituting for dD^*/dr using the previous result yields

$$
\begin{aligned}
\frac{dA^*}{dr} &\propto -\frac{\partial A^*}{\partial F}\frac{\partial \phi}{\partial D} + \frac{\partial A^*}{\partial D}\frac{\partial \phi}{\partial F} \\
&= \frac{\partial A^*}{\partial F}\left(-\frac{\partial A^*}{\partial D} + R\right)\gamma z F^* \;+\; 2\gamma z F^* \frac{\partial A^*}{\partial F}\frac{\partial A^*}{\partial D} - \gamma z F^* D^* \frac{\partial A^*}{\partial D}\frac{-\frac{\partial A^*}{\partial D} + R}{R - F^*} \\
&= \frac{\gamma z F^*}{R - F^*}\left(-\frac{\partial A^*}{\partial D} + R\right)\left(\frac{\partial A^*}{\partial F}(R - F^*) - D^* \frac{\partial A^*}{\partial D}\right) \;+\; 2\gamma z F^* \frac{\partial A^*}{\partial D}\frac{\partial A^*}{\partial F} \\
&= \frac{\gamma z F^*(1+\gamma z)F^*}{R - F^*}\left(\frac{\partial A^*}{\partial F}(R - F^*) - D^* \frac{\partial A^*}{\partial D} - \frac{2D^*(R - F^*)}{F^*}\frac{\partial A^*}{\partial D}\right) \\
&\propto RD^*\left(F^* + (1+\gamma z)F^* - 2R\right) \;=\; (2+\gamma z)RD^*(F^* - \underline{F}),
\end{aligned}
$$

where $\underline{F} \equiv \frac{R}{1+\frac{\gamma z}{2}}$. The second line follows from using optimality condition (A5) to substitute for $G(A^*)/g(A^*)$ and substituting the expressions for $\partial\phi/\partial F$ and $\partial\phi/\partial D$. The fifth line follows after substituting the expressions for $\partial A^*/\partial D$ and $\partial A^*/\partial F$ and re-arranging.

Observe that $\underline{F} \in (R/(1+\gamma z), R)$. Evaluating (A5) at $\underline{F}$ yields the corresponding debt level $\underline{D}$. Evaluating the participation constraint at $(\underline{F}, \underline{D})$, we obtain a critical value $\underline{r} \in (\underset{\sim}{r}, \tilde{r})$ that solves $V(\underline{D}, \underline{F}; \underline{r}) = 0$. Observe further that $\underline{F}$ and $\underline{D}$ are independent of r. Since F^* strictly increases in r, it follows that $F^* > \underline{F}$ and therefore $\frac{dA^*}{dr} > 0$ if and only if $r > \underline{r}$. ■

A5 Proof of Proposition 6

The failure threshold follows from substituting the generalized balance sheet identity $I \equiv D + E$ into equation (3). Moreover, the proof of Proposition 2 continues to apply except for the generalized bounds $\underset{\sim}{r}_E$ and $\tilde{r}_E$ because E enters A^* and E_2 linearly, so the first-order conditions derived in the proof of Proposition 2 hold. To calculate the bounds $\underset{\sim}{r}_E$ and $\tilde{r}_E$, observe that for $F = \frac{R}{1+\gamma z}$, the first-order conditions still imply $D^* = \omega$ and thus $A^* = RE$. The investor participation constraint implies $\underset{\sim}{r}_E = \frac{RG(RE)}{1+\gamma z}$. By the same logic, for $F \geq R$, the FOCs imply $D^* = 0$ and thus $\tilde{r}_E = RG(RE)$. Hence $\underset{\sim}{r} < \underset{\sim}{r}_E$ and $\tilde{r} < \tilde{r}_E$ because $G(\cdot)$

monotonically increases. The comparative statics of these bounds with respect to equity follow immediately.

To compute the comparative statics of (D^*, F^*, A^*) w.r.t. equity, note that E enters only via A^*, so equations (A9) and (A10) still apply. The partial derivatives with respect to E are given by $\frac{\partial \phi}{\partial E} = \frac{d(G/g)}{dA^*}\frac{\partial A^*}{\partial E}(R-F)$ and $\frac{\partial V}{\partial E} = gF\frac{\partial A^*}{\partial E}$. Hence:

$$\frac{dD^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & \frac{\partial V}{\partial E} \\ \frac{\partial \phi}{\partial F} & \frac{\partial \phi}{\partial E} \end{vmatrix}}{|\mathbf{J}|} > 0, \qquad \frac{dF^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial E} & \frac{\partial V}{\partial D} \\ \frac{\partial \phi}{\partial E} & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} < 0,$$

where the sign of the latter expressions follows from $\frac{\partial V}{\partial E}\frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D}\frac{\partial \phi}{\partial E} = -g(F^*)^2\gamma z(1+\gamma z)R < 0$. Finally, as E enters the participation constraint only via A^*, we have $\frac{dA^*}{dE} = -\frac{G}{gF}\frac{dF^*}{dE} > 0$.
■

A6 Derivation of Prediction 3

We first consider the comparative statics of D^* and F^* with respect to the market liquidity parameter ψ. Note that ψ appears via z, i.e., $z \equiv \frac{1-\psi}{\psi}$ with $dz/d\psi < 0$. Since $\frac{\partial A^*}{\partial z} = -\gamma FD < 0$ and $\frac{\partial^2 A^*}{\partial D \partial z} = -\gamma F < 0$, we have $\frac{\partial V}{\partial z} = Fg\frac{\partial A^*}{\partial z} < 0$ and $\frac{\partial \phi}{\partial z} = (R-F)\frac{d\left(\frac{G}{g}\right)}{dA^*}\frac{\partial A^*}{\partial z} + \gamma FD(\frac{\partial A^*}{\partial D} - R) + \gamma zF\frac{\partial A^*}{\partial z} < 0$. Applying the implicit function theorem yields $\frac{dD^*}{dz} < 0$ and because z depends negatively on ψ, we have $\frac{dD^*}{d\psi} > 0$. The sign of $\frac{dF^*}{dz}$ is ambiguous:

$$\frac{dF^*}{dz} = -\frac{\left(\frac{\partial V}{\partial z}\frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D}\frac{\partial \phi}{\partial z}\right)}{|\mathbf{J}|} = \frac{g(A^*)(F^*)^2(1+\gamma z)^2\frac{\partial A^*}{\partial z}\left(\frac{R(1+2\gamma z)}{(1+\gamma z)^2} - F^*\right)}{|\mathbf{J}|} \gtreqless 0 \Leftrightarrow F^* \lesseqgtr \overline{F} \tag{A12}$$

where

$$\overline{F} \equiv \frac{R(1+2\gamma z)}{(1+\gamma z)^2}. \tag{A13}$$

Observe that $\overline{F} \in \left(\frac{R}{1+\gamma z}, R\right)$. Evaluating equation (A4) at $\overline{F}$ yields the corresponding debt level $\overline{D}$. Evaluating the participation constraint at $(\overline{F}, \overline{D})$ yields $\overline{r} \in (\underset{\sim}{r}, \tilde{r})$ which solves $V(\overline{D}, \overline{F}; \overline{r}) = 0$. $\overline{F}$ and $\overline{D}$ are independent of r. Since F^* strictly increases in r, it follows that $\frac{dF^*}{d\gamma} < 0$ if and only if $r > \overline{r}$, where $\overline{r} > \underline{r}$. Since z depends negatively on ψ, we have $\frac{dF^*}{d\psi} \gtreqless 0 \Leftrightarrow r \gtreqless \overline{r}$.

As for the derivative dA^*/dr, we can decompose the impact of a marginal increase in ψ on fragility as follows:

$$\frac{dA^*}{d\psi} = \underbrace{\frac{1}{1 + \frac{\partial A^*}{\partial F}\frac{g(A^*)}{G(A^*)}F^*}}_{\text{Amplification effect }(+)} \times \left(\underbrace{\frac{\partial A^*}{\partial D}\frac{dD^*}{d\psi}}_{\text{Scale effect }(-)} + \overbrace{\frac{\partial A^*}{\partial \psi}}^{\text{Price effect }(+)} \right). \tag{A14}$$

In contrast to dA^*/dr, the scale effect increases fragility as $\frac{dD^*}{d\psi} > 0$, while the price effect reduces fragility as $\frac{\partial A^*}{\partial \psi} > 0$. The price effect is proportional to total funding costs since $\partial A^*/\partial\psi = -\gamma FD/\psi^2$. Thus, its magnitude increases in the total debt burden, FD. This implies that a larger face value is required to equalize scale and price effects. Thus the critical threshold becomes larger, i.e., $\overline{F} > \underline{F} \Leftrightarrow \bar{r} > \underline{r}$. ■

Real Interest Rates, Bank Borrowing, and Fragility (Online Appendix)

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B1 Scale-dependent shock and asset-side risk

The analysis in the main text assumed a scale-invariant (or additive) shock to the banker's balance sheet. This allowed us to derive the key implications of the funding liquidity risk channel by fully abstracting from the effect of the bank's decisions on asset-side risk. In this section, we touch upon the robustness of our analysis by relaxing this assumption and introduce a scale-dependent (or multiplicative) shock whose impact depends on the scale of the banker's borrowing. Specifically, we suppose that the investment return, R, is a random variable with distribution $G(R)$ (density $g(R)$) with an increasing hazard rate. Thus, the size of the shock experienced by the banker depends on the scale of his investments and hence on the banker's choice of borrowing.

Allowing for equity $E > 0$ on the banker's balance sheet, the failure condition is given by $RI - \ell\frac{FD}{\psi} < (1-\ell)FD$. The global games analysis continues to apply, giving rise to a threshold such that the bank fails if and only if $R < R^*$. As in Rochet and Vives (2004), this threshold becomes

$$R^* = \frac{DF}{D+E}(1+\gamma z), \tag{B1}$$

where we substituted the balance sheet identity $I \equiv D + E$ into the denominator on the right-hand side. More borrowing increases leverage and unambiguously increases bank fragility, $\frac{\partial R^*}{\partial D} > 0$, but at a diminishing rate, $\frac{\partial^2 R^*}{\partial D^2} > 0$.[1]

[1] The concavity of the threshold is a consequence of the scale-dependency of the shock. In contrast, the scale-invariant threshold A^* decreased linearly in D only if the the face value of debt was sufficiently large (i.e. $r > \underline{r}$).

The banker's optimization problem at $t = 0$ is

$$\max_{F,D} \int_{R^*}^{\infty} [R(D+E) - FD] \, dG(R) \quad \text{s.t.} \quad F(1 - G(R^*)) = r. \tag{B2}$$

The analogue of condition (6) is now

$$\mathbf{E}\left[\tilde{R}\middle| R \geq R^*\right] - F = (1+\gamma z) F D H(R^*) \frac{dR^*}{dD} \tag{B3}$$

where $H(x) \equiv \frac{g(x)}{1-G(x)}$ is the hazard rate of the investment return and $dR^*/dD = \partial R^*/\partial D + \partial R^*/\partial F \times dF/dD$. The key trade-off underlying the funding liquidity risk channel, i.e. balancing the benefit of scaling up investments with the cost of more expensive debt and losing equity due to higher rollover risk, remains largely unchanged. What differs, however, is that the benefit (left-hand side) now also depends on the scale of the bank's investment, I, via the dependency of the left-hand side on R^*. In particular, since the investment itself is the source of risk, the benefit from borrowing more is the expected return conditional on the bank not failing. As a consequence, the left-hand side of equation (B3) depends on the failure threshold, R^*.

What is the influence of this additional effect on the marginal benefits from borrowing? The expected return on investment increases when bank fragility increases, i.e. $\frac{d\mathbf{E}\left[\tilde{R}\middle|R\geq R^*\right]}{dR^*} > 0$, which strengthens the bank's incentives to increase its borrowing. This suggests that the analysis with a scale-invariant shock in the main text provides a lower bound for the effects on bank borrowing. Moreover, the scale-invariant shock allowed us to study the funding liquidity risk channel in isolation without having to account for asset-side risk adjustments arising from limited liability.

B2 Endogenous equity

We endogenize equity on the bank's balance sheet in this section. We show that restricting the analysis in the main paper to exogenous equity, $E^* \in \{0, E\}$, is without loss of generality under fairly mild assumptions.

Assumption B1 (Equity Issuance)**.** *(i) The per-unit cost of issuing equity are given by a constant mark-up, $\rho > 1$, over the risk-free rate r. (ii) The maximal amount of equity that the bank can issued is finite, $\overline{E} \in (0, \infty)$.*

Proposition B1. *Given Assumption B1, the bank never issues an interior equity level, $E^* \notin (0, \overline{E})$.*

Proof of Proposition B1. Under Assumption B1, the bank's objective becomes:

$$\Pi(D, F, E) = \int_{\infty}^{A^*} [R(D+E) - FD - A] \, dG(A) - \rho r E \tag{B4}$$

where

$$A^* \equiv R(D+E) - (1+\gamma z)FD.$$

The bank chooses $\{D, F, E\} \in [0, \omega] \times [r, R] \times [0, \overline{E}]$ to maximize $\Pi(D, F, E)$ subject to the participation constraints of creditors,

$$FG(A^*) = r,$$

and of outside equity owners,

$$\Pi(D, E, F) \geq 0.$$

For the sketch of the argument, we can ignore the participation constraint of equity for the moment. Letting μ denote the Lagrange multiplier for the participation constraint of creditors, the first-order necessary conditions for an interior optimum are given by:

$$D: \quad \frac{\partial \mathcal{L}}{\partial D} = (R - F)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial D} = 0 \tag{B5}$$

$$F: \quad \frac{\partial \mathcal{L}}{\partial F} = (\mu - D)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial F} = 0 \tag{B6}$$

$$E: \quad \frac{\partial \mathcal{L}}{\partial E} = G(A^*)R + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial E} - \rho r = 0 \tag{B7}$$

$$\mu: \quad \frac{\partial \mathcal{L}}{\partial \mu} = -r + FG(A^*) = 0 \tag{B8}$$

From (B8) follows that the bank chooses an interior equity issuance $E^* \in (0, \overline{E})$ only if

$$R < \rho F^*. \tag{B9}$$

Now assume that this condition holds and suppose towards a contradiction that there is an interior maximum such that $E^* \in (0, \overline{E})$. The interior E^* satisfies condition (B7). Combining (B7) with (B5) and substituting $r = FG(A^*)$ yields a closed-form solution for the face value of debt, given by

$$F^* = \frac{(\rho + \gamma z)R}{\rho(1 + \gamma z)}. \tag{B10}$$

Note, for consistency, that $\rho F^* > R$ (so (B9) indeed holds). The solution to the bank's program can be completed by using F^*, (B5) and (B6) to solve for μ^* and substituting μ^* and F^* back into (B5) in order to solve for D^*. Finally, by substituting F^* and D^* into (B8), one can obtain E^*. For $\{D^*, F^*, E^*, \mu^*\}$ to constitute an interior maximum, the second-order conditions must be satisfied, i.e., the third principle minor of the bordered Hessian must be positive and its fourth principle minor, i.e., the determinant of the bordered Hessian, must be negative (Takayama, 1985, Theorem 1.E.17).

The bordered Hessian matrix is given by:

$$\mathbf{H} = \begin{pmatrix} 0 & \frac{\partial V}{\partial D} & \frac{\partial V}{\partial F} & \frac{\partial V}{\partial E} \\ \frac{\partial V}{\partial D} & \frac{\partial^2 \mathcal{L}}{\partial D^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial D \partial E} \\ \frac{\partial V}{\partial F} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial F^2} & \frac{\partial^2 \mathcal{L}}{\partial F \partial E} \\ \frac{\partial V}{\partial E} & \frac{\partial^2 \mathcal{L}}{\partial E \partial D} & \frac{\partial^2 \mathcal{L}}{\partial E \partial F} & \frac{\partial^2 \mathcal{L}}{\partial E^2} \end{pmatrix}$$

The third principle minor is strictly positive, as shown in the main paper. For an interior solution to constitute a maximum, we must have $|\mathbf{H}| < 0$. We next use an indirect way of showing that the determinant of $\mathbf{H}$ must be strictly *positive* at $\{D^*, F^*, E^*, \mu^*\}$, implying that the interior solution cannot constitute a maximum. The indirect proof is less tedious because it involves computing only the determinant of a single 3×3 matrix, rather than the determinant of the entire Hessian $\mathbf{H}$. Specifically, given that $\mathbf{H}$ is non-singular, we first use the implicit function theorem to compute the derivative $\frac{dF^*}{d\rho}$ in terms of the determinant $|\mathbf{H}|$. Second, we compute $\frac{dF^*}{d\rho}$ from the closed-form solution (B10). Because both approaches must produce the same sign, we can infer the sign of the determinant of the bordered Hessian matrix at the interior solution.

By totally differentiating the set of first-order conditions we obtain:

$$\mathbf{H} \begin{pmatrix} d\mu & dD & dF & dE \end{pmatrix}^T + \begin{pmatrix} 0 & 0 & 0 & -r \end{pmatrix}^T d\rho = 0.$$

Letting $\mathbf{H}^{(j)}$ denote the matrix where column j in the bordered Hessian is replaced by the vector $\begin{pmatrix} 0 & 0 & 0 & -r \end{pmatrix}^T$. Hence, by the implicit function theorem (Takayama, 1985, p. 150):

$$\frac{dF^*}{d\rho} = -\frac{|\mathbf{H}^{(3)}|}{|\mathbf{H}|}.$$

Because $|\mathbf{H}| < 0$ at an interior maximum, if follows that the sign of $\frac{dF^*}{d\rho}$ must equal the sign of $|\mathbf{H}^{(3)}|$.

Computing $|\mathbf{H}^{(3)}|$ by cofactor expansion is straightforward:

$$\begin{aligned}
|\mathbf{H}^{(3)}| &= (-1)^7(-r)\begin{vmatrix} 0 & \frac{\partial V}{\partial D} & \frac{\partial V}{\partial E} \\ \frac{\partial V}{\partial D} & \frac{\partial^2 \mathcal{L}}{\partial D^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial E} \\ \frac{\partial V}{\partial F} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial F \partial E} \end{vmatrix} \\
&= r\left[\frac{\partial V}{\partial D}\left(\frac{\partial V}{\partial E}\frac{\partial^2 \mathcal{L}}{\partial D \partial F} - \frac{\partial V}{\partial D}\frac{\partial^2 \mathcal{L}}{\partial F \partial E}\right) - \frac{\partial V}{\partial F}\left(\frac{\partial V}{\partial D}\frac{\partial^2 \mathcal{L}}{\partial D \partial E} - \frac{\partial V}{\partial E}\frac{\partial^2 \mathcal{L}}{\partial D^2}\right)\right] \\
&= rFg(A^*)\left[\frac{\partial V}{\partial D}\left(-\frac{G(A^*)}{g(A^*)} + \gamma z F\frac{\partial A^*}{\partial D} + (\mu + \gamma z D)F\frac{\partial^2 A^*}{\partial D \partial F}\right) - \frac{\partial V}{\partial F}\gamma z F\frac{\partial A^*}{\partial E}\frac{\partial A^*}{\partial D}\right] > 0,
\end{aligned}$$

where we used the fact that at any interior solution, we must have $\frac{\partial V}{\partial D} < 0 < \frac{\partial V}{\partial F}$. The second and cross-partial derivatives for D and F are provided in the paper. The derivatives involving E are given by

$$\frac{\partial^2 \mathcal{L}}{\partial D \partial D \partial E} = (R - F)\frac{\partial A^*}{\partial E}\frac{\partial (G(A^*)/g(A^*))}{\partial A^*} > 0,$$

$$\frac{\partial^2 \mathcal{L}}{\partial D \partial F \partial E} = (\mu - D)\frac{\partial A^*}{\partial E}\frac{\partial (G(A^*)/g(A^*))}{\partial A^*} > 0,$$

and

$$\frac{\partial V}{\partial E} = Fg(A^*)\frac{\partial A^*}{\partial E} > 0.$$

Thus, at the interior maximum, we must have $\frac{dF^*}{d\rho} > 0$.

However, if $E^* \in (0, \overline{E})$, then the face value of debt is given in closed form by equation (B10). Differentiating F^* yields

$$\frac{dF^*}{d\rho} = \frac{(\rho(1+\gamma z) - (1+\gamma z)(\rho+\gamma z))R}{\rho^2(1+\gamma z)^2} = -\frac{\gamma z}{\rho}\frac{R}{\rho(1+\gamma)} < 0.$$

But this contradicts the fact that the interior solution constitutes a maximum. Put differently, since the signs of the implicitly derived derivative and the closed-form derivative must agree, it follows that for $dF^*/d\rho < 0$ at an interior maximum, the sign of $|\mathbf{H}|$ must be the same as the sign of $|\mathbf{H}^{(3)}|$. Thus, the fact that $|\mathbf{H}| > 0$ immediately contradicts that $\{D^*, F^*, E^*, \mu^*\}$ is an interior maximum. As a consequence, the bank never considers it optimal to choose $E^* \in (0, \overline{E})$. ■

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